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Morphology of great Geomagnetic Storms recorded at Alibag during 1905—1945 : Part I—The Storm-Time Disturbance Variation, D_{st}

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ABSTRACT. Following Sugiura and Chapman, a study of the morphology of 74 great geomagnetic storms, recorded at Alibag during 1905 to 1945, is made. The D_{st} variation in H, D and Z for the average storm is derived. D_{st} (H) is very similar to that obtained by Moos for Colaba for the period 1872–1904. The results for Alibag fit in with those for other latitudes obtained by Sugiura and Chapman. The D_{st} field is found to be almost entirely confined to the geomagnetic meridian plane.

1. Introduction

The geomagnetic field now and then experiences violent changes for a period of time extending to several days. These violent disturbances have been called geomagnetic storms and are broadly classified into two types. One type is characterized by a sudden onset; the field value (in the horizontal component) in low and moderate latitudes sharply increases and remains above the quiet day value for a few hours. A rapid decrease in the field value then sets in, reaching a maximum in about 10 to 20 hours from the onset of the storm, after which the depressed field gradually recovers to its normal level in the course of a few days. The other type begins gradually; its different phases are not well marked; its intensity of disturbance is usually much less compared with that of the sudden-commencement type, while its duration is very often much more prolonged. The disturbance effects and the time-variation of such effects associated with each storm are so varied that no two storms of either type are alike. Any average characteristics attributable to each type of storm can only be arrived at by a statistical study of the features of a large number of storms.

Moos (1910) undertook a statistical study of geomagnetic storms registered at the Colaba (Bombay) Observatory during the period 1872 to 1904. He derived the average character of sudden commencement storms as well as all storms in the element H, for the first 24 hours after the commencement of the storm and also (for all storms) the trend for the second day of storm. He found that the trends were alike whether only sudden commencement storms or all storms of large intensity were considered. He arrived at the average characteristics of the sudden commencement storms, *viz.*, (1) a small initial rise, (2) a large rapid fall which attained its lowest value, indicating the maximum effect of the disturbance from about 10 to 11 hours after the commencement of a storm, and (3) a slow and laboured rise back to normal conditions. He observed the same features for all storms of large intensity. Moos also derived the mean local-time disturbance variation in H by arranging the first 24-hourly values from the commencement of the S. C. storms as well as all storms for the same period (1872-1904) according to local time.

A detailed study of the morphology of geomagnetic storms in the elements H, D and Z was carried out by Chapman (1918, 1927 and 1952). Using data from a number of observatories he derived average features of the disturbance variation reckoned according to storm-time (time from the commencement of the storm), denoted by D_{st}, the disturbance daily variation, *SD*, reckoned according to local time, and the local-time disturbance variation, *DS*, which is local-time disturbance variation for shorter intervals of storm-time than a day.

Sugiura and Chapman (1956, 1957, 1958) have made a thorough investigation of the morphology of geomagnetic storms of the sudden commencement type, using the hourly values of *H*, *D* and *Z* from as many as 26 observatories distributed from geomagnetic latitude 80°N to 48°S. The geomagnetic storms selected by them were 346 in all, which were grouped into weak storms (136), moderate storms (136) and great storms (74). For each intensity group the mean D_{st}, *SD* and *DS* were derived for different mean geomagnetic latitudes.

In the case of great storms Sugiura and Chapman (1958) grouped the observatories (13 in all) according to geomagnetic latitude in the following way— (1) 80° (Godhavn), (2) 65° (Tromso, Sodankyla and Lerwick), (3) 58° (Sitka, Eskadalemuir, Lovo and Rude Skov), (4) 52° (De Bilt, Greenwich, Val Joyeux, Cheltenham), (5) 42° (Ebro, Tucson), (6) 28° (Porto Rico, Kakioka), (7) 21° (Honolulu, Zikawei) and (8) —1° (Huancayo). For no latitudinal grouping of observatories complete data for all the 74 great storms were available to them.

It may be noticed that the interval between the 7th and 8th groups of observatories is 22° geomagnetic latitude, which is rather a big gap for a study involving latitudinal variation in the morphology of geomagnetic storms, especially for the low latitude region. This gap can be filled by the inclusion of an observatory with a geomagnetic latitude of about 10°. The Alibag Observatory (geomagnetic latitude 9°.5) suited this very well and this study was undertaken for the group of storms classified as great by Sugiura and Chapman, using Alibag data for *H*, *D* and *Z* for the period 1905 to 1945 (Alibag started functioning from October 1904). Examination of the records of Alibag showed that data for all the 74 storms classified as great were available in the *H* and *Z* components and for 73 storms in the *D* component. The list comprising the 74 S. C. storms classified as great by Sugiura and Chapman is given in Table 1.

In this paper only the results of mean storm-time variation (D_{st}) in the elements *H*, *D* and *Z* for great storms are given and discussed. Results of local time disturbance variations *SD* and *DS* will be given in a separate paper.

2. Method of Analysis

The method followed in the derivation of the mean D_{st} is entirely that of Sugiura and Chapman (1956). In brief the method is as follows. For any storm S_n the four pre-storm hourly values of *H*, *D* and *Z* and the first seventytwo hourly values after the storm commencement are included in the analysis. The tabulated hourly values of the elements *H*, *D* and *Z* for Alibag are mean values for intervals of sixty minutes centred at full-hours GMT. The hourly values are considered as representing the values at exact full hours GMT. The full-hour closest to the time of commencement of storm is the 0^h storms time. The four pre-storm hours are labelled -4^h, -3^h, -2^h and -1^h storm time and the hours after the commencement are labelled 1^h, 2^h ......71^h storm time. Hour-to-hour differences of the hourly values are formed starting from the "-1^h which is actually taken as the zero level throughout the analysis. The series of hour-to-hour differences so formed for storms S_n, say for the element *H*, are $H_{n(-4)} - H_{n(-3)}$, $H_{n(-3)} - H_{n(-2)}$, $H_{n(-2)} - H_{n(-1)}$, 0, $H_{n0} - H_{n(-1)}$, $H_{n1} - H_{n0}$, $H_{n2} - H_{n1}$ .... $H_{n71} - H_{n70}$, where the subscripts to H_n refer to storm time. These hour-to-hour differences correspond respectively to -4^h, -3^h, -2^h, -1^h, 0^h, 1^h, 2^h, ...71^h storm time. Next, hour-to-hour differences of the mean hourly values of the element *H* for the five international quiet-days of the month

TABLE 1

List of great storms for the period 1905 to 1945

Serial No.	Year	Month	Date of S.C.	Time of S.C. (GMT)	Serial No.	Year	Month	Date of S.C.	Time of S.C. (GMT)
1	1905	Nov	15	15·3	38	1929	Feb	26	19·4
2	1906	Dec	21	21·5	39		Mar	11	13·9
3	1907	Feb	9	14·2	40		Mar	15	8·5
4	1908	Sep	11	7·9	41		Dec	3	12·1
5		Sep	28	9·2	42	1930	Sep	18	8·8
6	1909	May	14	4·9	43		Dec	3	1·1
7		May	18	5·1	44	1933	Sep	8	21·4
8		Sep	25	8·5	45	1936	Jun	18	9·7
9		Sep	30	4·0	46		Nov	28	23·6
10	1915	Jun	16	13·0	47		Dec	27	3·5
11		Nov	5	14·6	48	1937	Apr	24	12·0
12	1916	Aug	26	19·7	49		May	4	16·9
13	1917	*Jan	4	5·0	50		Jun	4	14·4
14		Aug	9	4·2	51		Aug	1	21·8
15		Dec	16	9·2	52		Aug	22	3·1
16	1918	Mar	17	21·2	53		Sep	30	13·8
17		Jan	9	23·1	54		Oct	7	5·3
18	1919	May	1	22·9	55	1938	Jan	16	22.6
19		Aug	11	7·0	56		Jan	25	11·9
20		Oct	1	16·2	57		Mar	21	22·7
21	1920	Mar	4	11·6	58		Apr	16	5·8
22		Mar	22	9·2	59		May	11	15·9
23	1921	May	19	20·1	60		Sep	13	18·6
24	1926	Jan	26	16·3	61		Oct	7	6·2
25		Feb	23	16·4	62	1939	Feb	24	17·1
26		Apr	14	14·1	63		Apr	17	1·9
27		Jun	1	11·2	64		Aug	12	1·7
28		Oct	13	19·4	65		Aug	22	0·7
29	1927	Apr	13	23·8	66		Oct	13	2·1
30		Jul	21	21·0	67	1940	Mar	29	16·1
31		Aug	20	6·6	68		Nov	12	7·1
32		Oct	22	6·7	69	1941	Mar	1	3·9
33	1928	May	27	14·8	70		Jul	4	3·7
34		Jul	7	23·5	71		Sep	18	4·2
35		Sep	7	13·8	72		Oct	31	3·7
36		Oct	18	7·4	73		Dec	1	6·0
37	1929	Feb	16	23·1	74	1945	Dec	13	12·7

* Record for *D* (Declination) was lost

in which the storm S_n occurred are formed. Twentyfour such differences are obtained which may be labelled $S_{qn\,24} - S_{qn\,23}$, $S_{qn1} - S_{qn0}$, $S_{qn2} - S_{qn1}$ .... $S_{qn23} - S_{qn22}$, where the subscripts to S_{qn} refer to hours GMT. These hour-to-hour differences correspond respectively to 0^h ....23^h GMT. Let a particular difference, say $S_{qn3} - S_{qn2}$ correspond to the same GMT hour as the hour-to-hour difference $H_{n0} - H_{n\,(-1)}$ (which is the difference corresponding to 0^h storm time). The 24 consecutive differences $S_{qn3} - S_{qn2}$, $S_{qn4} - S_{qn3}$, $S_{qn5} - S_{qn4}$, ... $S_{qn24} - S_{qn23}$, $S_{qn1} - S_{qn0}$, $S_{qn2} - S_{qn1}$ are subtracted respectively from the 24 hour-to-hour differences of the first day of storm, *viz.*, $H_{n0} - H_{n\,(-1)}$, $H_{n1} - H_{n0}$, $H_{n2} - H_{n1}$, ... $H_{n23} - H_{n22}$, to give the residuals d_{n0}, d_{n1}, d_{n2} .....d_{n23} , corresponding to 0^h , 1^h ,........23^h storm time. Similarly the same sequences of hour-to-hour differences of S_{qn} values are subtracted from the hour-to-hour differences of storm-time hourly values for the 2nd and 3rd 24 hours of storm to give the residuals d_{n24} , d_{n25} , d_{n26} .... d_{n47} and d_{n48} , d_{n49} , d_{n50} ....d_{n71} corresponding to 24^h , 25^h , ...71^h storm-time. The series of residuals represent hour-to-hour differences of disturbance effects, freed from the quiet-day values. In addition, the residuals $d_{n\,(-4)}$, $d_{n\,(-3)}$ and $d_{n(-2)}$ [$d_{n(-1)}$ being equal to zero] corresponding to -4^h , -3^h , -2^h and -1^h storm time are also obtained for storm S_n by subtracting $S_{qn23} - S_{qn24}$, $S_{qn0} - S_{qn1}$ and $S_{qn1} - S_{qn2}$ from the storm time hour-to-hour differences $H_{n(-4)} - H_{n(-3)}$, $H_{n(-3)} - H_{n(-2)}$, and $H_{n(-2)} - H_{n(-1)}$. Thus for the storm S_n we finally have the sequence of residuals $d_{n(-4)}$, $d_{n(-3)}$, $d_{n(-2)}$, 0, d_{n0}, d_{n1} .... d_{n71} corresponding to — 4^h ....0^h ......71^h storm time. For the N storms there will be N such sequences for each of the elements H, D and Z. The mean storm time residuals of the form

$$\frac{1}{N} \sum_{n=1}^{N} d_{nh}$$

(where $h = -4, -3, \ldots\ldots 0, \ldots\ldots\ldots\ldots 71$) are then computed, which give the mean storm-time disturbance variation, D_{st}, on an hour-to-hour difference basis. The actual storm time hourly disturbance inequalities are obtained by cumulatively summing the mean residuals

$$\frac{I}{N} \sum_{n=1}^{N} d_{nh}$$

from -1^h storm time (where the residual is zero) backwards upto -4^h storm time and forwards upto 71^h storm time. In this way we obtain the D_{st} (H, D, Z) for the four pre-storm hours and the first three storm days, the variation being reckoned from the level of -1^h storm time. In arranging the storms according to storm-time and arriving at a mean D_{st} variation, the type of disturbance that varies according to local time is averaged out, provided the number of storms is fairly large and the commencement times of these storms are distributed fairly evenly from 0 to 24 hours of the day. In the case of Alibag the number of storms is 74 for the elements H and Z and 73 for D and their commencement times in local time are evenly distributed from 0^h to 23^h of the day, as may be seen from Table 1.

In freeing the S_q values from the storm-time hourly values it is supposed that S_q values are invariably present on storm days also, and therefore S_q hourly values uncorrected for non-cyclic change are used for eliminating the quiet-day component.

The mean storm-time variation D_{st} (D) which is in minutes of arc is converted to the magnetic-east force component in γ by multiplying the values in minutes of arc by H_m tanl', where H_m is the mean value of H for the years in which the storms occurred. D_{st} (D) in γ is positive when it is easterly and negative when westerly. D_{st} (H) is the magnetic north component of the storm time disturbance force variation in γ. It is positive when directed in the magnetic-north direction and negative when directed in the magnetic-south direction. D_{st} (Z) is the vertical component of storm time disturbance force variation

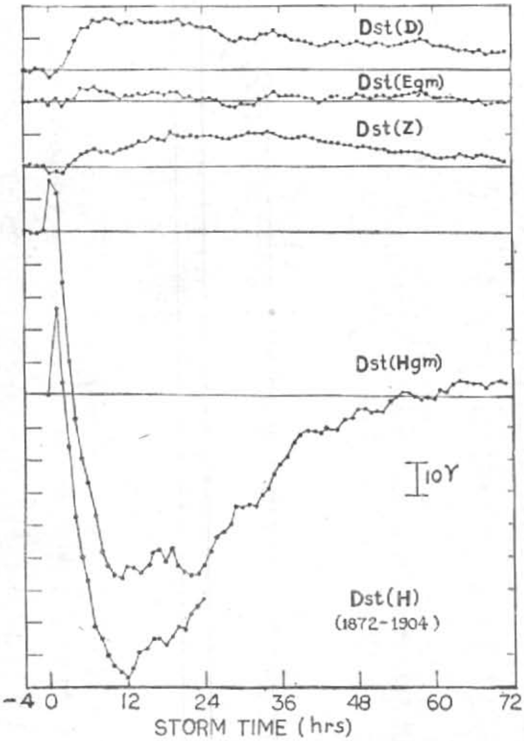

Fig. 1. Mean storm-time disturbance variations D_{st} (H_{gm}), D_{st} (D), D_{st} (E_{gm}) and D_{st} (Z) for 74 great storms recorded at Alibag during 1905 to 1945. The D_{st} (H) for the first 24 hr. storm time, as derived by Moos for Colaba (Bombay), for the period 1872 to 1904 is shown in the lower-most diagram

in γ, positive when directed downwards and negative when directed upwards.

In this analysis the components of the storm-time disturbance variation in the geomagnetic-east direction and in the geomagnetic-north direction are also derived as was done by Sugiura and Chapman. For this purpose the angle (ψ—D) between the magnetic and geomagnetic meridians at Alibag, based on the mean value of declination D at Alibag for the years in which the storm occurred, is determined. For the period under consideration mean D for Alibag is $+0° 1{\cdot}4'$ and ψ, the angle between the geomagnetic and geographic meridians is $-7°10'$. Thus ($\psi - D$) works out to be $-7°11{\cdot}4'$ which indicates that the geomagnetic meridian is $7°11{\cdot}4'$ to the west of the magnetic meridian. Using this value the D_{st} (H) and D_{st} (D) are split up to give the geomagnetic north and east components, D_{st} (H_{gm}) and D_{st} (E_{gm}).

3. Results and Discussion

The mean storm-time disturbance variations D_{st} (H_{gm}), D_{st} (E_{gm}) and D_{st} (Z) for Alibag, for the period 1905 to 1945, for great storms, are shown in Fig. 1. The magnetic east component D_{st} (D) is also shown in Fig. 1. The storm-time disturbance variation, for the first 24 hours after the commencement of S. C. storms, as obtained by Moos (1910) for Colaba (Bombay), for the period 1872 to 1904, is also plotted in Fig. 1, for comparison. The magnetic-north component, D_{st} (H), is not significantly different from D_{st} (H_{gm}) and is, therefore, not shown separately. The exact storm-time hourly inequalities of D_{st} (D, Z, H_{gm}, E_{gm}), for the four pre-storm hours and the three storm days are given as an appendix to the paper. In Figs. 2, 3 and 4 respectively are shown D_{st} (H_{gm}), D_{st} (E_{gm}) and D_{st} (Z) for Alibag together with those for other geomagnetic latitudes as derived by Sugiura and Champan (1958). For the first 12 hours of storm time, hourly inequalities of D_{st} (H_{gm}, E_{gm}, Z) have been shown and, therefore, only mean inequalities for periods of 12 or 24 hours storm-time are shown.

It is at once seen that for Alibag D_{st} (E_{gm}) or D_{st} (D) and D_{st} (Z) are insignificant compared with D_{st} (H_{gm}), which is in conformity with observations at similar latitudes as may be noticed in Figs. 2, 3 and 4. The mean D_{st} (H_{gm}) is positive only for about $1{\cdot}5$ hours after which it dips sharply below the zero level to attain a minimum in about 11 hours from the commencement. For Alibag the minimum D_{st} (H_{gm}) is rather broad, for, after small fluctuations, a second minimum of about the same value as at 11^h storm-time, is attained at 22^h storm-time. The maximum value of D_{st} (H_{gm}) is $+16\,\gamma$ and the minimum

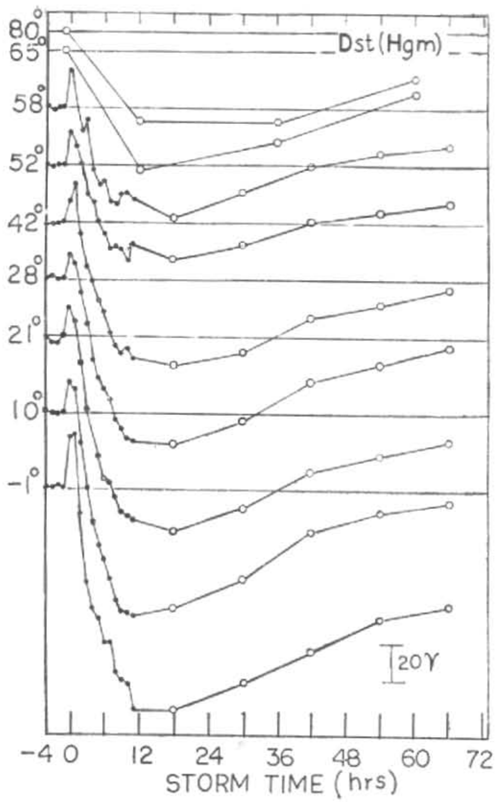

Fig. 2. D_{st} **variations in the geomagnetic-north component** H_{gm} **at different geomagnetic latitudes for the average great storm**

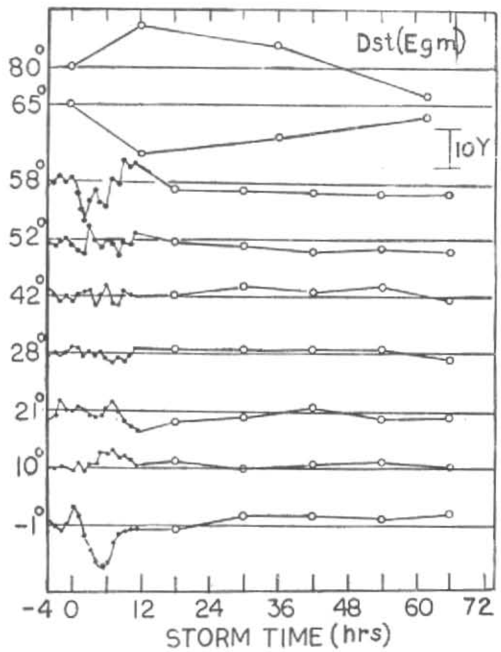

Fig. 3. D_{st} **variations in the geomagnetic-east component** E_{gm} **at different geomagnetic latitudes for the average great storm**

The variations for all the latitudes except for 10° (Alibag) have been reproduced from the analysis by Sugiura and Chapman (1958)

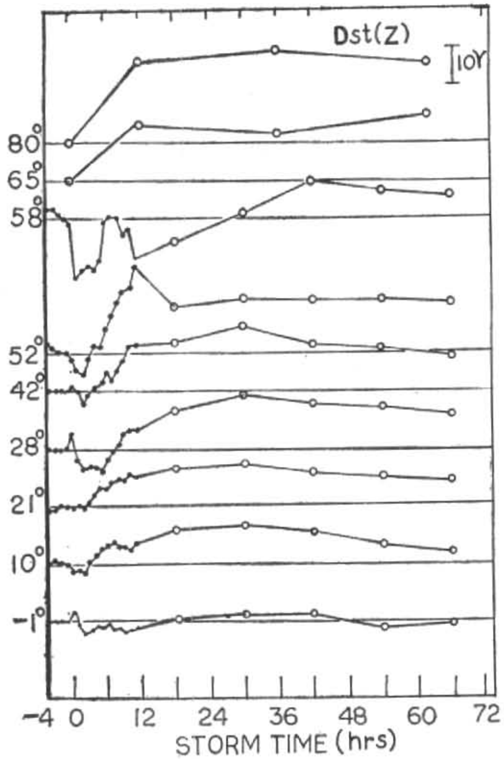

Fig. 4. D_{st} **variations in the vertical force,** Z**, at different geomagnetic latitudes for the average great storm. The variations for all the latitudes except for 10° (Alibag) have been reproduced from the analysis by Sugiura and Chapman (1958)**

value —106γ. The quantitative characteristics of the mean D_{st} (H_{gm}) for Alibag are given in Table 2 together with results for other latitudes. Though Alibag is midway between the observatory groups 7 (mean Lat. +21°) and 8 (Lat. —1°) its D_{st} (H_{gm}) characteristics are closer to those of the group 7, but a transition is definitely indicated. There is a clear tendency for the amplitude of depression in H or the range in D_{st} (H_{gm}) to steadily increase from about the geomagnetic latitude of 52° to the geomagnetic equator, with a fairly conspicuous maximum at the geomagnetic equator. This is in conformity with Vestine's (1963) finding of a bulge of increased disturbance amplitude in H_{gm} (of about 3 γ) from + 20° to — 20° geomagnetic latitude with a maximum at the geomagnetic equator, which he arrives at by studying the distribution, according to geomagnetic latitude, of the difference in disturbed day values and quiet day values (5 days for each month) for the period 1922-1933. He attributes the effect to possible refraction of hydromagnetic waves, in the region of the geomagnetic equator or to ring currents established at distances from the earth comparable with the lateral extent of the enhanced effects noticed. The same causes may also be contemplated for the large amplitude of D_{st} (H_{gm}) noticed in the region of the geomagnetic equator.

The mean D_{st} (H), which is practically the same as D_{st} (H_{gm}) is very similar in all its phases to that derived by Moos (1910) for Colaba (Bombay) from 113 S.C. storms for the period 1872 to 1904. In both cases the maximum and minimum D_{st} (H) are attained at about the same time after the commencement. The range in D_{st} (H) obtained in this analysis is 122 γ while the range from the analysis of Moos is 113 γ. In his analysis, Moos had included all principal S.C. storms, irrespective of their intensity, whereas the present analysis is for great storms only. It is perhaps on this account that the range in D_{st} (H) obtained by Moos is slightly less.

TABLE 2

Maximum D_{st} (H_{gm}) in the initial phase, the approximate storm-time at which H_{gm} crosses its pre-storm level and values of minimum $D_{st}(H_{gm})$ at different latitudes for the average great storm

	Mean geomagnetic latitude of observatory group								
	80°	65°	58°	52°	42°	28°	21°	10°	—1°
Max. D_{st} (H_{gm}) in the initial phase (in γ)			19	17	20	12	15	16	29
Approx. storm-time at which H_{gm} crosses to pre-storm level (in hours)			1·0	2·0	1·8	1·5	1·2	1·3	1·5
Minimum D_{st} (H_{gm}) (in γ)	—48	—62	—58	—52	—76	—88	—102	—106	—116

The values given in the table, except for the geomagnetic latitude of 10° (Alibag) are from the results of Sugiura and Chapman (1958)

The D_{st} (Z) for Alibag is negative (directed upwards) when D_{st} (H_{gm}) is positive (directed northwards) and is positive (directed downwards) when the latter is negative (directed southwards). But the growth and decay of D_{st} (Z) and D_{st} (H_{gm}) are not in phase. The maximum D_{st} (Z) is attained 7 to 8 hours after D_{st} (H_{gm}) reaches its minimum (first minimum) and the recovery phase of the former commences about 12 hours later than the latter. The maximum D_{st} (Z) is $+10\gamma$ attained at about 32^h storm time and the minimum is -2γ attained at 2^h storm time. In almost all respects D_{st} (Z) for Alibag agrees closely with that for observatory group 7 (mean Lat. +21°).

The mean D_{st} (D), expressed as magnetic easterly force in gammas (10^{-5} cgs)—Fig. 1—is easterly when D_{st} (H) or D_{st} (H_{gm}) is negative (southerly) and westerly when the latter is positive (northerly). The minimum D_{st} (D) attained is -3γ at 0^h storm time and the maximum is $+16\gamma$ attained at about 9^h storm time. The maximum phase is very broad lasting for several hours. The recovery sets in gradually from about 21 hours storm time.

What is of particular significance in the east-west component of disturbance variation is the fact that the magnitude of D_{st} (E_{gm}) is negligible throughout the entire period of 3 storm days, being only a few gammas in value (Fig. 1). This feature for Alibag is in agreement with the findings of Sugiura and Chapman (1958) for the low and moderate latitudes, which may be noticed in Fig. 4. An important conclusion which follows from this finding is that the D_{st} field for the low and moderate latitudes is almost entirely confined to the geomagnetic meridian plane. In other words the orientation of the ring current, that is responsible for the main phase of the geomagnetic storm, is in a geomagnetic east-west direction. This is in a way to be expected since the longitudinal drift of the charged particles, that constitute the ring current, is in a direction perpendicular to both the field-line and its gradient and since at distances of several earth radii, where the ring current is formed, the geomagnetic field distribution conforms to the centred dipole model. Vestine (1963) also finds the geomagnetic easterly component of disturbance (disturbed days *minus* quite days) for the period 1922-1933, to be almost zero for low and moderate geomagnetic latitudes, which should be the case if the ring current is due to radiation trapped in the geomagnetic field.

4. Conclusion

The mean D_{st} field, at Alibag, of 74 sudden commencement great storms for the period 1905 to 1945, fits in very well with the latitudinal distribution of such fields observed by

Sugiura and Chapman (1958). This analysis has served its purpose of filling the large gap in latitude between the 7th group (+21° geomagnetic lat.) and the 8th group (−1° geomagnetic lat.) of observatories in the analysis by Sugiura and Chapman, while, at the same time, arriving at average characteristics of great S.C. storms for Alibag. The average characteristics for Alibag are about the same as for Colaba (Bombay), arrived at by Moos (1910) for the period 1872-1904. The main features observed are : (1) D_{st} (H_{gm}) is the most prominent component of the D_{st} field for low and moderate latitudes, (2) the range in D_{st} (H_{gm}) tends to increase towards the magnetic equator and (3) almost the entire D_{st} field is confined to the geomagnetic meridian plane.

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APPENDIX

Hourly values in gammas of the different D_{st} components for the 74 great storms (Table 1) recorded at Alibag during 1905 to 1945

Storm time (hrs)	D_{st} (H_{gm}) in γ			D_{st} (D) in γ			D_{st} (E_{gm}) in γ			D_{st} (Z) in γ		
	1st	2nd	3rd	1st	2nd	3rd	1st	2nd	3rd	1st	2nd	3rd
−4	+1			0			0			0		
−3	0			0			0			0		
−2	0			0			0			0		
−1	0			0			0			0		
0	+16	−102	−55	−3	+13	+9	−1	+1	+2	−2	+9	+7
1	+12	−98	−54	−1	+13	+8	+1	+1	+1	−1	+10	+6
2	−16	−94	−55	+1	+12	+8	−1	+1	+1	−2	+9	+5
3	−40	−92	−55	+5	+11	+8	0	−1	+1	0	+9	+5
4	−57	−90	−55	+9	+10	+8	+1	−2	+1	+2	+9	+5
5	−70	−84	−52	+13	+9	+7	+4	−2	+1	+4	+9	+5
6	−77	−85	−51	+13	+10	+7	+4	−1	+1	+4	+10	+4
7	−87	−84	−49	+15	+9	+8	+5	−1	+2	+5	+10	+5
8	−98	−84	−50	+15	+10	+8	+3	−1	+2	+4	+10	+5
9	−103	−81	−51	+16	+11	+9	+3	+1	+3	+4	+10	+4
10	−105	−79	−51	+15	+11	+9	+2	+1	+3	+4	+11	+4
11	−106	−74	−51	+14	+12	+9	+1	+3	+2	+5	+10	+4
12	−103	−71	−51	+15	+11	+7	+2	+2	+1	+5	+9	+3
13	−103	−70	−49	+15	+10	+7	+2	+1	+1	+6	+9	+3
14	−105	−65	−49	+15	+9	+7	+2	+1	+1	+8	+9	+3
15	−103	−62	−47	+15	+9	+7	+2	+1	+1	+8	+9	+3
16	−99	−61	−46	+15	+9	+6	+2	+1	0	+9	+9	+3
17	−97	−62	−46	+15	+8	+6	+3	+1	0	+9	+9	+3
18	−101	−62	−46	+15	+7	+6	+2	0	0	+8	+8	+3
19	−97	−61	−46	+15	+8	+6	+3	0	0	+10	+7	+3
20	−103	−61	−47	+16	+9	+5	+3	+1	−1	+10	+7	+3
21	−104	−60	−46	+14	+9	+5	+1	+1	−1	+9	+7	+3
22	−105	−58	−46	+15	+8	+5	+1	+1	0	+10	+6	+2
23	−105	−57	−46	+13	+8	+5	0	+1	0	+9	+7	+1