550·34 : 550·38

Sensitivity Adjustments of Electromagnetic Seismographs

H. M. CHAUDHURY

Meteorological Office, New Delhi

(*Received 10 July 1963*)

ABSTRACT. The paper presents a study of the limitations imposed by the seismometer and galvanometer constants on the attenuation factor of the circuit. It is seen that the full range from zero to one of this factor can be used only if (*i*) $Z_{11} = Z_{22}$ and (*ii*) R_1/Z_{11} and R_2/Z_{22} are both less than or equal to 1/2. The special case of a seismograph satisfying the Galitzin Conditions has been studied in the light of the above limitations, and a few possible ways of adjusting the various parameters of the seismometer to obtain the optimum magnification considered.

1. Introduction

Electromagnetic seismographs are nowadays extensively used throughout the world for high magnification recording of earth movements. Though the most sensitive seismographs now available, the Benioff instruments are of the variable reluctance type, a very large number of electromagnetic seismographs in operation in the seismological observatories employ the moving coil transducers. The comparatively lower magnification of these seismograhps is perhaps offset by their simpler design and operation. Even in studies of vibration problems in engineering, moving coil pick-ups are widely used.

While for engineering studies very high magnifications are not so much demanded as ruggedness of the pick-ups and the associated instruments, the study of seismological problems has been asking for the detection of more and more feeble movements. Consequently, the design and construction of these seismographs have been changing to achieve higher and higher magnifications. In practice, the electromagnetic seismograph is set to operate at some magnification, which changes but little with time. However, it becomes necessary, at times, to either increase or decrease the magnification of the instrument to suit particular conditions and hence an attenuator has to be introduced into the circuit. With the help of this attenuator it is possible, as pointed out by Neumann (1956) and Hagiwara (1958), to alter the sensitivity of the seismograph. The range of variation of this sensitivity or the attenuation factor, however, depends on some of the parameters of the seismometer and the galvanometer. A discussion of these parameters is given below.

2. Magnification of an electromagnetic seismograph

Since the very beginning, the solution of the equations of motion of the electromagnetic seismograph and the determination of its transient and steady state responses have attracted the attention of various workers. Unlike a mechanical seismograph, an electromagnetic seismograph employs a galvanometer to record the movements. The seismometer and galvanometer combination brings into play the effect of the latter on the former, the so called "reaction" of the galvanometer, and this complicates the equation of motion of the seismograph. After Galitzin, Wenner (1929), Coulomb and Grenet (1935), Schmerwitz(1936), Rybner (1937), Chakrabarty (1949, 1960), Eaton (1957), Eaton and Byerly (1957) and Chakrabarty and Tandon (1961) have studied the problem at various times. Though the solution for the transient response has been worked out only for the case of no reaction and for some special combinations of seismograph parameters, the steady state solution has been derived for the general case. Methods, based on the above solutions for the practical determination of the response curve have also been indicated by the above authors. Recently an elegant method for the determination of the magnification and response of the electromagnetic seismograph has been given by Willmore (1959). Willmore uses the Maxwell bridge and an oscillator giving sine wave outputs at different frequencies in this method. Espinosa, Sutton and Miller (1962) have also used Willmore's method and have calibrated a number of seismographs. Besides, they have also used electrical analogs of seismographs and have compiled an album of seismograph responses to some standard known inputs for a number of combinations of seismograph parameters. With this album, the problem of calibration of an instrument is reduced to that of referring to a dictionary. Another practical method, without the use of any electronic equipment, has been worked out by Hagiwara (1958). Hagiwara has introduced in the seismograph circuit an attenuator and has worked out the final expression in terms of easily determinable quantities. He has also introduced the attenuation factor μ. While, it is also possible to alter μ with only a shunt across the seismometer and galvanometer, it is possible to do so without altering the damping conditions only with such an attenuator. In the following discussion, therefore, the Hagiwara's form of the expression for the magnification has been used.

According to this, the magnification for displacement of a hinged e.m. seismograph is given by—

$$\text{Mag} = \left\{ (M\,H\,G\,g\,L\,\mu) / (K\,k\,n_1\,Z_{11}) \right\} \times \times f\,(h_1, h_2, \nu, \sigma, u) \qquad (1)$$

where M=Mass of the pendulum, K=Moment of inertia of the pendulum about the axis, H=Distance of the centre of mass of the pendulum from the axis, G=Electrodynamic constant of the seismometer, g=Electrodynamic constant of the galvanometer, k=Moment of inertia of the galvanometer coil, $n_1 = 2\pi / T_1$ = Circular frequency of the seismometer, L=Length of the optical lever of the galvanometer, μ = Attenuation factor and is defined as the ratio of the current passing through the galvanometer to the current through the seismometer, Z_{11}= the sum of the resistance of the seismometer coil and the resistance of the external circuit, f=a function of the parameters, where h_1=damping constant of the seismometer and h_2=damping constant of the galvanometer, $\nu = T_2 / T_1$ = ratio of galvanometer period to seismometer period, σ = the coupling factor, giving a measure of the reaction, $u = T_e / T_1$ = ratio of earth period to seismometer period, and f is related to the other parameters as given by—

$$\frac{u^2}{f^2} = \left[1 - \left\{ \left(1 - \frac{1}{\nu^2} \right) + + 4\,h_1\,h_2\,\frac{1}{\nu}\,(1-\sigma^2) \right\} u^2 + + \frac{1}{\nu^2} u^4 \right]^2 + \left[-2\left(h_1 + \frac{1}{\nu}\,h_2 \right) u + + 2\left(\frac{1}{\nu}\,h_1 + h_2 \right) \frac{1}{\nu}\,u^3 \right]^2$$

(*see* Fig. 1)

In short, Mag $= Q \times f$, where $Q = (M\,H\,G\,g\,L\,\mu) / (K\,k\,n_1\,Z_{11})$. The function f gives the nature of the frequency response curve and Q the measure of the magnification of the seismograph. The peak magnification is given by—

$$(\text{Mag})_{\text{peak}} = Q \times f_{\max} \qquad (1a)$$

Computations show that f_{max} is higher for (*i*) lower value of h_1, (*ii*) lower value of h_2 and (*iii*) higher value of σ. Its dependance on ν is not considered here. In practice, however, values of h_1, h_2 and ν are decided upon by the nature of responses curve required for the particular type of study. The value of σ does, however, vary with the attenuation, as will be shown later. Hence after h_1, h_2 and ν are fixed for the seismographs, the peak magnification is seen to depend, among other factors, on μ. Though theoretically

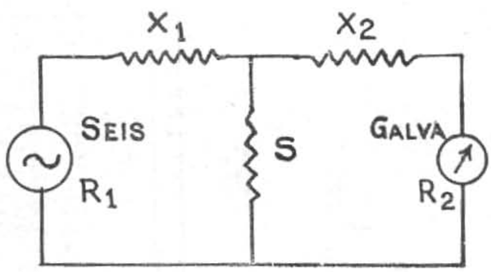

Fig. 1

μ can take values from zero to one, the constants of the seismometer and the galvanometer restrict, in practice, its range. These limitations, thus, prevent a continuons adjustment of the sensitivity of the seismograph. Neumann (1956) has studied this problem while providing sensitivity controls to Sprengnether seismographs. These seismographs, however, are provided with extra damping magnets and so Neumann's studies were restricted to this special case. When no extra damping magnets are provided, as in most electromagnetic seismographs, and when the damping of the seismometer is provided only by the resistances in the circuit, these limitations take a different form.

3. Limits of μ

In the general case considered, the seismometer and galvanometer are connected as shown in Fig. 1.

In Fig. 1, R_1 = Resistance of the seismometer, R_2 = Resistance of the galvanometer, X_1, X_2 and S are resistances which constitute the T-type attenuator.

Further, let
$$\left.\begin{aligned} R_1 + X_1 &= Z_1 \\ \text{and} \quad R_2 + X_2 &= Z_2 \end{aligned}\right\} \qquad (2)$$

then, the attenuation factor μ is given by

$$\left.\begin{aligned} \mu &= S \,/\, (S + Z_2) \\ \text{Also} \quad Z_{11} &= Z_1 + (S\, Z_2) \,/\, (S + Z_2) \\ \text{and} \quad Z_{22} &= Z_2 + (S\, Z_1) \,/\, (S + Z_1) \end{aligned}\right\} \qquad (3)$$

Z_{11} is as defined earlier and Z_{22} is the corresponding quantity for the galvanometer.

In terms of Z_{11}, Z_{22} and μ, we have for the circuit—

$$\left.\begin{aligned} S &= (\mu Z_{22}) / \left\{1 - \mu^2 (Z_{22} / Z_{11})\right\} \\ \text{or } S \,/\, Z_{22} &= \mu \,/\, (1 - \alpha\, \mu^2), \\ Z_1 &= (Z_{11} - \mu\, Z_{22}) \,/ \\ &\quad \left\{1 - \mu^2 (Z_{22} / Z_{11})\right\} \\ \text{or } Z_1 / Z_{11} &= (1 - \alpha\, \mu) \,/\, (1 - \alpha\, \mu^2), \\ \text{and } Z_2 &= \left\{Z_{22} (1 - \mu)\right\} / \\ &\quad \left\{1 - \mu^2 (Z_{22} \,/\, Z_{11})\right\} \\ \text{or } Z_2 \,/\, Z_{22} &= (1 - \mu) \,/\, (1 - \alpha\, \mu^2) \end{aligned}\right\}$$

(4 a, b, c)

In the above equations $\alpha = Z_{22}/Z_{11}$. Since the dampings of the seismometer and galvanometer are provided mostly by the resistances Z_{11} and Z_{22}, their values depend on the values of h_1 and h_2. The equations (4 a,b,c) enable the calculation of the resistances X_1, X_2 and S of the attenuator for any desired value of μ.

In practice, however, none of the above resistances can be less than zero. Hence Z_1 cannot be less than R_1 and Z_2 cannot be less than R_2. If for any value of μ, X_1 and X_2 come out to be negative, that value of μ cannot be attained. Thus, the three conditions which set the limits to the value of μ are—

$$S \nless 0 \quad \text{or} \quad \mu \,/\, (1 - \alpha\, \mu^2) \nless 0 \qquad 5(a)$$

$$Z_1 \nless R_1 \text{ or } (1 - \alpha\, \mu) \,/ (1 - \alpha\, \mu^2) \nless R_1 \,/\, Z_{11} \qquad 5(b)$$

$$Z_2 \nless R_2 \text{ or } (1 - \mu) \,/\, (1 - \alpha\, \mu^2) \nless R_2 \,/\, Z_{22} \qquad 5(c)$$

Let us consider them one by one.

(I) $S \nless 0$ or $\mu \,/\, (1 - \alpha\, \mu^2) \nless 0$

Since μ is never greater than one and α by its very nature cannot be negative, it is seen that S can become negative only if $\alpha > 1$. This happens when $\alpha\, \mu^2 > 1$ or $\mu > (1 \,/\, \alpha)^{\frac{1}{2}}$. If $\alpha < 1$, S is positive for all possible values

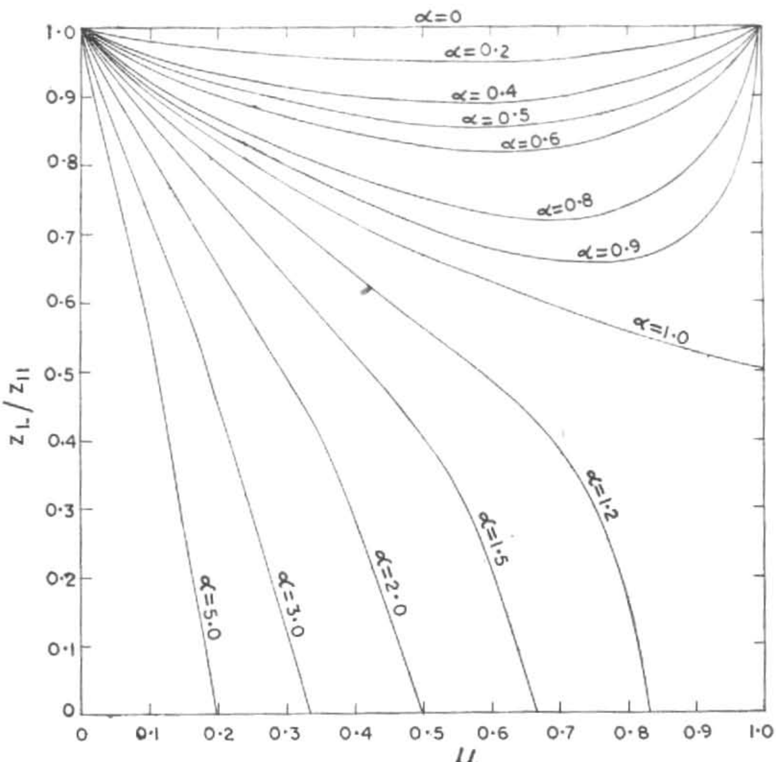

Fig. 2

of μ. Hence the condition (5a) gives the restriction—

$$\mu \not> (1/\alpha)^{\frac{1}{2}} \text{ when } \alpha > 1 \qquad (6)$$

It will be shown later, however, that this limitation does not operate in practice.

(II) Consider the condition (5 b), *i.e.*,

$$(1 - \alpha\mu) / (1 - \alpha\mu^2) \not< R_1 / Z_{11}$$

Put $\alpha_1 = R_1 / Z_{11}$; then at the limiting values of μ

$$1 - \alpha\mu = \alpha_1 (1 - \alpha\mu^2)$$

which gives two values of μ say, μ_1. μ_2

$$\mu_{1,2} = \frac{1}{2}\left[\frac{1}{\alpha_1} \pm \sqrt{\frac{1}{\alpha_1{}^2} - \frac{4(1-\alpha_1)}{\alpha\alpha_1}}\right] \qquad (7)$$

(It will be seen later that the two values do not always come in the domain of interest. Whenever they do, the lower value will be denoted by μ_1 and the higher value by μ_2. If we get only one value, it will be denoted by μ_1). At these two values of μ, Z_1 is equal to R_1 and so $X_1 = 0$. It can be shown that if μ takes a value lying between the two values given by eqn. (7), Z_1 becomes less than R_1; hence μ cannot be set to values in this range.

These restrictions on the value of μ may be seen also from Fig. 2 which shows the variation of Z_1/Z_{11} with μ for various values of α. These curves have been drawn on the basis of (4b), and show that when $\alpha<1$, Z_1/Z_{11} is equal to one at $\mu=0$, decreases as μ increases, passes through a minimum value and then increases again to the value one when $\mu=1$. Z_1/Z_{11} attains a minimum value when—

$$\mu = (1/\alpha)(1 - \sqrt{1-\alpha})$$

and the minimum value of Z_1/Z_{11} is given by—

$$(Z_1/Z_{11})_{\text{min.}} = \alpha/\left\{2\,(1-\sqrt{1-\alpha})\right\} \quad (8)$$

If, however, $\alpha \geqslant 1$, Z_1/Z_{11} decreases continuously from one as μ increases. Further, Z_1/Z_{11} has a minimum value of 0·5 at $\mu=1$, if $\alpha=1$; whereas when $\alpha > 1$, it reaches zero at a value of μ given by $1/\alpha$. If, therefore, a line is drawn parallel to the μ axis giving $Z_1/Z_{11} = \alpha_1$, the intersection of this line with the curves gives the limiting values of μ. All values of μ at which the curves are below this line are not possible in practice.

Another point of interest which can be seen from Fig. 2 is that while μ cannot be set equal to one when $\alpha > 1$, there may not be any restrictions on its value when $\alpha \leqslant 1$, if R_1/Z_{11} at the same time happens to be less than $(Z_1/Z_{11})_{\text{min.}}$ as given by eqn. (8).

The restrictions imposed by the condition (5b) on μ may, therefore, be summed up as below—

(*a*) $\alpha > 1$. μ cannot exceed the value given by eqn. (7). It may be pointed out here that this value of μ is less than $(1/\alpha)^{\frac{1}{2}}$ (eqn. 6). Hence it turns out that the restriction imposed by eqn. (6) is practically inoperative.

(*b*) $\alpha = 1$. (*i*) μ cannot exceed the value given by eqn. (7) if R_1/Z_{11} is greater than 0·5, (*ii*) μ can have all values if $R_1/Z_{11} \leqslant 0.5$.

(*c*) $\alpha < 1$. (*i*) μ cannot take values between those given by eqn. (7) if $R_1/Z_{11} > (Z_1/Z_{11})_{\text{min.}}$ and (*ii*) μ can have all values if $R_1/Z_{11} \leqslant (Z_1/Z_{11})_{\text{min.}}$

(III) Let us now consider the third condition (5c), which stipulates that $(1-\mu)\,/\,(1-\alpha\mu^2) \not< R_2/Z_{22}$

Putting $\alpha_2 = R_2/Z_{22}$, we have in the limit

$$1-\mu = \alpha_2\,(1-\alpha\mu^2)$$

which gives the two values, which we shall denote by μ_3 and μ_4

$$\mu_{3,4} = \tfrac{1}{2}\left[\frac{1}{\alpha\alpha_2} \pm \sqrt{\frac{1}{\alpha^2\alpha_2^2} - \frac{4(1-\alpha_2)}{\alpha\alpha_2}}\,\right] \quad (9)$$

(The same convention will be applied to μ_3 and μ_4 as for μ_1 and μ_2). At these two values of μ, $Z_2 = R_2$ and so $X_2 = 0$. As in the case of X_1, X_2 is also seen to become negative when μ takes a value in between the two given by eqn. (9) above. Here again, therefore, μ cannot be set to values in this range.

Curves of Z_2/Z_{22} against μ have been drawn, on the basis of eqn. (4c) and are shown in Fig. 3. This figure brings out the above points. Fig. 3 is, in many respects, similar to Fig. 2 and shows that when $\alpha < 1$, Z_2/Z_{22} decreases continuously from one at $\mu=0$ to zero when $\mu=1$. If $\alpha=1$, Z_2/Z_{22} also decreases continuously but attains the lowest value equal to 0·5 at $\mu=1$. This curve for $\alpha=1$ is identical to the corresponding curve in Fig. 2 and shows that if $\alpha=1$, $Z_1/Z_{11} = Z_2/Z_{22}$ for the same value of μ. For values of $\alpha > 1$, Z_2/Z_{22} decreases from one at $\mu=0$, attains a minimum value and then increases to the value one at $\mu=1$. The minimum value of Z_2/Z_{22} is given by—

$$(Z_2/Z_{22})_{\text{min}} = (1/2\alpha)\left\{1/(1-\sqrt{1-1/\alpha}\,)\right\} \quad (10)$$

and is attained when $\mu = 1-(1-1/\alpha)^{\frac{1}{2}}$.

Hence if a line is drawn giving $Z_2/Z_{22} = \alpha_2$, it is seen that there is always one intersection, giving one value of μ if $\alpha < 1$. Values of μ greater than this value result in negative values of X_2 and so are forbidden. Thus , there is invariably an upper limit to the usable value of μ when $\alpha < 1$. This limit approaches the theoretical maximum, *i.e.*, one, as α_2 $(=R_2/Z_{22})$ approaches zero. If $\alpha=1$, Z_2/Z_{22} has a minimum value of 0·5 in the domain of interest. Hence there can be no restriction on μ if $\alpha_2 \leqslant 0{\cdot}5$. If, however, $\alpha_2 > 0{\cdot}5$, there will be an intersection of the line $Z_2/Z_{22} = \alpha_2$ with the

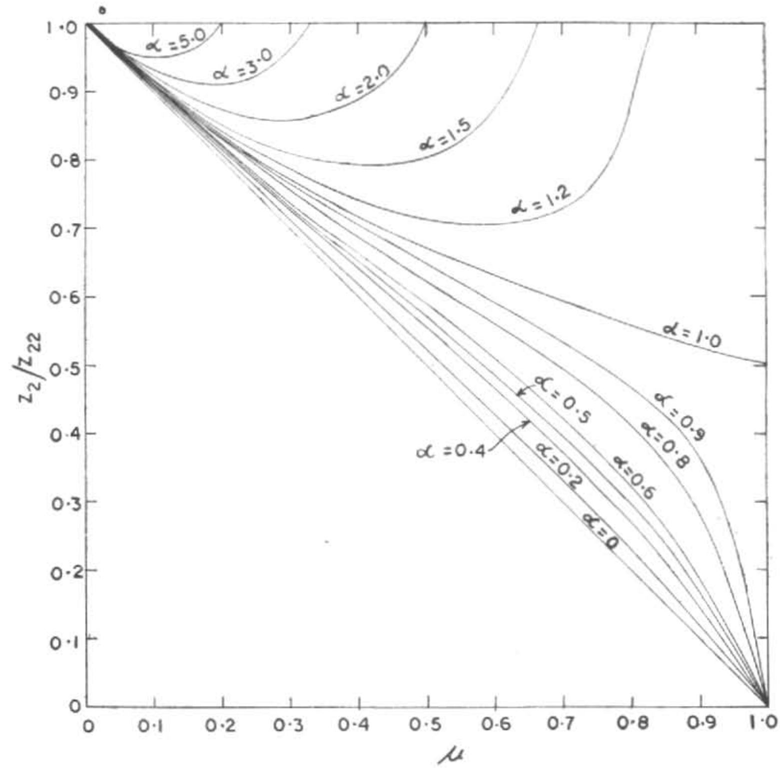

Fig. 3

curve and this would impose an upper limit to μ. If $\mu > 1$, there can again be no restrictions imposed on μ in the range 0 to $1/\alpha$ if $\alpha_2 \leqslant (Z_2/Z_{22})_{\text{min.}}$ given by eqn. (10). If $\alpha_2 > (Z_2/Z_{22})_{\text{min.}}$, there will be two values of $\mu - \mu_3, \mu_4$ at which $X_2 = 0$ and intermediate values of μ result in negative values of X_2. In this case, therefore, μ is restricted to the ranges $0 \sim \mu_3$ and $\mu_4 \sim 1/\alpha$.

The restrictions imposed by this condition may, therefore, be summed up as follows (*see* also Table 1)—

(*a*) $\alpha < 1$. μ cannot exceed a value given by eqn. (9).

(*b*) $\alpha = 1$. (*i*) μ can have all values if $\alpha_2 \leqslant 0 \cdot 5$, and (*ii*) μ cannot exceed the value given by eqn. (9) if $\alpha_2 > 0 \cdot 5$.

(*c*) $\alpha > 1$. (*i*) μ can have all values from $0 \sim 1/\alpha$, if $\alpha_2 \leqslant (Z_2/Z_{22})_{\text{min.}}$ and (*ii*) μ is restricted to the ranges $0 \sim \mu_3$ and $\mu_4 \sim 1/\alpha$, if $\alpha_2 > (Z_2/Z_{22})_{\text{min.}}$

In practice μ can take only those values which are allowed by all the conditions at the same time. These restrictions have been summarised and shown in Table 1. In the last column of the table the combinations of parameters, which permit a wide range of variation of μ, have been indicated. It is seen that the most desirable combination is the case when $\alpha = 1$ and R_1/Z_{11} and R_2/Z_{22} are just equal to or less than $0 \cdot 5$. This, however, is a very special case and may not be always easy to attain and maintain. The next in order and one which could be attained more easily is to have α slightly less than one and both R_1/Z_{11} and R_2/Z_{22} less than $0 \cdot 5$.

TABLE 1

Summary of restrictions on the attenuation factor, μ

	First condition $S \nless 0$	Second condition $X_1 < 0$	Third condition $X_2 \nless 0$	Remarks
$Z_{22}/Z_{11} < 1$	No restriction	(i) No restriction if $(R_1/Z_{11}) \leq (Z_1/Z_{11})$min (ii) μ cannot take values between μ_1 and μ_2 if $(R_1/Z_{11}) > (Z_1/Z_{11})$min	(i) μ cannot exceed μ_3 for any value of R_2/Z_{22}	Desirable combination: Low values of R_1/Z_{11} and R_2/Z_{22}, in particular $R_1/Z_{11} \leq (Z_1/Z_{11})$min
$Z_{22}/Z_{11} = 1$	No restriction	(i) No restriction if $R_1/Z_{11} \leq 0{\cdot}5$ (ii) μ cannot exceed μ_1 if $R_1/Z_{11} > 0{\cdot}5$	(i) No restriction if $R_2/Z_{22} \leq 0{\cdot}5$ (ii) μ cannot exceed μ_3 if $R_2/Z_{22} > 0{\cdot}5$	Desirable combination: R_1/Z_{11} and R_2/Z_{22} either equal to or less than $0{\cdot}5$
$Z_{22}/Z_{11} > 1$	(i) μ cannot exceed $(1/\alpha)^{\frac{1}{2}}$	(i) μ cannot exceed μ_1 for any value of R_1/Z_{11}	(i) μ can have all values between $0 \sim 1/\alpha$ if $R_2/Z_{22} \leq (Z_2/Z_{22})$min	Restriction by first condition is always contained in the restriction by third condition
			(ii) μ can have values only in the ranges $0 \sim \mu_3$ and $\mu_4 \sim 1/\alpha$ if $(R_2/Z_{22}) > (Z_2/Z_{22})$min	Desirable combination: Low values of R_1/Z_{11} and R_2/Z_{22}

One often comes across statements that Z_{11} is adjusted to be nearly equal to Z_{22}, *i.e.*, α is made nearly equal to one for the sake of matching resistances. It is clear from the foregoing discussion that as far as the range of possible values of the attenuation factor is concerned, the mere adjustment of α nearly to unity does not give any substantial advantage. Equally necessary, in this case, is to see that R_1/Z_{11} and R_2/Z_{22} are low and at least not higher than $0{\cdot}5$.

4. Seismograph satisfying the Galitzin conditions

In the previous section, the limits imposed on the value of μ were considered for a seismograph in general. One of the combinations which is very widely used in operating e.m. seismographs is the well known Galitzin condition. It is, therefore, considered worthwhile to discuss this special case separately.

In this combination $T_1 = T_2$ and $h_1 = h_2 = 1$ (T_2 is the free period of the galvanometer). When these conditions are satisfied, Z_{11} represents the sum of the external critical damping resistance of the seismometer and its internal resistance. Z_{22} represents the corresponding quantity for the galvanometer. Further, under these conditions, the effect of the reaction of the galvanometer is very pronounced. Computation of the function f of eqn. (1) shows that as the coupling factor increases, the effect of resonance shows itself in the response curve. In practice, therefore, it is found undesirable to have the coupling factor σ larger than about 1/3.

The coupling factor σ, which is limited to the range $0 \sim 1$, is given by the relation—

$$\sigma^2 = \frac{(h_1 - h_{01})\ (h_2 - h_{02})}{h_1\ h_2} \cdot \frac{Z_{22}}{Z_{11}}\ \mu^2 \quad (11)$$

where h_1 and h_2 are the dampings of the

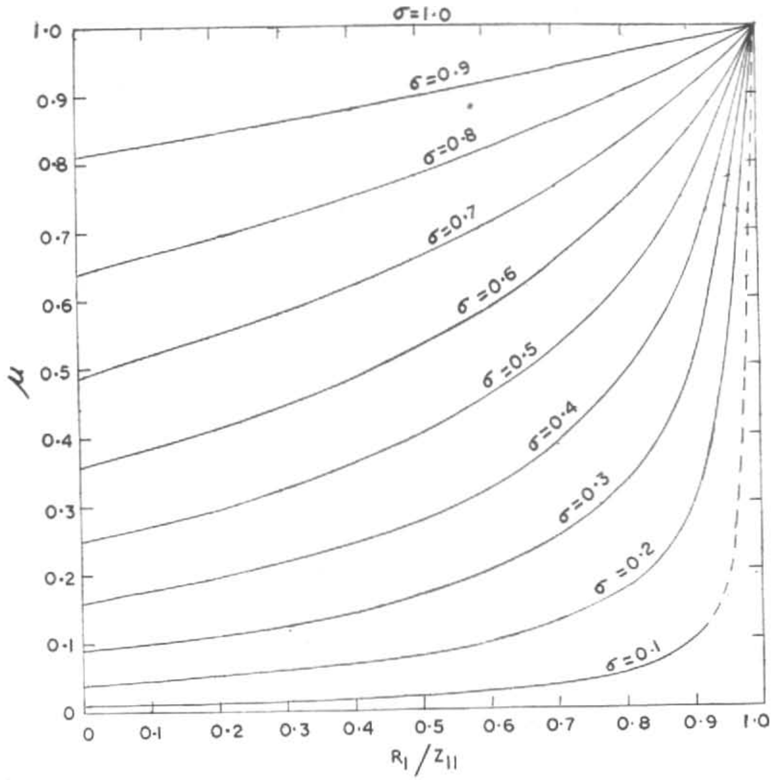

Fig. 4

seismometer and galvanometer (here both are equal to one), h_{01} = open circuit damping of seismometer; h_{02} = open circuit damping of galvanometer; Z_{11}, Z_{22}=critical damping of resistances of seismometer and galvanometer (external + internal) and μ = attenuation factor (Hagiwara 1958). When no extra damping magnets are provided in the seismometer, the open-circuit damping happens generally to be very small to be neglected in comparison with $h_1 = 1$. h_{02}, the open circuit damping of the galvanometer also happens to be negligible. Hence the coupling factor may be expressed as—

$$\sigma^2 \approx (Z_{22}/Z_{11})\,\mu^2 \qquad (12)$$

It was seen, however, that μ can take the value one only if Z_{22}/Z_{11} is equal to one; and then σ is also equal to one. It is, therefore, clear that if σ is kept less than one, μ cannot be set equal to one. The range of values in which μ can be set is, of course, dependant on the values of R_1/Z_{11}, R_2/Z_{22} and α as discussed earlier.

The above limitations and the calculation of the optimum values of the parameters may also be studied by the introduction of the coupling factor σ into the conditions 5 (a,b,c). When this is done in accordance with eqn. (12), the above conditions may be written as—

$$S/Z_{22} = \mu/(1-\sigma^2) < 0 \qquad (13a)$$

$$Z_1/Z_{11} = (1-\sigma^2/\mu)/(1-\sigma^2) \not< R_1/Z_{11} \qquad (13b)$$

$$Z_2/Z_{22} = (1-\mu)/(1-\sigma^2) \not< R_2/Z_{22} \qquad (13c)$$

As long as the value of σ has to be adjusted to a particular value, the conditions

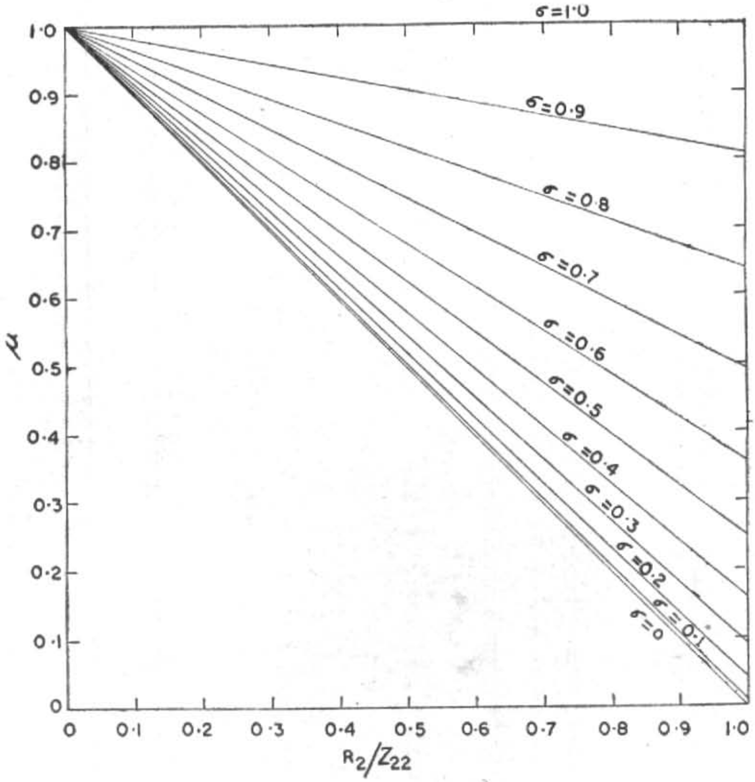

Fig. 5

13 (a,b,c) put the following restrictions on μ.

(*a*) No restriction

(*b*) $\mu \nless \sigma^2/\{1-\alpha_1(1-\sigma^2)\}$ where $\alpha_1=R_1/Z_{11}$

(*c*) $\mu \ngtr 1-\alpha_2(1-\sigma^2)$ where $\alpha_2=R_2/Z_{22}$.

It is, therefore, seen that the seismometer restricts μ on the lower side whereas the galvanometer does so on the higher side. These limitations may be found out from Figs. 4 and 5 where the variation with μ of Z_1/Z_{11} and of Z_2/Z_{22} has been shown for various values of σ.

The significance of these limits is that once α_1 is fixed, the seismometer cannot work with an attenuation less than a certain value

$$\mu = \sigma^2 / \{1-\alpha_1(1-\sigma^2)\}$$

and yet give the predetermined value of σ, no matter how we change the other parameters. Similarly, once α_2, *i. e.*, the galvanometer is fixed, μ cannot exceed the value

$$\mu = 1-\alpha_2(1-\sigma^2)$$

and yet give the value of σ, however we may change the other parameters. It is this latter case that is of interest in practice in calculating the optimum values of the constants of a seismometer. This is illustrated by the following example.

Let us start with a galvanometer with (say) $R_2 = 75$ ohms and $Z_2 = 300$ ohms and let us fix $\sigma=0\cdot3$. Then Fig. 5 give the upper limit of μ at $0\cdot77$. If we try to set the seismograph at μ equal to a higher value, say $0\cdot9$, to get the same value of σ, we must have—

$$Z_2/Z_{22} = 0\cdot11$$

which is less than α_2 and hence this value of μ is not physically possible.

It is to be noted that the above considerations are of interest only at the time of choosing the instrument. If it is decided, say, to operate the instrument at the maximum value of $\mu = 0\cdot77$ in the above example, then to get $\sigma = 0\cdot3$ we must have—

$$(Z_{22}/Z_{11})\ \mu^2 = \sigma^2$$

$$\text{or } Z_{11} = 1975 \text{ ohms.}$$

With these values, if the seismograph is set at a lower μ the value of σ will be less. The range of μ, in which the seismograph can be set, now depends on the value of α_1. If α_1 is less than $(Z_1/Z_{11})_{\min}$, *i.e.*, $0\cdot93$, μ can take all values from 0 to $0\cdot77$. This would require $R_1 \leqslant 1840$ ohms. In this way all the constants are specified.

The Peak Magnification—It may not be out of place here to see how the peak magnification is related to the above considerations. Going back again to the expression for this, we see that

$$(\text{Mag})_{\text{peak}} = \frac{gL}{k} \cdot \frac{MHG}{K\, n_1 Z_{11}}\ \mu f_{\max}$$

If σ is always maintained at a particular value, as discussed above, $f_{\max}$ is fixed and so—

$$(\text{Mag})_{\text{peak}} \propto \frac{gL}{k} \cdot \frac{MHG}{K\, n_1 Z_{11}} \cdot \mu$$

Under the assumption that h_{01} is negligible, it can be shown that

$$Z_{11} = G^2/2\, n_1 K \qquad (14)$$

Substituting this value of G and using eqn. (12) we get—

$$(\text{Mag})_{\text{peak}} \propto \frac{gL}{k} \times MH\ (2/n_1 K)^{\frac{1}{2}} \times \sigma/(Z_{22})^{\frac{1}{2}} \qquad (15)$$

It is thus seen that the magnification of the seismograph remains the same at whatever value of μ it is set, as long as the same galvanometer is used and the same value of σ is maintained. This is clarified by the example given below.

Let us again take the galvanometer with resistance $R_2 = 75$ ohms and $Z_{22} = 300$ ohms and fix $\sigma = 0\cdot3$. We have seen that the maximum value of μ that can be used in this case is $0\cdot77$ and we can set the seismograph at its peak magnification at this value of μ by choosing $Z_{11} = 1975$ ohms.

With the same galvanometer, it is also possible to set the seismograph at its peak magnification with the attenuation at a different value. If say, we set $\mu = 0\cdot5$, to get the same peak magnification we should have—

$$(300/Z_{11}) \times (0\cdot5)^2 = (0\cdot3)^2$$

$$\text{or } Z_{11} = 833 \text{ ohms.}$$

From the above examples it is seen that it is possible to choose the seismometer constant Z_{11} to give the peak magnification at any possible value of μ. How the value of Z_{11} can be adjusted to a desired value is indicated by its relation to the moment of inertia of the pendulum K and the electrodynamic constant G of the seismometer in eqn. (14) and also from the discussion in Sec. 5. Once the value of Z_{11} is fixed, the magnification of the seismograph can be reduced by reducing the value of μ, when the coupling factor σ will also decrease.

5. Adjustments for maximum peak magnification

We have just discussed about the peak magnification of a seismograph operating in the Galitzin condition. Let us now consider the case in general without any restriction on the value of σ. It was pointed out earlier that the peak value of function f in eqn. (1)

is higher for higher values of σ and since σ is directly proportional to μ, the value of the product $\mu f_{\max}$ is maximum at the maximum value of μ. This maximum value of μ, as we have seen, is fixed by the value of Z_{11}. (We assume here that the parameters of the galvanometer and the values of n_1 and h_1 are fixed). If Z_{11} is also fixed, magnification given by

$$(\text{Mag})_{\text{peak}} = (gL/k) \times (\mu f_{\max}/Z_{11}) \times \times (1/n_1) \times (MHG/K)$$

is seen to depend on the value of (MHG/K). (K/MH) is the well-known length of the equivalent simple pendulum. The expression thus indicates that of two instruments that will give the higher magnification which has (1) the smaller value of this length of the equivalent simple pendulum or (2) the larger value of G. The condition (1) above is the same as obtains in mechanical seismographs. Let us look further into the condition, number (2) above. If the coil is wound on a cylindrical bobbin, we may write, for a hinged seismometer—

$$G = 2\pi r n F l \qquad (16)$$

where, n=Number of turns of wire in the coil, r=Mean radius of the coil, F=Strength of the magnetic field in the air gap, and l= Distance of the centre of the coil from the axis. If further we assume that the volume V of material used in the coil (say copper) is held constant, G can be expressed in terms of R_1 as

$$G = F\, l\, (V/s)^{\frac{1}{2}} \times (R_1)^{\frac{1}{2}} \qquad (17)$$

Here s is the specific resistance of the material of the wire (say copper).

G can be increased, therefore, either by (*i*) increasing the resistance of the coil or by (*ii*) fixing the coil farther away from the hinge. However, with a fixed value of Z_{11}, any increase in R_1 would increase the value of R_1/Z_{11}. This tends to reduce the range of μ. On the other hand, an increase of l could give the same increase in G without restricting the range of μ. The increase in G does, however, increase Z_{11} of the pendulum, which will need slight alteration to keep Z_{11} constant. The other consideration in the latter case, is the dimension of the seismometer, which has got to be kept within reasonable limits. It may still be pointed out that the presence of l in the eqns. (16) and (17) gives an additional factor with which G could be altered. This appears to be an advantage of the hinged seismometer over the linear motion seismometer.

6. Application to linear motion seismometers

The discussions till now have related to a hinged seismometer. Even if, the seismometer involves only linear motion, as in the case of instruments employing the Willmore suspension system, the conclusions drawn above mostly apply. In this case the equations of motion of the seismograph change only in so far as $H \to \infty$, $K \to MH^2$ and the linear motion $x \to H\theta$. Consequently the magnification is given by—

$$\text{Mag} = (G g \mu L)/(k n_1 Z_{11}) \times f(h_1, h_2, v, \sigma, u)$$

The nature and behaviour of the function f remains the same and the main discussions and results of Sections 2, 3 and 4 hold good.

Results of Sec. 5 are, however, different. Since now,

$$(\text{Mag})_{\text{peak}} = (gL/k) \times (1/n_1) \times (\mu f_{\max}/Z_{11}) \times G$$

the mechanical quantities relating to the pendulum which brought the length of the equivalent simple pendulum into consideration are not explicitly present. Hence the only way of raising the overall magnification is by raising the value of G. Here

$$G = 2\pi r n F$$
$$= F\,(V/s)^{\frac{1}{2}}\,(R_1)^{\frac{1}{2}}$$

and if the magnetic field in the air gap of the transducer is fixed, G can be increased only by increasing the resistance of the coil, R_1. This suffers from the same limitations which were pointed out in the case of the hinged seismometer. As already mentioned, the additional factor l present in the hinged seismometer, is not available.

7. Summary and Conclusions

The discussions in the preceeding pages reveal that in an electromagnetic seismograph, without extra damping magnets and recording with the help of a galvanometer, there are restrictions on the value of the attenuation factor that can be used. The most favourable combination of parameters, which permit the use of the full range of μ from 0 to 1 are seen to be (*i*) $Z_{11} = Z_{22}$ and (*ii*) R_1/Z_{11} and R_2/Z_{22} are equal to or less than 0·5. If Z_{22} is less than Z_{11}, the optimum conditions are found to lie in having low values of R_1/Z_{11} and R_2/Z_{22}. The full range of μ is, however, not available in this case. It does not appear desirable to have Z_{22} greater than Z_{11}.

From practical considerations, it is seen that in a seismograph operating in the Galitzin condition, the full value of μ cannot be used and that by suitably choosing the values of Z_{11} and R_1 it is possible to get the optimum magnification of the instrument at any value of μ within a range depending on the values of R_1/Z_{11} and R_2/Z_{22}.

The conditions relating to the seismometer for increased magnification of an e.m. seismograph in general are found to be (1) a low value of the length of the equivalent simple pendulum and (2) a high value of the transducer constant. Even though a higher value of G could be attained by increasing the coil resistance R_1, it is seen that it is more advantageous to get the same effect by increasing, if possible, the distance of the coil from the hinge. The discussions mostly apply also to linear motion seismometers. This latter type of seismometer, however, suffers from the limitation, in that the transducer constant can be increased only at the risk of restricting the range of variation of μ.

8. Acknowledgement

The author is thankful to Dr. A. N. Tandon, Director (Seismology) for the stimulating discussions he had with him and for his suggestions during the study.

REFERENCES

Chakrabarty, S. K.	1949	*Bull. seismol. Soc. Amer.*, **39**, pp. 205-218.
	1960	*Proc. Nat. Inst. Sci.*, India, 26, A (Suppl. II), 133-142.
Chakrabarty, S. K. and Tandon, A. N.	1961	*Bull. seismol. Soc. Amer.*, **51**, pp. 111-125.
Coulomb, J. and Grenet, G.	1935	*Ann. Phys., Paris*, Ser. 11, 321-369.
Eaton, J. P.	1957	*Bull. seismol. Soc. Amer.*, **47**, pp. 37-75.
Eaton, J. P. and Byerly, P.	1957	*Ibid.*, **47**, pp. 155-166.
Espinosa, A.F., Sutton, G. H. and Miller, H. J.	1962	*Ibid.*, **52**, pp. 767-779.
Hagiwara, T.	1958	*Bull. Earthq. Res. Inst. Tokyo*, **36**, pp. 139-162.
Neumann, F.	1956	*Trans. Amer. geophys. Un.*, **37**, pp. 483-490.
Rybner, J.	1937	*Beitr. Geophys.*, **51**, pp. 375-401.
Schmerwitz, G.	1936	*Z. Geophys.*, **12**, 206.
Wenner, F.	1929	*Bur. Stand. J. Res., Wash.*, **2**, Res. Pap. 66, pp. 963-999.
Willmore, P. L.	1959	*Bull. seimol. Soc. Amer.*, **49**, pp. 99-114.