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Curvilinear Study on the effect of weather on growth of Sugarcane

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ABSTRACT. The technique of curvilinear correlation study has been applied in studying the effect of meteorological factors on the growth of sugarcane. It has been found that at Poona of all the meteorological factors the maximum and minimum temperatures influence elongation most and that their optimum values are equal to 87·5° F and less than or equal to 68°F respectively. Rainfall has slight effect as the crop is irrigated. Curves for estimating the height of crop by taking into account the combined effect of these three factors have been worked out by this technique.

1. Introduction

Crop-weather relationships by simple correlation methods have been studied by Mallik *et al*. (1960). Similar studies by partial correlations have also been made by Hooker (1907) and by Gangopadhyaya and Sarker (1963). However, in the linear correlation (simple or partial) and regression studies, there is an inherent assumption that the change in the dependent variable accompanying unit changes in each independent variable is of exactly the same amount, no matter how large or how small the independent variable is. Crop-weather relations, being not so simple, cannot be expressed by linear correlations. Also there may be an optimum point of a particular crop-characteristic with regard to one or more of the weather factors upon which the crop-characteristic depends. Linear regressions are obviously unable to bring out such relations. Such information may be brought out by determining the curvilinear relation between the dependent and each independent variable, while simultaneously eliminating the effects of the other factors.

If X_1 be the crop-characteristic and X_2, X_3 etc be the independent factors (meteorological factors and time factor) upon which X_1 depends, our problem is to find out a regression equation of the type

$$X_1 = a + f_2(X_2) + f_3(X_3) + f_4(X_4) + \ldots \quad (1)$$

The expression $f_2\ (X_2)$ is a general function meaning any regular change in X_1 with changes in X_2 whether describable by a straight line or a curve.

In the present study an attempt has been made to see how the height of the sugarcane crop at Poona depends upon the weather factors of the elongation phase. The technique of analysis is similar to what has been described by Ezekiel and Fox (1959) and used by Gangopadhyaya and Sarker (1964) in developing a formula connecting yield and crop growth of sugarcane.

The partial regression curves have been found by the method of "successive graphic approximations" in preference to fitting mathematical expressions as the former involves no prior assumption as to the shapes of the curves.

2. Procedure

The height values of sugarcane at Poona along with the associated meteorological factors have been taken from 1945-46 to 1960-61, collected under the Crop-Weather

TABLE 1

V_1—CO·419 V_2—POJ 2878

Year	Variety	Daily average min. temp. (X_2) (°F)	Daily average max. temp. (X_3) (°F)	Total rainfall (X_4) (in.)	Height (X_1) (cm)	Estimated height (X_1') (cm)	X_1-X_1' $=Z'$
1945-46	V_1	70·5	87·2	21·19	338	331·26	6·74
	V_2	70·2	86·2	20·06	303	331·55	—28·55
46-47	V_1	—	—	—	—	—	—
	V_2	—	—	—	—	—	—
47-48	V_1	69·9	88·1	23·49	343	344·47	—1·47
	V_2	69·1	88·1	23·79	399	355·73	43·27
48-49	V_1	70·7	88·8	25·58	293	337·88	—44·88
	V_2	70·7	88·0	25·58	350	335·96	14·04
49-50	V_1	70·7	87·5	18·24	353	325·56	27·44
	V_2	69·7	86·5	18·12	367	336·62	30·38
50-51	V_1	68·3	86·4	22·81	349	361·34	—12·34
	V_2	68·3	86·4	22·81	356	361·34	— 5·34
51-52	V_1	71·5	88·9	20·68	309	321·08	—12·08
	V_2	70·8	88·9	21·78	335	331·99	3·01
52-53	V_1	70·4	87·0	20·15	341	330·84	10·16
	V_2	70·2	85·7	20·15	333	330·46	2·54
53-54	V_1	—	—	—	—	—	—
	V_2	70·5	85·9	16·46	325	322·22	2·78
54-55	V_1	71·0	86·0	25·32	323	326·76	— 3·76
	V_2	71·0	85·2	25·32	306	324·84	—18·84
55-56	V_1	—	—	—	—	—	—
	V_2	70·6	86·8	29·12	359	338·89	20·11
56-57	V_1	69·8	84·4	38·83	366	356·21	9·79
	V_2	69·6	83·5	38·83	364	356·79	7·21
57-58	V_1	69·4	87·6	19·55	333	345·15	—12·15
	V_2	69·8	89·1	19·10	326	342·72	—16·72
58-59	V_1	69·3	88·3	17·06	346	345·05	0·95
	V_2	71·4	87·6	15·84	325	313·26	11·74
59-60	V_1	70·7	83·1	30·00	331	329·78	1·22
	V_2	70·7	83·1	30·00	325	329·78	— 4·78
60-61	V_1	69·8	85·5	24·36	327	340·70	—13·70
	V_2	68·9	85·5	24·93	335	353·67	—18·67

TABLE 2

X_2 value	No. of cases	Average of X_2	Average of Z'
Upto 69·1	4	68·65	1·73
69·2—69·7	4	69·50	6·60
69·8—70·2	6	69·95	—8·02
70·3—70·6	4	70·47	9·95
70·7—70·9	6	70·72	—0·66
71·0 and above	4	71·23	—5·73

Scheme—Sugarcane. On an examination by the curvilinear method described later in the paper it was found that so far as elongation is concerned the most important factors are maximum and minimum temperatures, rainfall has slight effect while sunshine has practically no effect. Sugarcane being an irrigated crop, the effect of rainfall on elongation is expected to be as has been found. During the elongation phase of sugarcane in Poona (May to October) daily sunshine duration ranges from 5 to 7 hours. This range, which itself is extremely small, is perhaps very near the optimum value of sunshine required for elongation and hence in the present study sunshine does not show any effect on growth. Also on plotting the data chronologically, no obvious trend of rise or fall from year to year in height values was found. So the time factor has been omitted in eq. (1). The independent factors considered accordingly in equation (1) are: average daily value of minimum temperature (X_2), average daily value of maximum temperature (X_3) and total rainfall amount (X_4). The values utilised are given in Table 1.

It will be seen from Table 1 that there are 13 values of variety CO–419 and 15 values of variety POJ–2878. These 28 values have been assumed to have come from the same population for our analysis, neglecting thereby the varietal characteristic, if any. This assumption has not introduced any serious error (Gangopadhyaya and Sarker 1963).

Because elongation should have an optimum value with each weather factor, each curve showing elongation *vs* a certain meteorological factor should have a single maximum. For the free-hand drawing of such curves, this logical nature of the relations between crop growth and meteorological factors has been imposed on each curve.

"First Approximation" Net Regression curves

Using Table 1 the linear multiple regression equation

$$X_1 = a_{1\cdot234} + b_{12\cdot34} X_2 + b_{13\cdot24} X_3 + b_{14\cdot23} X_4$$

was first worked out. This was found to be

$$X_1 = 1055\cdot95 - 13\cdot616\, X_2 + 2\cdot393\, X_3 + +1\cdot254\, X_4 \qquad (2)$$

with significant multiple correlation—$R_{1\cdot234} = 0\cdot574$.

Estimates X_1' from (2) and residuals $Z' = X_1 - X_1'$ are calculated and given in Table 1.

Now to find the regression relation of X_1 and X_2 a scatter diagram showing the relation between X_1 and X_2 is drawn. To do this the net regression line

$$X_1 = 1055\cdot95 - 13\cdot616\, X_2 + + 2\cdot393\, M_3 + 1\cdot254\, M_4 = 1292\cdot75 - 13\cdot616\, X_2 \qquad (3)$$

between X_1 and X_2 is drawn in Fig. 1.

(M_1, M_2, M_3, M_4 are used to denote mean values of X_1, X_2, X_3, X_4 respectively)

The residuals Z' are then plotted with X_2 values as abscissae and Z' values as ordinates with the net regression line as zero base. The residuals are then averaged for selected group values of X_2 and are given in Table 2.

These averages are similarly plotted and these, when connected by a broken line, called the line of averages, indicate the curvilinear relation between X_1 and X_2. A freehand smooth curve is then drawn, through the several group averages, keeping in mind the limiting condition of single optimum value mentioned above. This curve is the first approximation to the curvilinear

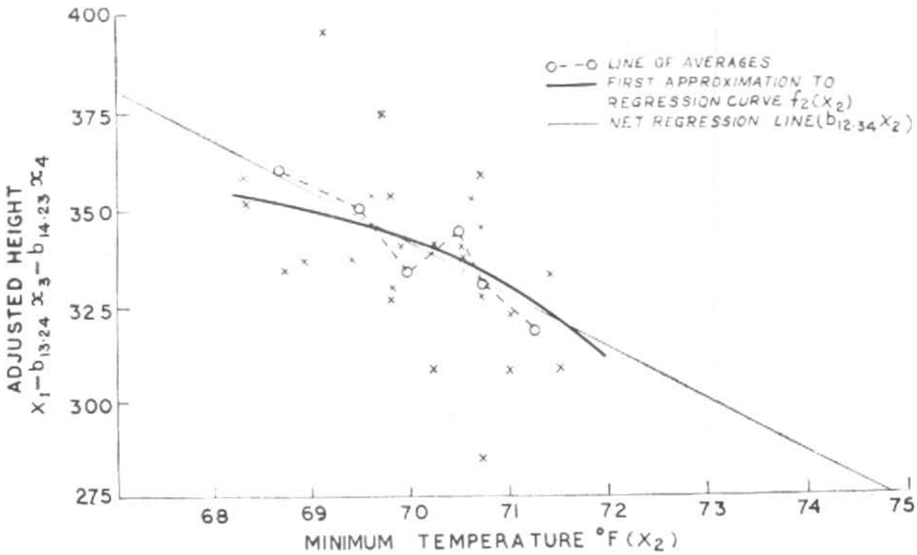
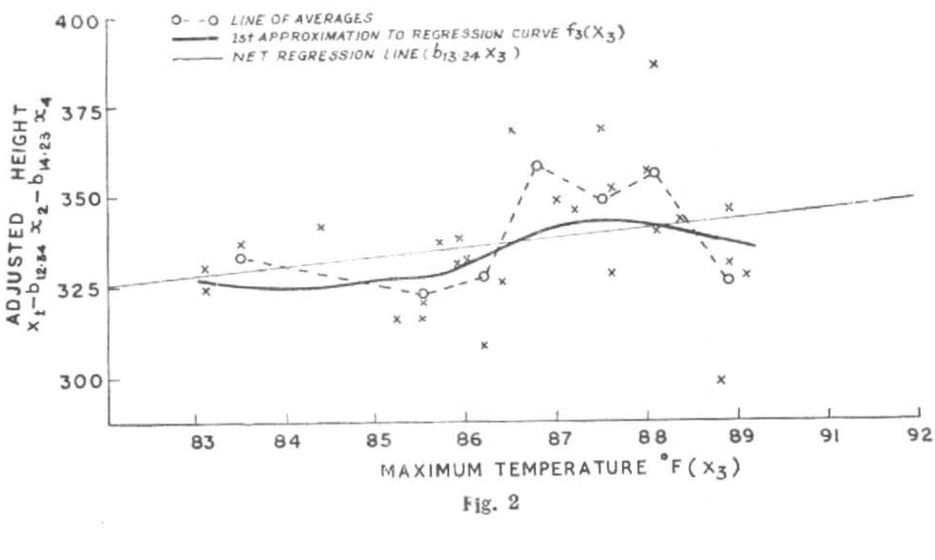

O- -O LINE OF AVERAGES
FIRST APPROXIMATION TO REGRESSION CURVE $f_2(x_2)$
NET REGRESSION LINE($b_{12\cdot34}x_2$)
ADJUSTED HEIGHT
$x_1 - b_{13\cdot24}x_3 - b_{14\cdot23}x_4$
400
375
350
325
300
275
68
69
70
71
72
73
74
75
MINIMUM TEMPERATURE °F (X_2)

Fig. 1


O- -O LINE OF AVERAGES
1st APPROXIMATION TO REGRESSION CURVE $f_3(X_3)$
NET REGRESSION LINE ($b_{13\cdot24}X_3$)
ADJUSTED HEIGHT
$x_1 - b_{12\cdot34}x_2 - b_{14\cdot23}x_4$
400
375
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83
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92
MAXIMUM TEMPERATURE °F (X_3)

Fig. 2

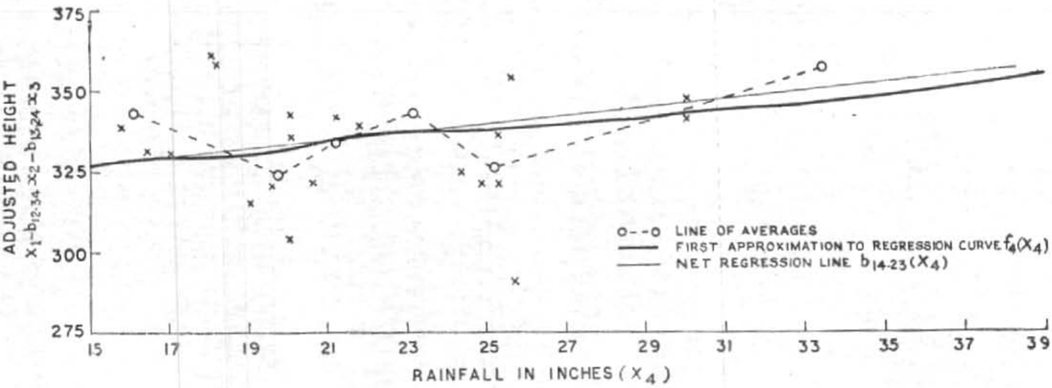

Fig. 3

function $X_1 = f_2(X_2)$. This is only a first approximate as it has been determined by allowing for only the net *linear* effects of the other variables.

In exactly the same manner the first approximation curves for the functions $X_1 = f_3(X_3)$ and $X_1 = f_4(X_4)$ are drawn. These are shown in Figs. 2 and 3. The group average values used for the purpose are given in Tables 3 and 4.

It will be seen from first approximation curve itself that the curve for elongation with rainfall is almost a horizontal straight line. This is probably due to the fact that the crop being irrigated depends little on the rainfall amount.

Estimates of X_1 from the first approximating curves

Denoting the first approximate curves by $f_2'(X_2)$, $f_3'(X_3)$ and $f_4'(X_4)$, the estimates X_1'' of X_1 from the first approximating curves are worked out by the equation—

$$X_1''=a'_{1\cdot234}+f_2'(X_2)+f_3'(X_3)+f_4'(X_4) \quad (4)$$

where,

$$a'_{1\cdot234}=M_1-\frac{\Sigma[f_2'(X_2)+f_3'(X_3)+f_4'(X_4)]}{n} \quad (5)$$

The residuals $Z'' = X_1-X_1''$ are then computed which are given in column 2 of Table 5.

It will be seen from Table 5 that the new residuals Z'' are in general smaller than the previous ones Z' and there are 17 cases in which the new residuals are smaller and 11 in which they are larger than the previous ones. A more accurate comparison can be had by comparing the adjusted standard deviations of the two sets of residuals. The linear multiple regression uses 4 degrees of freedom and so the adjusted standard deviation of Z' is $[\Sigma Z'^2/(28-4)]^{\frac{1}{2}}$ $=19\cdot5$ cm. The results of the first approximation curves indicate that the curvilinear relations of X_2 and X_3 with X_1 are each of second degree and the relation of X_1 and X_4 is of first degree. Assuming this, the regression equation (4) uses 6 degrees of freedom and so the adjusted standard deviation of Z'' is $[\Sigma Z''^2/(28-6)]^{\frac{1}{2}} = 18\cdot8$ cm as compared with the value $19\cdot5$ cm of Z'. Apparently the new estimates do come nearer the observed values, on the average, than did the first set of estimates.

Second approximate net regression curves

The first approximation curves from Figs. 1 to 3 are first drawn. Each of the last residuals Z'' is then plotted as a deviation just as before, except that the residuals now are plotted as deviations from the regression curves, instead of from the regression lines. Averaging the residual values, the second approximation curves are drawn as before.

TABLE 3

Average values of Z' for corresponding X_3 values

X_3 values	No. of cases	Average of X_3	Average of Z'
Upto 85·0	4	83·5	3·36
85·1—85·7	4	85·5	—12·17
85·8—86·4	5	86·2	— 9·25
86·5—87·1	3	86·8	20·22
87·2—87·8	4	87·5	8·44
87·9—88·5	4	88·1	14·20
88·6 and above	4	88·9	—17·67

TABLE 4

Average values of Z' for corresponding X_4 values

X_4 values	No. of cases	Average of X_4	Average of Z'
Upto 18·2	5	16·06	14·66
18·3—20·2	5	19·80	— 8·94
20·3—22·2	3	21·22	— 0·78
22·3—24·2	4	23·23	6·03
24·3—26·2	6	25·18	—14·30
26·3 and above	5	33·36	6·69

These are shown in Figs. 4 to 6. The new estimates X_1''' are worked out from—

$$X_1'''=a''_{1\cdot234}+f_2''(X_2)+f_3''(X_3)+f_4''(X_4) \quad (6)$$

The new residuals $Z'''=X_1-X_1'''$ are given in col. 3 of Table 5.

Even though the residuals have decreased in 13 cases and increased in 15, the new residuals have decreased on the average and the adjusted standard deviation of Z''' values is 18·3 cm compared to 18·8 cm of the Z'' values and so the third estimates are apparently nearer the actuals than the second estimates.

Further successive approximations

Using the residuals from the second approximations and proceeding exactly as before the third approximation curves are drawn and the heights estimated therefrom and the residuals computed. The process was continued upto the fifth approximation when the estimated standard deviation of the residuals showed about steady value. The final curves are given in Figs. 7 to 9. Results of these successive approximations are given in cols. 4 to 6 of Table 5.

From cols. 3 and 4 of Table 5 it would be seen that residuals from third approximation have decreased in 15 cases and increased in 13 cases as compared to those from second approximations and the adjusted standard deviation of the residuals decreased to 17·1 cm from 18·3 cm. Residuals from 4th approximations (col. 5 of Table 5) have decreased in 20 cases and increased in 8 only and the adjusted standard deviation has decreased from 17·1 to 16·8 cm. Further approximation (col. 6 of Table 5) shows that the residuals have decreased in 14 cases and increased in 14 and the adjusted standard deviation has decreased from 16·8 to 16·5 cm. Apparently the estimates from the fifth approximation curves do come nearer the observed values than the previous estimates.

3. Inferences of the curves

The final regression curves shown in Figs. 7 to 9 represent the net relation between the dependent variable and each independent variable with the net variation associated with the other independent variables held constant. Following are the inferences from them—

(*i*) The height of the sugarcane crop at Poona decreases as the average daily minimum temperature during the elongation phase increases from 68°F. The optimum value of the minimum temperature is less than or equal to 68°F.

(*ii*) The height appears to remain stationary so long the average daily maximum temperature during the elongation phase remains within 86°F and increases as the maximum temperature increases from 86 to 87·5°F and decreases as the

TABLE 5

	Residuals from					
	Linear Regression Z'	First approximation curve Z''	Second approximation curve Z'''	Third approximation curve Z''''	Fourth approximation curve Z'''''	Fifth approximation curve Z''''''
	6·7	— 2·4	— 7·4	— 7·4	— 6·4	— 3·5
	— 1·5	— 5·4	—12·9	—10·5	— 9·9	— 9·2
	—44·9	—45·0	—36·9	—34·6	—36·3	—38·7
	27·4	19·6	15·1	11·9	11·5	12·4
	—12·3	— 4·5	—18·4	— 9·7	— 7·9	—10·2
	—12·1	—10·6	— 0·7	2·8	0·9	0·8
	10·2	2·9	0·2	— 2·0	— 1·2	2·0
	— 3·8	— 4·6	1·9	3·2	1·4	2·3
	9·8	16·5	23·8	18·5	16·7	16·9
	—12·1	—14·9	—25·4	—25·8	—23·9	—20·4
	0·9	2·4	— 2·6	— 2·3	— 1·4	— 0·2
	1·2	1·5	7·7	4·1	2·4	0·8
	—13·7	— 9·4	— 9·7	—10·6	— 8·6	— 9·2
	—28·5	—29·3	—28·5	—26·1	—25·0	—15·4
	43·3	43·7	34·4	38·0	40·7	38·4
	14·0	8·0	7·8	9·4	8·2	6·4
	30·4	28·9	24·2	23·6	24·5	25·4
	— 5·3	2·5	— 6·9	— 2·5	— 0·9	— 3·2
	3·0	2·5	8·1	12·0	11·3	12·4
	2·5	4·5	6·5	7·0	10·3	9·6
	2·8	2·2	7·1	4·2	3·3	4·6
	—18·8	—16·8	—10·1	—12·7	—13·1	—14·7
	20·1	14·7	17·3	14·5	11·2	10·6
	7·2	12·9	20·2	14·0	13·4	13·3
	—16·7	—11·3	— 6·0	— 5·1	— 5·4	— 3·4
	11·7	4·0	0·6	— 2·8	— 3·6	— 2·5
	— 4·8	— 4·5	1·7	— 1·9	— 3·6	— 5·2
	—18·7	— 8·4	—11·3	— 9·8	— 7·4	—10·6
Adj. S.D.	*19·5*	*18·8*	*18·3*	*17·1*	*16·8*	*16·5*

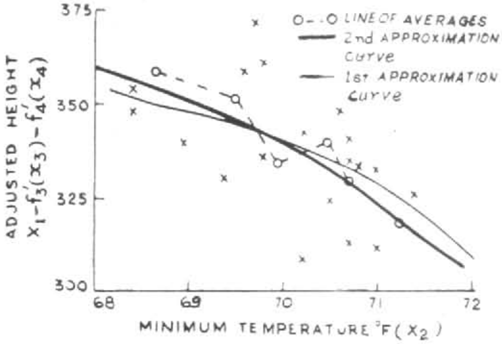

Fig. 4

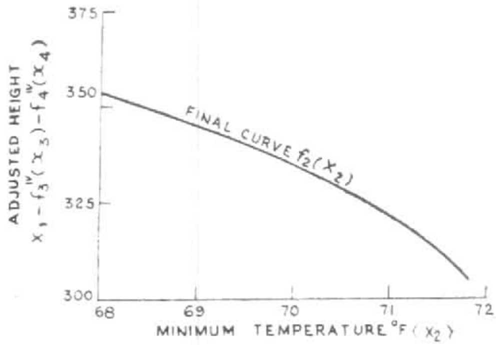

Fig. 7

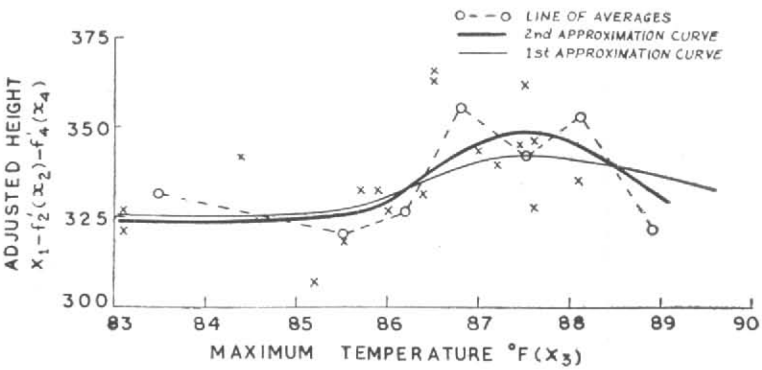

Fig. 5

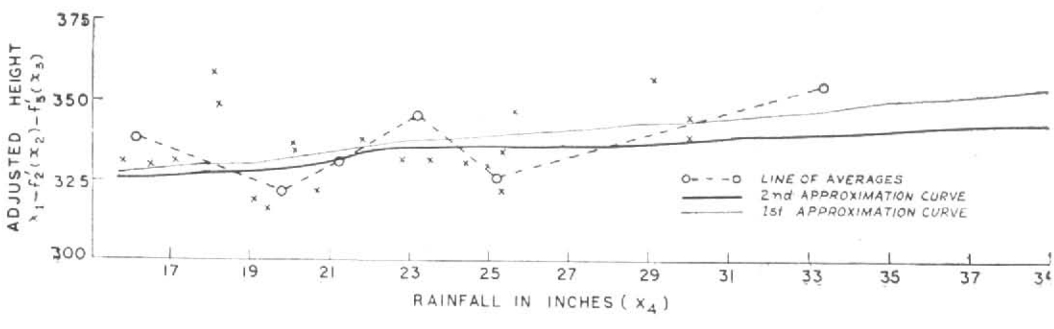

Fig. 6

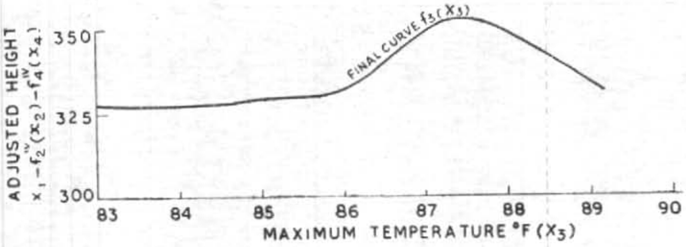

Fig. 8

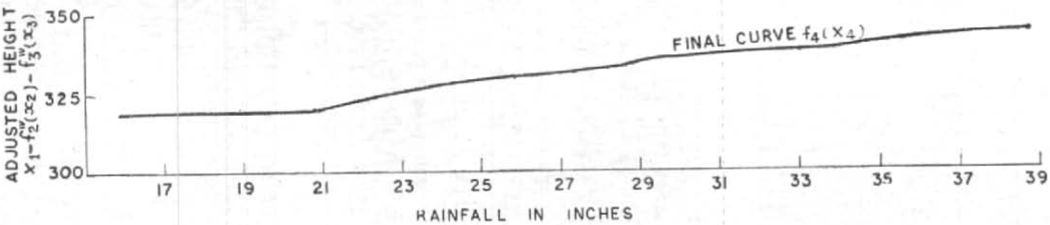

Fig. 9

temperature exceeds the limit 87·5° F. The optimum value of the daily maximum temperature is 87·5°F.

(*iii*) The height does not appear to rise or fall much with the variations in rainfall. This is quite probable as the crop is irrigated.

4. Use of the curves

The final curves are then utilised to estimate the values of the dependent factor. It would be clear from the curves in Figs. 7 to 9 that minimum temperature is the most important factor, then come the maximum temperature and rainfall in order.

The height value X_1 is estimated from the most important factor X_2 (minimum temperature) by the equation—

$$X_1' = F_2(X_2) = f_2(X_2) - M_{f(2)} + M_1$$
$$= f_2(X_2) + 6 \cdot 47 \quad (7)$$

[$M_{f(2)}$ is the mean of the values obtained from the final curve f_2 (X_2)]

The values estimated from eq. (7) for different values of X_2 are tabulated in Table 6.

The estimated values are then adjusted for the other independent factors. The *correction* for the maximum temperature (X_3) is made by the equation—

$$X_1' = F_3(x_3) = f_3(X_3) - M_{f(3)}$$
$$= f_3(X_3) - 337 \cdot 66 \quad (8)$$

These *corrections* for various maximum temperatures are given in Table 7.

Corrections for rainfall (X_4) are found by the equation—

$$X_1' = F_4(x_4) = f_4(X_4) - M_{f(4)}$$
$$= f_4(X_4) - 324 \cdot 92 \quad (9)$$

These corrections are given in Table 8.

The height values may now be estimated based on all the independent factors—minimum temperature, maximum temperature and rainfall—by the equation

$$X_1' = F_2(X_2) + F_3(x_3) + F_4(x_4) \quad (10)$$

These estimates for a particular value of rainfall have been made and given in Table 9.

In Table 9 values have not been entered for the combinations of factors which were not represented in the data from which the relations have been found out.

The final results of our curvilinear study, as represented in Tables 6 to 8 or in Table 9 may be represented in simple form graphically. These have been done in Figs. 10 to 12.

TABLE 6

Average height of sugarcane crop with varying minimum temperature keeping influences of maximum temperature and rainfall constant

Min. temp. X_2	Readings from final curve $f_2\ (X_2)$	Constant $M_1 - M_{f(2)}$	Average height $F_2(X_2)$
68·0	353·1	6·5	359·6
68·5	349·7	6·5	356·2
69.0	344·9	6·5	351·4
69·5	339·7	6·5	346·2
70·0	333·7	6·5	340·2
70·5	327·7	6·5	334·2
71·0	321·7	6·5	327 7
71·5	312·5	6·5	319·0

TABLE 7

Corrections to heights due to differences in maximum temperature

Average maximum temp. X_3	Readings from final curve $f_3\ (X_3)$	Constant $-M_{f(3)}$	Correction to expected height $F_3(x_3)$
83·0	327·5	—337·7	—10·2
83·5	327·5	—337·7	—10·2
84·0	327·5	—337·7	—10·2
84·5	328·3	—337·7	— 9·4
85·0	329·0	—337·7	— 8·7
85·5	329·2	—337·7	— 8·5
86·0	331·9	—337·7	— 5·8
86·5	341·0	—337·7	3·3
87·0	349·6	—337·7	11·9
87·5	352·5	—337·7	14·8
88·0	347·5	—337·7	9·8
88·5	340·1	—337·7	2·4
89·0	332·7	—337·7	—5·0

TABLE 8

Correction to heights due to differences in rainfall

Total rainfall X_4	Readings from final curve $f_4(X_4)$	Constant $-M_{f(4)}$	Correction to expected height $F_4(x_4)$
16	318·3	—324·9	—6·6
17	318·3	—324·9	—6·6
18	318·3	—324·9	—6·6
19	318·6	—324·9	—6·3
20	319·2	—324·9	—5·7
21	319·2	—324·9	— 5·7
22	322·4	—324·9	—2·5
23	324·5	—324·9	—0·4
24	326·5	—324·9	1·7
25	327·5	—324·9	2·6
26	327·5	—324·9	2·8

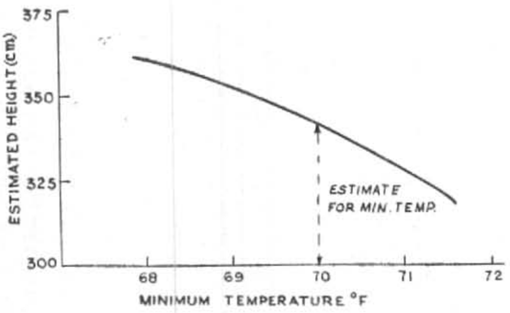

Fig. 10

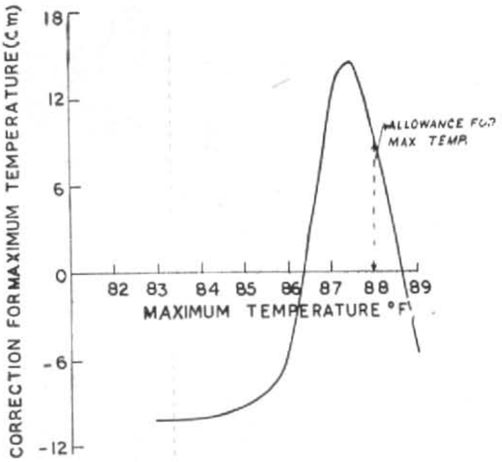

Fig. 11

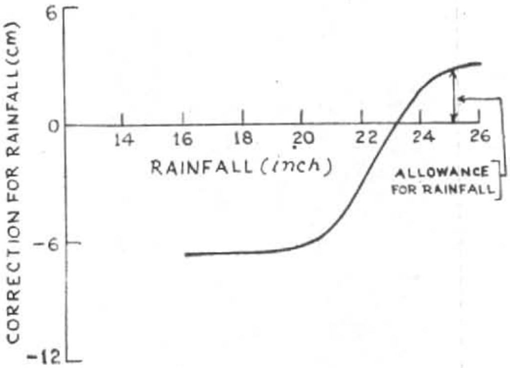

Fig. 12

TABLE 9

Expected heights with varying maximum and minimum temperature for rainfall 25″

Minimum temperature	Maximum temperature 83	84	85	86	87	88	89
68	—	—	—	356·4	—	—	—
69	—	—	345·3	348·2	365·9	363·8	—
70	332·2	332·6	334·1	337·0	354·7	352·6	327·8
71	320·1	320·1	321·6	324·5	342·2	340·1	325·3

Corrections for rainfall have been confined for rainfall upto 26″ (*vide* Table 8 and Fig. 12) even though the rainfall amount upto 31″ were present in the data analysed. This is because the observations beyond 26″ were very few and the final curve (Fig. 9) for height and rainfall could not be drawn beyond $X_4 = 26''$ with much confidence.

The reliability of the final curves was examined by estimating the heights from the curves 10 to 12 corresponding to the observed meteorological factors. Only in 2 cases out of 23 the estimates were outside 10 per cent and two thirds of the estimates were within 2 to 3 per cent of the actuals.

5. Conclusion

The curvilinear study has brought out how the height of sugarcane crop at Poona depends on the meteorological factors of the elongation phase. It has also brought out the optimum value, of a weather factor, if any. Such detailed information cannot be had from a correlation or linear regression study.

Also it may be noted that Table 9 is not merely a table of average heights for various maximum and minimum temperatures. To begin with, there were only 3 observations with rainfall 25″. The table gives the most probable height to be associated with any of the 19 different combinations of maximum and minimum temperatures for 25″ of rainfall. Similar other estimates can be made for a large number of other combinations.

The present study thus illustrates the ability of the curvilinear analysis both to bring out a series of crop weather relationships which are not observable on the surface and to provide a basis for estimating the probable effect of new combinations of independent factors upon the dependent one.

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