Rainfall persistence in India during May-October

T. R. SRINIVASAN

Meteorological Office, Poona

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ABSTRACT. Ten years' (1951-1960) daily rainfall data of India for the summer season (May-October) for about 93 representative stations have been analysed with a view to study the persistency pattern of the occurrence of rainfall. The observed frequencies of rain spells of various lengths are found to conform closely to the logarithmic model $S_r = \alpha x^r/r$ (where S_r is the number of sequences of length r ; α and x being parameters determinable from the data). A chart showing the spatial distribution of the parameter x designated as the "persistence parameter" is presented. A nomogram that can be used for finding the probability of rain on the rth day if the preceding rain-spellis of r-1 days duration has been prepared. Some sample charts giving rainfall probability on the 2nd, 6th and 10th day with preceding spell of 1, 5, 9 days duration respectively are also presented and results discussed.

1. Introduction

It is well-known that weather sequences at any location are not entirely unrelated to one another and that certain weather types exhibit a tendency to persist. Great emphasis is being placed in these days on objective methods of forecasting. In searching for objective methods of forecasting weather, it is natural to look upon the theory of persistence as a possible basis. A good deal of work on these lines has been done as far back as 1916. Newnham (1916) has studied the persistence of wet and dry spells of weather. Gold (1929) advanced a method of examining persistence of one type of weather and subsequently Cochran (1938) extended Gold's investigation to types of weather having unequal probability. Srinivasan (1954) gave the analogue of Cochran's formula when probabilities vary with respect to time. The studies of Gold, Cochran and Srinivasan are based on the theory of runs applied to the distribution function of the combined data of wet and dry sequences. Jorgensen (1949) investigated the persistence of rain and no-rain periods during the winter at San Francisco and defines a "Skill score" for forecasts based on persistency. Williams (1952) applied the logarithmic model to the sequence of wet and

dry spells. Ramabhadran (1954) applied the Williams' logarithmic model to the daily rainfall data of Poona during the monsoon season. Srinivasan (1959) gave a generalised model that is suitable for describing weather persistence. These studies indicated that it is possible to give an objective forecast of the probability of rainfall persisting for one or more days.

A very important factor to be reckoned with in the agricultural economy of India is the summer monsoon. The date of onset of the southwest monsoon, its duration and the date of withdrawal as also the intensity of the monsoon rainfall vary considerably from year to year. A special feature of the monsoon is that it is not made up of a long continuous period of rainy days but the rainfall in many parts of the country occurs in spells lasting for a few days and is interspersed by spells of rainless days. The durations of wet spells and dry spells during monsoon and the probability of further continuance of rain after it has occurred continuously over a certain number of days are factors of great interest to the Indian farmer to plan the agricultural operations. Though the period June to September is taken as the southwest monsoon season in India, the monsoon rainfall will be

fully covered only if rainfall from May to October is taken into consideration. The purpose of this paper is to apply the logarithmic model suggested by Williams to the 10 years' daily rainfall data (May-October) of about 93 selected stations in India, with a view to forecasting the continuance of a spell.

2. Theoretical representation of the frequencies of rainspells of various lengths

Cochran (1938) studied the problems of runs for two weather types and gave the $formula -$

$$
f_{r,m} = N [p^r q \{2 + q(m - r - 1)\}]
$$

for determining the frequency of rain period of various lengths expected primarily on chance.

In the above equation

 $f_{r,m}$ = frequency of spells of length r out of m

 $r =$ length of spell

- $m =$ number of days in the season
- $p =$ the random probability of occurrence

 $q = (1-p)$

 $N =$ number of years of data

An alternative approach to this problem is based on the assumption of *weather persis*tency, *i.e.*, the longer a spell of weather of one type, the more likely it is to last another day. Williams (1952) made use of the logarithmic series $\alpha x^r/r$ which has been found suitable to describe a series which exhibits such characteristics.

Determination of the parameters α , x

Let $s_1, s_2, s_3, \ldots, s_r$ represent the frequency of rain spell of length $1, 2, 3$ $\ldots \ldots$ respectively and

$$
S = \frac{\sum_{r=1}^{n} s_r \text{ and } T_1 = \frac{\sum_{r=1}^{n} s_r \times r, \quad i.e., \text{ the}
$$

first moment (total number of rainy day.)

$$
T_2 = \frac{1}{2} s_r \times r^2
$$
, the second moment.
We have $s_r = a \, \text{ar } |r, \quad 0 < x < 1$

$$
\Sigma s_r = S = -a \log_e(1-x) \tag{1}
$$

$$
T_1 = \Sigma s_r \times r = \Sigma a \, \text{ar}
$$

$$
= a x/(1-x) \qquad (2)
$$

 \mathcal{E}

and
$$
T_2 = \sum s_r \times r^2
$$

= $a x/(1-x)^2$ (3)

From (2) and (3) we get

$$
a = T_1^2/(T_2 - T_1)
$$

and $x = (T_2 - T_1)/T_2$ (Yule 1944)

Williams has given the solution

$$
S = a \log_e (1 + T_1/a)
$$

$$
x = T_1/(T_1+a)
$$

Method of obtaining α and x for given S and T has been given by Williams (1949).

The quantities α and x have been computed for all the 93 stations and are given in Table 1.

Significance of the parameter x

Since
$$
S = \sum_{1}^{n} s_r = -a \log_{e} (1-x)
$$

and $T_1 = \sum_{1}^{n} r \times s_r = a x/(1-x)$

We have $T_1/S = x/\{-1-x\} \log_e (1-x)$

i.e., the quantity x depends on T_1/S , *i.e.*, the average number of rainy days per sequence and hence the quantity x will be the same for stations having the same number of rainy days per spell. The quantity x can be designated as "persistence parameter" since it depends on the average number of rainy days per sequence.

After determining the values of the parameters α and x we now proceed to utilise these in obtaining the probability of rainfall on the rth day if preceding $r-1$ days have

rained. Let $s_1, s_2, s_3, \ldots, s_r$ be the actual frequencies of rain-spells of length $1, 2, 3, \ldots$ reto and $S_1, S_2, S_3, \ldots, S_r$ be the corresponding cumulative frequencies, *i.e.*,

$$
S_r = \sum_{r}^{n} s_r
$$

$$
S = \sum_{r}^{n} s_r
$$

Total number of rain-spells of at least r days duration is S_r and of $(r-1)$ days duration is $S_{(r-1)}$. Hence out of $S_{(r-1)}$ occasions, on S_r occasions rain has persisted till the rth day. Hence the probability of rain on the rth day if it had rained during the preceding $r-1$ days $is-$

$$
Pr = S_r / S_{r-1}
$$

= $\frac{a}{a} \frac{x^r/r + a}{x^{r-1}/(r-1) + a} \frac{x^r}{r^2 + \cdots}$
= $\frac{x^r}{x^{r-1}} \frac{[1/r + x/(r+1) + \cdots]}{[1/(r-1) + x/r + \cdots]}$ (4)

 \rightarrow *x* for large values of *r*

Thus we get an interesting and important result that the persistence probability P_r tends to be the value of the parameter x ; in other words the maximum value of P_r is the parameter value x itself.

Equation (4) can be written as

$$
P_r = \frac{\left(x + \frac{x^2}{2} + \dots\right) - \left(x + \frac{x^2}{2} + \dots \frac{x^{r-1}}{r-1}\right)}{\left(x + \frac{x^2}{2} + \dots\right) - \left(x + \frac{x^2}{2} + \dots \frac{x^{r-2}}{r-2}\right)}
$$

\n
$$
- \log_e\left(1 - x\right) - \sum_{r=1}^{r-1} x^r/r
$$

\n
$$
= \frac{1}{\left(1 - x\right) - \sum_{r=2}^{r-2} x^r/r}
$$

\n
$$
= \log_e\left(1 - x\right) - \sum_{r=1}^{r-2} x^r/r
$$

\n(5)

Fig. 1. Distribution of parameter

 P_r has been computed for various values of $x = 40, -50, -60, -70, -80, -90, -95$ and are given in Table 2. A nomogram for finding the value of P_r for known r has also been given.

3. Results and Discussion

1. Daily rainfall data of 93 stations in India during the season (May—October) for the years, 1951-1960, were used for the present study. A day with rainfall 2.5 mm or more is designated as a rainy day (consistent with the criteria adopted by the India Meteorological Department). From this data, the frequency of spells of rainy days of various lengths was found. These are presented in Table 1. It will be seen that the length of rain-spell in a place is directly related to the total number of rainy days there.

2. The parameter x has been estimated for each one of the 93 stations and are presented in Table 1 and plotted in Fig. 1. The parameter x as already mentioned earlier is the stochastic probability of the persistence of rain. Consistent with the known climatological features of the country,

TABLE 1

| Station | | Length of rain-spell in days per year | | | | | | | | | | | | |
|--------------|--------------|---------------------------------------|-------------------------|-----------------------|------------------------|-----------------------|------------------------------|----------------------------|----------------------------|------------------------------|-----------------------------|------------------------|---------------|---------------|
| | | $\mathbf{1}$ | $\frac{1}{2}$ | $\overline{3}$ | $\ddot{+}$ | 5 | 6 | 7 | $\hat{\mathbf{S}}$ | 9 | 10 | -10 | α | \mathcal{X} |
| Raichur | Obs. Cal. | $16 - 2$ $19 - 31$ | ふっさ $5 - 50$ | $3 - 3$ $2 - (19)$ | $1 - 1$ $(1 - S!)$ | (1.4 $(1 - 4)$ | $() \cdot 2$ $(1 - 1.9)$ | $() - ()$ $0 - 09$ | $(1 - 1)$ $0 - 0.5$ | $(1 - 1)$ $(1 - (1)^3)$ | $(1 - 1)$ $(1 - 0)$ | $0 - 0$ $(1 - 4) +$ | 33.88 | .57 |
| Bangalore | Obs. Cal. | 18.4 $21 - 26$ | $6 - 5$ $6 - 38$ | 2.9 2:56 | 1.8 $1 - 1.5$ | 0.5 $(1 - 55)$ | 0.2 0.28 | $0 - 1$ $(1 - 1)$ | $0 \cdot 1$ $0 - 07$ | $0 - 0$ 0.04 | $0 - 0$ $0 - 02$ | $0 - 0$ 0.02 | 35.43 | -60 |
| Masulipatam | Obs. Cal. | $15 - 2$ $17 - 91$ | 5.2 $6 - 18$ | $4 \cdot 0$ 2.84 | 1.7 1.47 | $1 - 2$ (1.8) | 0.3 (1.47) | $() - 2$ $0 - 28$ | $() \cdot 2 $ $0 - 17$ | $(1 - 1)$ $0 - 10$ | $0 - 0$ 0.06 | $0-1$ $0 - 11$ | $25 - 95$ | .69 |
| Madras | Obs. Cal. | $16 - 5$ $18 - 86$ | $6 - 3$ $5 - 09$ | 1.8 1.83 | 0.8 $0 - 74$ | $(1 \cdot 2)$ 0.32 | $(1 \cdot 2)$ $(1 - 1) +$ | $0 - 1$ $0 - 07$ | $0 - 0$ 0.03 | () () $0 - 01$ | $0 - 0$ 0.01 | $(1 - 1)$ $0 - 02$ | 34.92 | .51 |
| Trichy | Obs. Cal. | $13 - 6$ 14.76 | $+5$ 3.54 | 0.7 1.13 | $(1 - 7)$ $0 - 41$ | $0 - 1$ $() - 16$ | $(1 - 1)$ 0.06 | $(1 - 1)$ 0.03 | 0.0 0.01 | $0 - 0$ < 0.01 | 0.0 | 0.0 < 0.1 | $30 - 75$ | .48 |
| Pamban | Obs. Cal. | $5 - 2$ 4.89 | $1-7$ $1 - 56$ | $(1 - 1)$ 0.66 | $(1 - 1)$ (1.32) | $(3 - 1)$ 0.16 | $() \cdot 2$ $0 - 09$ | $() - $ $(1 - 1)$. | $() - $ 0.03 | $(1 - 1)$ | $0 - 0$ 0.01 < 0.01 | $(1 - 1)$ 0.0 | 7.59 | .64 |
| Calicut | Obs. Cal. | $9 - 6$ $8 - 73$ | $4 - 7$ 4.02 | $2 - 0$ $2 - 46$ | $1 - 2$ 1.70 | 0.3 1.25 | $1-3$ 0.96 | 0.8 0.76 | $(1 \cdot 9)$ 0.61 | $1 \cdot 1$ 0.50 | 0.4 0.41 | $1-9$ 2.57 | 9.49 | -92 |
| Trivandrum | Obs. Cal. | 12.7 $12 - 71$ | $5 - 8$ 5.27 | 2.8 $2 - 92$ | $1-3$ $1 - 82$ | $1 - D$ 1.21 | 0.8 0.83 | $() \cdot 5 $ $0 - 60$ | 0.5 0.43 | 0.5 0.32 | 0.5 0.24 | 0.9 0.79 | $15 - 31$ | .83 |
| Mangalore | Obs. Cal. | $S-3$ $6 - 49$ | 3.0 $3 - 0.5$ | $2 - 2$ $1 - 91$ | $ - $ $1 - 3.5$ | $(1 - 7)$ $1 - 01$ | $0 - 7$ 0.79 | $1-0$ 0.64 | 0.4 0.53 | $0 - 4$ $0 - 44$ | 0.4 $0 - 37$ | $2 - 7$ $2\cdot 84$ | $6 - 90$ | -94 |
| Hyderabad | Obs. Cal. | 14.7 $17 - 51$ | 4.3 $5 - 52$ | $3 - 1$ $2 - 32$ | 1.8 $1 - 10$ | $1 - 1$ $(1 - 55)$ | $0 - 3$ (1.29) | $(1 - 1)$ $0 - 16$ | 0.0 $0 - 09$ | $() \cdot ()$ $0 - 0.5$ | $(0 - 0)$ $0 - 03$ | $0-0$ 0.03 | $27 - 80$ | .63 |
| Calingapatam | Obs. Cal. | $12 - 7$ $16 - 95$ | 5.9 $5 - 68$ | 4.0 2.54 | $1 - 3$ $1 - 28$ | 0.7 $0 - 68$ | $(1 \cdot 2)$ 0.38 | 0.4 0.22 | $0-1$ 0.13 | $(1 - 1)$ 0.08 | $0 \cdot 1$ $0 - 0.5$ | $0 - 0$ $0 - 07$ | 25.30 | -67 |
| Honavar | Obs. Cal. | $7 - 4$ $4 - 95$ | 3.3 $2 - 38$ | $1 - 5$ 1.52 | $(1 - 9)$ $1 - 09$ | $1-0$ 0.84 | 0.3 0.67 | $0 - 2$ 0.55 | 0.5 0.46 | $0 - 4$ $0 - 40$ | 0.4 0.34 | $2 - 8$ 3.39 | 5.15 | .96 |
| Cuddappah | Obs. Cal, | 13.2 14.58 | $+1$ 4.23 | 2.9 1.63 | $0\cdot 8$ $0 - 71$ | $0 - 3$ 0.33 | $0 - 4$ $0 - 16$ | 0.0 $0 - 0S$ | 0.0 0.04 | () () 0.02 | $0 - 0$ $0 - 01$ | $0 - 0$ 0.03 | $25 \cdot 14$ | .58 |
| Bellary | Obs. Cal. | $13 - 4$ $14 - 71$ | $4 \cdot 1$ $3 - 90$ | 0.8 1.38 | 1.2 $(1 - 55)$ | $(1 \cdot)$ 0.23 | $(1 - 1)$ $0 - 10$ | () ($(1 - 1)$. | 0.0 0.02 | $(1 - 1)$ | 0.0 0.01 < 0.01 | () () 0.01 | $27 - 76$ | .53 |
| Aurangabad | Obs. Cal. | $11-6$ $14 - 28$ | 5·4 $4 - 83$ | 2.8 $2\cdot 26$ | $(1 - 1)$ $1 - 17$ | 0.7 () (i) | $(1 - 5)$ 0.37 | 0 ¹ 0.22 | $0-1$ $0 - 13$ | $0 \cdot 1$ $0 - 08$ | $() \cdot$] $0 - 0.5$ | 0.0 | $20 - 69$ | .69 |
| Ahmedabad | Obs. Cal. | $7 - 5$ $8 - 24$ | 3.4 $3 - 13$ | $1-3$ 1.59 | 1.4 $() \cdot 90$ | 0.3 0.55 | $(1 - 2)$ 0.35 | 0.3 $0 - 23$ | $0-1$ 0.15 | 0.3 0.10 | 0.0 0.07 | $(1 - 2)$ | 10.84 | .76 |
| Surat | Obs. Cal. | $7 - S$ $9 - 13$ | 4.4 3.69 | $1 - 9$ $1 - 66$ | $1 - 4$ 1:21 | 0.6 0.69 | $0 - 5$ 0.53 | $0 - 1$ 0.37 | 0.5 0.26 | 0.5 $0 - 19$ | 0.3 | 0.4 | $11 - 27$ | .81 |
| Bhuj | Obs. Cal. | $6 \cdot 4$ $5 - S(1)$ | $1 - 9$ $2 - 00$ | $0 - S$ 0.92 | $0 - 6$ 0.48 | $0-1$ 0.26 | $0 - 1$ 0.15 | $0-1$ $0 - 09$ | 0.1 | $0 - 0$ | 0.15 $0 - 0$ | 0:1 | 8.41 | .69 |
| Veraval | Obs. Cal. | $8 - 6$ 7.92 | $3 - 3$ $3 - 0.5$ | $1 - 0$ 1.57 | 1.2 0.90 | $(1 - 7)$ 0.56 | 0.2 0.36 | $0 - 2$ 0.24 | 0.05 (1, 9) 0.16 | 0.03 $0 - 2$ $0 - 11$ | 0.02 $0 - 0$ $0 - 08$ | (0.2) | $10 - 29$ | .77 |

Obs.--Observed

TABLE 1 (contd)

 $Obs.$ --Observed

Cal.-Calculated

TABLE 1 (contd)

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 $Obs.$ --Observed

 $Cal. -Calculated$

TABLE 1 (contd)

 $Obs.$ --Observed

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 $Cal. - Calculated$

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TABLE 1 (contd)

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 $Obs.$ --Observed

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 $Cal. -Calculated$

we see that the distribution of x is more or less the same as that of the precipitation pattern during the season. Following features are noticed

- (i) Higher values of the parameter occur along the west coast, Assam and sub-montane areas of the Himalayas. Pockets of high values exist in and near Mt. Abu, Pachmarhi, Koraput, suggesting thereby the influence of orography,
- (ii) Lower values occur over northwest India, south Peninsula outside west coast, and
- (iii) Generally moderate values occur over rest of the country.

To summarise, this distribution shows that the parameter is highly related to the rainfall of the place.

3. As an illustration probabilities of occurrence of rainfall on the 2nd, 6th and 10th day (if rain fell on the previous day, previous five days continuously and previous nine days continuously respectively) have been worked out and shown in Figs. 2 (a) to 2 (c).

It is interesting to see from Fig. 2 (a) that the probability of rain on the 2nd day is very low, *i.e.*, about 60 per cent along west coast and upper Assam, and as low as 40-50 per cent over rest of the country except northwest India and southeast peninsula where the chance is even less than 40 per cent.

It is seen from Fig. 2(b) that the chance of rain on the sixth day is over 70 per cent along west coast, Assam, Sub-Himalayan West Bengal, hills of west Uttar Pradesh and of Punjab (India) and the three high pockets, viz., Mt. Abu, Pachmarhi and Koraput.

The probability is over 60 per cent over the rest of the country except northwest India and southeast peninsula where the chance decreases rapidly from 60 per cent northwards in northwest India and southwards in southeast peninsula.

Fig. 2(a). Probability of rain on 2nd day $(r=1)$

Fig. 2(c). Probability of rain on 10th day $(r=9)$

Fig. 2 (c) shows that the chance of rain on the 10th day is over 70 per cent along west coast, Assam, Sub-Himalayan West Bengal, the hills of west Uttar Pradesh and of Punjab (India), central parts of the country, Mt. Abu and Koraput areas; and less than 70 per cent and more than 50 per cent over the rest of the country except northwest India and extreme southeast peninsula, where the chances are almost of the values of the parameter x .

Use of Figs. 2(a) to 2(c) for objective forecasting

Where the quantity x is known for a place, we can with the aid of Table 2 (or the nomogram) find the probability of rain on any day following a spell of certain days duration. To find the probability of rain on the 6th day (preceding spell being of 5 days' duration) at a place having $x = -75$; erect ordinate at .75 and see where the ordinate meets the Then read the corresponding curve $r=5$. probability value along Y-axis, i.e., 65 per cent in this case (Fig. 3). Charts similar to Figs. 2 (a) to 2(c) can be drawn for various values of the length of the preceding spell of wet weather. These charts can be readily consulted for issuing an objective forecast of rain on any given day.

The basic data considered in the above study is for 10 seasons only. Though conclusions arrived at on the basis of ten years' data may not be absolutely conclusive, the major features can be seen from the above analysis. It will be useful to utilise a longer series of data, say for a 50-year period for this purpose.

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