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# Constant response sensitivity adjustments of electromagnetic seismographs

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ABSTRACT. The conditions under which the magnification of an electromagnetic seismograph can be changed without altering its response characteristics are discussed. It is seen that this can be done only if (1) the seismograph is made up of a seismometer and galvanometer of equal free periods and (2) the dampings of the seismometer and the galvanometer are also suitably altered. When sensitivity adjustments are so made, the maximum magnification is seen to be attained when the damping of the seismometer is equal to that of the galvanometer and this does not generally go with the maximum value of $\mu$, the attenuation factor.

## 1. Introduction

In a recent study, Chaudhury (1965) has discussed the use of an attenuator in the circuit of an electromagnetic seismograph and the limits of the values which $\mu$, the attenuation factor, can take in the normal practice of maintaining the damping constants of the galvanometer and seismometer unchanged during attenuation. These values were found to depend on the internal and critical damping resistances of the galvanometer and seismometer. The favourable combinations of these parameters which would enable a continuous variation of $\mu$ within wide limits were also indicated.

Although the practice of keeping the seismometer and galvanometer dampings constant while changing the attenuation factor serves the main purpose of adjusting the magnification of the seismograph, it does not strictly maintain the response characteristics of the instrument. Hence, if the response of the instrument has been determined for a particular value of $\mu$, and the same response curve is used for the other values of $\mu$ also, the evaluation of the magnitudes of ground motions becomes erroneous, though generally only to a small extent. The present study is related to the use of the attenuator in the circuit of an electromagnetic seismograph under constant response characteristics. In this, the notations used in the author's earlier study have been retained and the method adopted is on the lines of that used by Tobyas (1963) in his study of the influence of the galvanometer reaction on the response characteristics of electromagnetic seismographs.

## 2. Attenuation under constant damping

The circuit in Fig. 1 shows the T-type attenuator normally used in the circuit of a seismograph. $R_1$ is the internal resistance of the seismometer and $R_2$ that of the galvanometer. The attenuation factor $\mu$ is defined by

$$\mu = S/(S+Z_2) \qquad (1)$$

where $Z_2 = X_2 + R_2$, and the magnification for displacement of the seismograph is given by

$$\text{Mag} = \left(\frac{MHGgL\mu}{Kkn_1Z_{11}}\right) \times f\,(h_1,\, h_2, v, \sigma, u) \qquad (2)$$

In the above equation, the factor within the brackets on the R.H.S. gives a measure of the magnitude of the magnification and the function gives the nature of the response curve.

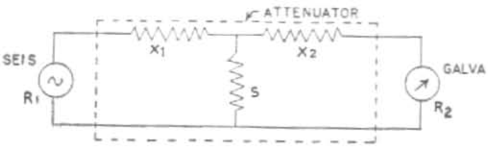


Fig. 1

It may, therefore, be seen that even if $h_1$, $h_2$ and $v$ are kept constant, when $\sigma$ is altered, the nature of the response curve is changed. The reaction $\sigma$ given by

$$\sigma^2 = \frac{(h_1-h_{01})\ (h_2-h_{02})}{h_1\ h_2}\ \frac{Z_{22}}{Z_{11}}\ \mu^2 \qquad (3)$$

changes in direct proportion to the attenuation factor, $\mu$. The extent of the change in the response curve of the instrument due to this change in $\sigma$ has been shown in a set of curves given by Hagiwara (1958) for various combinations of parameters. They clearly show that this change happens to be pronounced in the case of the most widely used combination, *viz.*, $h_1=h_2=1$ and $n_1=n_2$ (Fig. 2). However, for the very combinations in which $n_1=n_2$, it is seen that attenuation can be made under constant response. This is worked out in the next section.

### 3. Attenuation under constant response

Let us take a seismograph with the following constants: seismometer free period $T_1=2\pi/n_1$, seismometer damping $h_1$, galvanometer free period $T_2=2\pi/n_2$ and damping $h_2$, attenuation factor $\mu$ and reaction $\sigma$. The equations of motion of the seismometer and the galvanometer may be written in the standard form

$$\left.\begin{aligned} \frac{d^2\theta}{dt^2}+2\epsilon_1\frac{d\theta}{dt}+n_1^2\theta-2\epsilon_1\sigma_1\frac{d\phi}{dt} &= -\frac{MH}{K}\ \frac{d^2x}{dt^2} \\ \frac{d^2\phi}{dt^2}+2\epsilon_2\frac{d\phi}{dt}+n_2^2\phi-2\epsilon_2\sigma_2\ \frac{d\theta}{dt} &= 0 \end{aligned}\right\} \qquad (4)$$

Eliminating $\theta$ from (4), we get for the response of the galvanometer the equation

$$\frac{d^4\phi}{dt^4}+A\ \frac{d^3\phi}{dt^3}+B\frac{d^2\phi}{dt^2}+C\ \frac{d\phi}{dt}+D\phi=-E\ \frac{d^3x}{dt^3} \qquad (5)$$

$$\left.\begin{aligned} \text{where, } A &= 2\ (\epsilon_1+\epsilon_2) \\ B &= n_1^2+n_2^2+4\epsilon_1\epsilon_2\ (1-\sigma^2) \\ C &= 2\ (\epsilon_1 n_2^2+\epsilon_2 n_1^2) \\ D &= n_1^2\ n_2^2 \\ E &= \frac{MH}{K}\cdot 2\ \epsilon_2\sigma_2 \end{aligned}\right\} \qquad (6)$$

The solution of (5) giving $\phi$ as a function of time is the response of the galvanometer for any ground motion $x(t)$.

If we take a second seismograph whose parameters are denoted by primed quantities ($n_1'$, $n_2'$ etc), its response for the same ground motion is given by the solution of an equation like (5), *viz.*,

$$\frac{d^4\phi}{dt^4}+A'\ \frac{d^3\phi}{dt^3}+B'\frac{d^2\phi}{dt^2}+C'\frac{d\phi}{dt}+D'\ \phi=-E'\ \frac{d^3x}{dt^3}$$

Where $A'$, $B'$, $C'$, $D'$ and $E'$ are also given by (6) with primes. The response of the two systems will be same if $A=A'$, $B=B'$, $C=C'$ and $D=D'$. The quantities $E$ and $E'$ only affect the absolute magnification. The condition for conformable response of the two systems may, therefore, be put down as

$$\left.\begin{aligned} \epsilon_1+\epsilon_2 &= \epsilon_1'+\epsilon_2' \\ n_1^2+n_2^2+4\epsilon_1\epsilon_2(1-\sigma^2) &= n_1'^2+n_2'^2+ \\ &\quad +4\epsilon_1'\epsilon_2'(1-\sigma'^2) \\ \epsilon_1 n_2^2+\epsilon_2 n_1^2 &= \epsilon_1' n_2'^2+\epsilon_2' n_1'^2 \\ n_1 n_2 &= n_1' n_2' \end{aligned}\right\} \qquad (7)$$

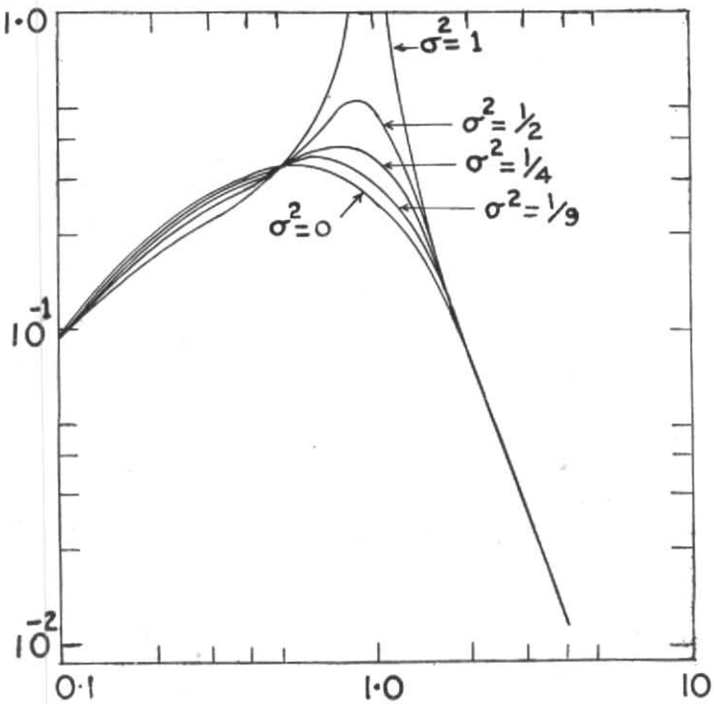


**Fig. 2. Response curves of seismograph for various values of the reaction ($\sigma$) for the case $n_1 = n_2$, $h_1 = h_2$**

Applied to the case of the same seismograph with different values of $\mu$ (and so of $\sigma$), we have $n_1 = n_1'$ and $n_2 = n_2'$ and equations (7) then give the two following solutions—

$$(i)\ \epsilon_1 = \epsilon_1',\ \epsilon_2 = \epsilon_2' \text{ and } (ii)\ n_1 = n_2$$

It may be mentioned that the solution (*ii*) is one of the conditions which were found necessary (Chakravarty 1960) for the analytical evaluation of the response.

The solution (*i*), *i.e.*, $\epsilon_1 = \epsilon_1'$ and $\epsilon_2 = \epsilon_2'$ gives $\sigma = \sigma'$; and with $n_1 = n_1'$ and $n_2 = n_2'$ this is identical to the first system and so is of no interest.

The other solution $n_1 = n_2$ (*i.e.*, the free periods of seismometer and galvanometer equal) leads to the following:

$$\left.\begin{aligned} h_1 + h_2 &= h_1' + h_2' = K_1 \quad \text{(say)} \\ h_1 h_2 (1-\sigma^2) &= h_1' h_2' (1-\sigma'^2) = K_2 \end{aligned}\right\} \quad (8)$$

In terms of $K_1$ and $K_2$, the parameters of the second system are given by

$$\left.\begin{aligned} h_2' &= K_1 - h_1' \\ \sigma'^2 &= 1 - \frac{K_2}{h_1'(K_1 - h_1')} \end{aligned}\right\} \quad (8a)$$

It is thus seen that for different values of $h_1'$, there are different corresponding values of $h_2'$, and $\sigma'$ and by adjusting the parameters to these values, it is possible to set the second system with response conformable to that of the first. It should be noted, however, that this can be done only if the free periods

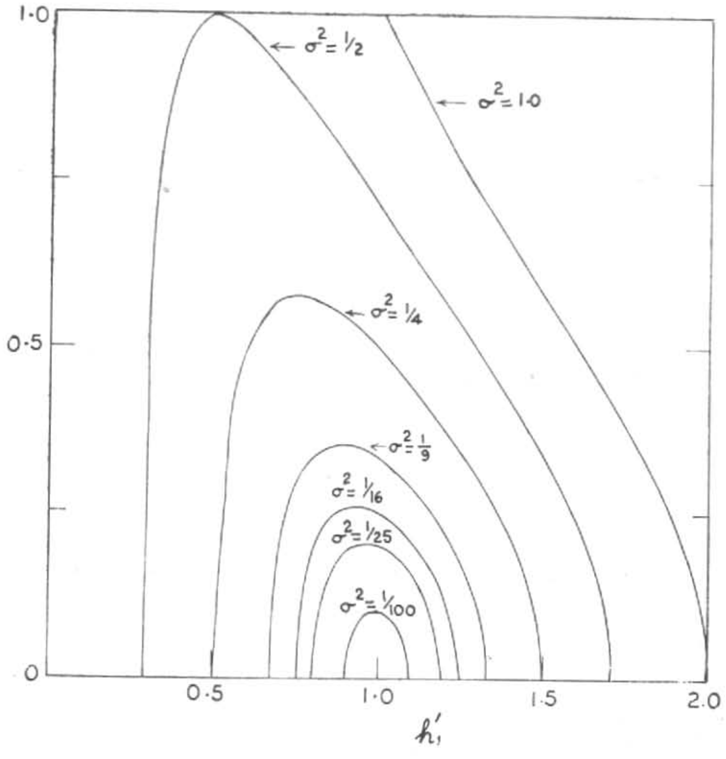


**Fig. 3. Variation of $\mu_1'$ with $h_1'$**
$h_1 = h_2 = 1, K_1 = 2, K_3 = 1$

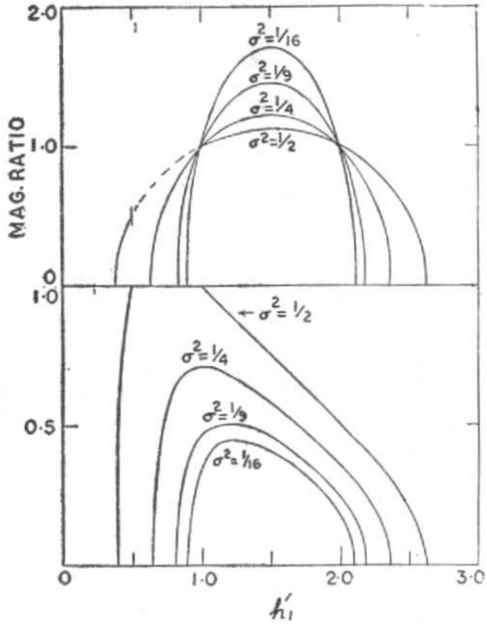


**Fig. 4. Variation of $\mu_1'$ and magnification ratio with $h_1'$**
$h_1 = 1$, $h_2 = 2$, $K_1 = 3$, $K_3 = 1$

of the seismometer and galvanometer are equal.

Let us introduce $\mu'$ in (8 a) by using equation (3). If it is assumed that the impedance in the seismograph circuit is only resistive, we may write

$$\left.\begin{aligned} h_1' &= h_{01} + \frac{a_1}{Z_{11}'} \\ h_2' &= h_{02} + \frac{a_2}{Z_{22}'}, \end{aligned}\right\} \quad (9)$$

where $a_1$ and $a_2$ are constants. $h_{01}$ and $h_{02}$ are respectively equal to $h_{01}'$ and $h_{02}'$ in the present case, and

$$\frac{Z_{22}'}{Z_{11}'} = K_3 \frac{h_1'-h_{01}}{h_2'-h_{02}} \text{ (where, } K_3 = a_2/a_1) \quad (10)$$

$$\text{Hence} \quad \sigma'^2 = K_3 \frac{(h_1'-h_{01})^2}{h_1'(K_1-h_1')} \mu'^2 \quad . (11)$$

and from (8 a) we get

$$\mu'^2 = \frac{1}{K_3} \frac{1}{(h_1'-h_{01})^2} \left[ h_1'(K_1-h_1') - K_2 \right] \quad (12)$$

In the absence of separate damping arrangements and except when the free periods of the seismometer and galvanometer are long $h_{01}$ and $h_{02}$, the open circuit dampings

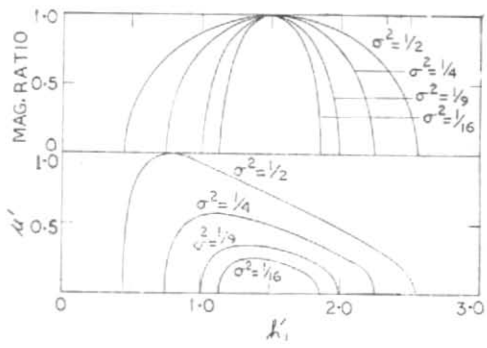


**Fig. 5. Variation of $\mu_1'$ and magnification ratio with $h_1'$**
$h_1 = h_2 = 1\cdot5$, $K_1 = 3$, $K_3 = 1$

of the seismometer and galvanometer are generally very small. If, therefore, in comparison with $h_1'$ and $h_2'$, they are neglected, we get

$$\mu'^2 \approx \frac{1}{K_3}\left[\frac{K_1 h_1' - K_2}{h_1'^2} - 1\right] \qquad (12a)$$

$$\text{Also, } K_3 = \frac{a_2}{a_1} = \frac{(Z_{22})_{h_2=1}}{(Z_{11})_{h_1=1}}$$

is then equal to the ratio of the total critical damping resistance of the galvanometer to that of the seismometer.

The values of $\mu'$ corresponding to different values of $h_1'$ have been calculated from equation (12a) for a few combinations of values of $K_1$ and $K_2$, and are shown in Figs. 3–6. $K_3$ has been taken equal to one in the above calculations, but as $\mu'$ is inversely proportional to $\sqrt{K_3}$, the curves can be used for any value of $K_3$. As $\mu'$ cannot be less than zero or greater than one, the curves have not been extended beyond these limits; values of $\mu'$ beyond the limits only indicating that the combinations are not physically possible. Apart from the above limits, the realisation of the attenuation factor $\mu'$ is also subject to the additional restrictions imposed by the values of $Z_{22}'/Z_{11}'$, $R_1/Z_{11}'$ and $R_2/Z_{22}'$ as discussed in the author's previous paper.

### 4. Relation between $\mu'$ and magnification

In the previous section, $\mu'$ was related to $h_1'$ and the other parameters. Since, however, along with the attenuation factor, the damping factors $h_1'$ and $h_2'$ and consequently $Z_{11}'$ and $Z_{22}'$ also need adjustments, the magnification of the seismograph does not vary directly as $\mu'$. Under constant response characteristics, from equation (2) we get for the ratio of the magnifications of the two systems

$$\frac{Mag'}{Mag} = \left[\frac{h_1'\,(K_1 - h_1') - K_2}{h_1\,(K_1 - h_1) - K_2}\right]^{\frac{1}{2}} \qquad (13)$$

The magnification ratio, given by (13), has been calculated for the combinations of $K_1$, $K_2$ and $K_3$ for which $\mu'$ was calculated and are also shown in Figs. 4–6 and in Fig. 7, which gives the ratio in the case of the most frequently used combination, $h_1 = h_2$, $K_1 = 2$. In these curves, the magnification ratio corresponding to values of $\mu'$ more than one, which are not physically possible, have been shown by the dashed portions. It may be seen from equation (13) as well as from these figures that the magnification ratio *versus* $h_1'$ curves form a family of ellipses with a common origin and passing through two common points, situated symmetrically with respect to the mag. ratio axis.

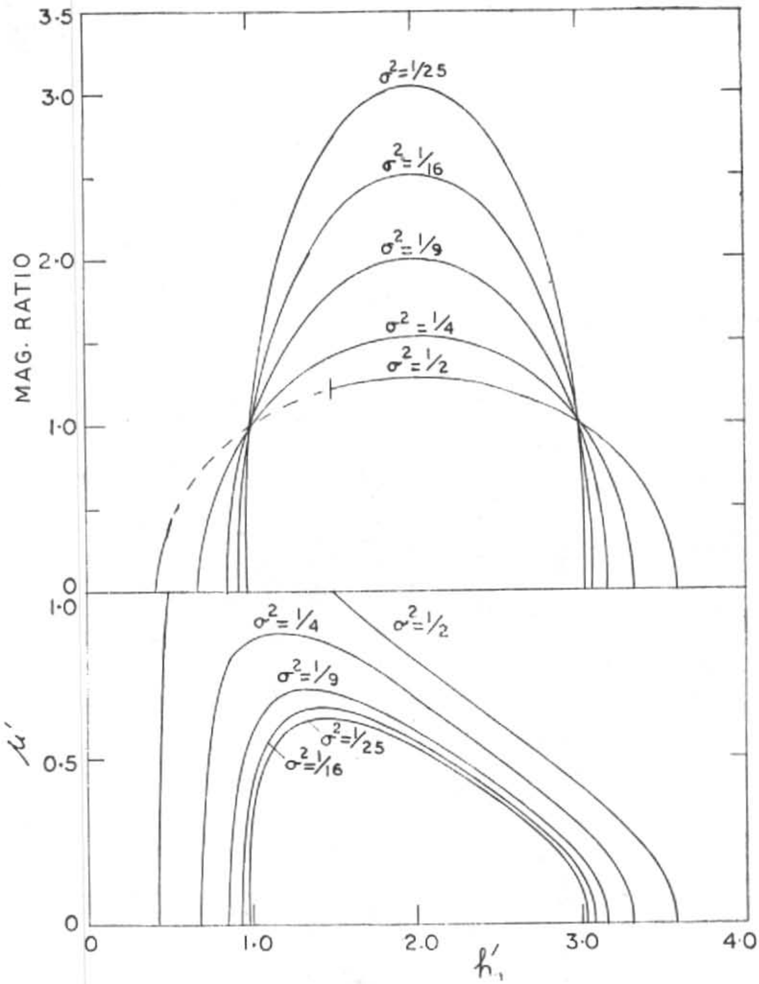


**Fig. 6. Variation of $\mu_1'$ and magnification ratio with $h_1'$**
$h_1 = 1,\ h_2 = 3,\ K_1 = 4,\ K_3 = 1$

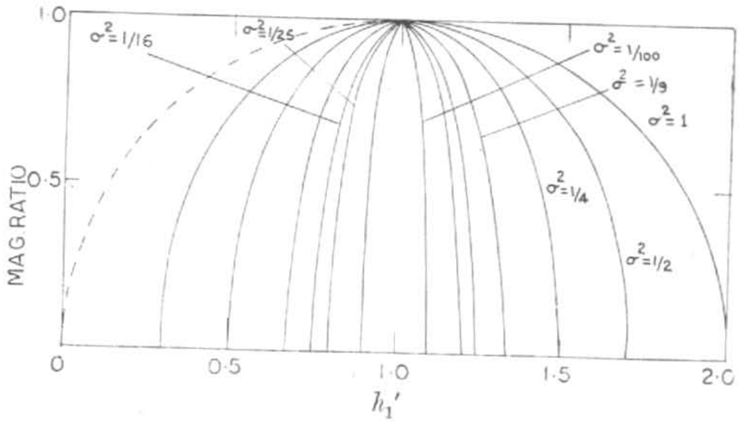


**Fig. 7. Variation of magnification ratio with $h_1'$**

$h_1 = h_2 = 1, \quad K_1 = 2, \quad K_3 = 1$

**5. Discussion**

It is seen from the curves that for any value of $K_3$, the $\mu'$ curve remains within the limits $0 \sim 1$ more for lower values of reaction. $\mu'$ is zero when

$$h_1' = \tfrac{1}{2}\left(K_1 - \sqrt{K_1^2 - 4K_2}\right),$$

increases with $h_1'$ upto a maximum value of

$$\frac{1}{K_3}\left(\frac{K_1^2}{4K_2} - 1\right) \quad \text{at } h_1' = 2K_2/K_1$$

and then decreases as $h_1'$ increases, reaching zero at

$$h_1' = \tfrac{1}{2}\left(K_1 + \sqrt{K_1^2 - 4K_2}\right).$$

The fall of $\mu'$ on the side of larger values of $h_1'$ is, however, less rapid than the rise upto the maximum. It may be shown from equation (12a) that in order that $\mu' \geqslant 0$,

$$\frac{K_1 - \sqrt{K_1^2 - 4K_2}}{2} > h_1' > \frac{K_1^2 + \sqrt{K_1^2 - 4K_2}}{2}$$

Similarly, for $\mu'$ not to be more than one,

$$\frac{K_1 - \sqrt{K_1^2 - 4K_2(1 + K_3)}}{2(1 + K_3)} \leqslant h_1' \geqslant$$

$$\geqslant \frac{K_1 + \sqrt{K_1^2 - 4K_2(1 + K_3)}}{2(1 + K_3)}.$$

If $4K_2\,(1 + K_3) > K_1^2$ the limits become imaginary and hence $\mu'$ does not exceed one.

The seismometer and galvanometer free periods being equal, $h_1'$ and $h_2'$ can be interchanged. We thus get two combinations for the same value of $\mu'$. The two values of $h_1'$ obtained from the curves for any value of $\mu'$ give just these systems.

The magnification ratio curves, on the other hand are symmetrical. The maximum gain in magnification is seen to be associated with $h_1' = h_2' = K_1/2$. The fall of the magnification ratio on either side of this maximum is same and the zero gain coincides, as it should, with zero $\mu'$. It is interesting to note that the maximum magnification is not associated with the maximum value of $\mu'$.

Just as for any value of $\mu'$, we get two combinations, similarly for any value of magnification ratio, we get two combinations. The values of $\mu'$ corresponding to these combinations are, however, not equal. Further, both these combinations may not always be physically possible to operate, due to the restrictions brought into play by the values of $R_1/Z_{11}'$, $R_2/Z_{22}'$ and $Z_{22}'/Z_{11}'$.

**TABLE 1**

Original combination : $h_1 = h_2 = 1.0$ ; $\sigma^2 = 1/9$ ; $R_1 = 100\Omega$ ; $(Z_{11})_{h_1=1} = 200\Omega$
$R_2 = 15\Omega$ ; $(Z_{22})_{h_2=1} = 50\,\Omega$ ; $K_1 = 2$ ; $K_2 = 8/9$ ; $K_3 = 1/4$

| Mag. ratio | First combination | | | | | | | Second combination | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | From curve | | | | | | Remarks | From curve | | | | | | Remarks |
| | $h_1'$ | $h_2'$ | $\mu'$ | $\alpha_1'$ | $\alpha_2'$ | $\alpha'$ | | $h'_1$ | $h_2'$ | $\mu'$ | $\alpha_1'$ | $\alpha_2'$ | $\alpha'$ | |
| 0·2 | 0·68 | 1·32 | 0·25 | 0·34 | 0·395 | 0·13 | Possible | 1·32 | 0·68 | 0·14 | 0·66 | 0·204 | 0·485 | Possible |
| 0·4 | 0·70 | 1·30 | 0·42 | 0·35 | 0·39 | 0·135 | ,, | 1·30 | 0·70 | 0·22 | 0·65 | 0·21 | 0·465 | ,, |
| 0·6 | 0·74 | 1·26 | 0·57 | 0·37 | 0·38 | 0·15 | ,, | 1·26 | 0·74 | 0·33 | 0·63 | 0·22 | 0·42 | ,, |
| 0·8 | 0·80 | 1·20 | 0·66 | 0·40 | 0·36 | 0·17 | ,, | 1·20 | 0·80 | 0·45 | 0·60 | 0·24 | 0·375 | ,, |
| 0·9 | 0·86 | 0·14 | 0·71 | 0·43 | 0·34 | 0·19 | Not possible | 1·14 | 0·86 | 0·53 | 0·57 | 0·26 | 0·33 | ,, |
| 1·0 | 1·0 | 1·0 | 0·66 | 0·50 | 0·3 | 0·25 | Possible | 1·0 | 1·0 | 0·66 | 0·50 | 0·3 | 0·25 | ,, |

These are

$$\left.\begin{aligned} \frac{1-\alpha'\mu'}{1-\alpha'\mu'^2} &\nless \frac{R_1}{Z_{11}'} \\ \frac{1-\mu'}{1-\alpha'\mu'^2} &\nless \frac{R_2}{Z_{22}'} \end{aligned}\right\} \qquad (14)$$

The calculations of the different combinations are shown in the example below.

*Example* : Let us take a seismograph in which the seismometer period = galvanometer period. $h_1 = h_2 = 1$; $\sigma^2 = 1/9$; $R_1 = 100$ ohms ; $(Z_{11})_{h_1=1} = 200$ ohms ; $R_2 = 15$ ohms; $(Z_{22})_{h_2=1} = 50$ ohms. So $K_1 = 2$, $K_2 = 8/9$ and $K_3 = 1/4$.

Table 1 shows the parameters of the different combinations. In the Remarks column, the physical possibility of operating the instrument with the parameters has also been included. This has been done by using the curves given in the author's previous paper. This could also be done by calculation.

It is easy to see that we cannot, in the above instrument, adjust the response curve to be identical with that of a reaction free Galitzin combination ($T_1 = T_2$, $h_1 = h_2 = 1$). This can be done only if $T_1 \neq T_2$ and by adjusting the parameters as given by Coulomb and Grenet (1935) for the 'Faux Galitzin'.

#### 6. Summary and Conclusions

The results of the discussions may be summed up as follows —

(*i*) The usual practice of adjusting the magnification of electromagnetic seismographs by keeping the damping conditions of the seismometer and galvanometer constant does change the response curves of the instruments. This change is seen to be more in the case of the widely used combination $h_1 = h_2 = 1$, $T_1 = T_2$.

(*ii*) It is possible to adjust the magnification without affecting its dynamical characteristics in the case of seismographs in which $T_1 = T_2$ by suitably changing the damping conditions along with the attenuation factor $\mu'$.

(*iii*) When sensitivity adjustments are made as in (*ii*) above the maximum sensitivity is attained when the dampings of the seismometer and galvanometer are equal. This possibly is one of the reasons why seismographs with equal damping constants of the seismometers and galvanometers are so widely used. This maximum does not coincide with the maximum value of the attenuation factor.

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