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Computation of soil temperature from air temperature

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ABSTRACT. Considering the thermal relationship between a bare soil and the atmosphere immediately above it, the mean daily soil temperatures at a depth of 5 to 10 cm have been calculated for the non-rainy months, from the daily mean Stevenson Screen temperature at three different stations in India. The result agrees well with the observed soil temperature.

1. Introduction

Soil temperature is of great importance in agriculture. Each stage of plant growth is influenced by it. But soil temperature is not available in most of the stations. Also recording of soil temperature by direct observation is not always possible. However, the air temperature is available in most of the stations. Based on these air temperature data, it may be possible to determine soil temperature indirectly.

In early investigations of soil temperature the equilibrium processes considered for the heat-budget did not take into account the air to ground inflow. Dorodnitsyn (1940) gave a realistic model in which he took into account the basic facts of thermal interactions between the surface layer of air and ground. Solutions to the problem under generalised conditions, *i.e.*, conductivity varying with depth, coefficient of turbulence varying with height etc were obtained by Shvets (1943). It is a well known fact that soil temperature variations which are sharp and relatively short lived are of main importance in agriculture and these take place when the temperature fluctuates aperiodically. Applying a double laplace transform method solutions concerning these aperiodic variations in a completely general form were obtained by Berlyand (1954). The next step in the studies of soil temperature was to consider the temporal variations of the physical characteristics of the soil and atmosphere and include the generalisations in the solution. Following Malkin (1944), Dyubyuk and Monin (1950) have developed a method for practical calculations of soil temperature by using Eigen functions. The soil temperature calculations for two-layer (air and soil) and three-layer (air, snow and soil) models are available and the results derived showed good agreement with the actual data. However, for the practical purpose the Eigen equation method is linked with some difficult computations.

Gutman (1954) developed a formula under some simplified assumptions regarding the vertical distribution of turbulence in the atmosphere, moisture in the soil and heat inflow from the sun at the earth's surface, to calculate the daily mean soil temperature from the daily mean air temperature of the instrumental shelter. The present attempt is to utilize the above solution to calculate the daily mean soil temperature at some selected Indian stations, in the non-rainy days and to observe whether the method could be used for tropical country like India. From the results of comparison of the computed values with the actual ones, it is observed that the method can be used for tropical regions as well. Accordingly, the method provides a means to calculate the daily mean soil temperature for stations where the screen temperatures etc are available but no soil temperature is recorded.

2. Theory

With sufficient accuracy for all practical purposes, the differential equations of conduction of heat in the surface layer of the air and in the ground can be given by—

$$\frac{\partial T}{\partial t} + u(z,t)\frac{\partial T}{\partial x} = \frac{\partial}{\partial z}\left\{K(z)\frac{\partial T}{\partial z}\right\} \quad (1)$$

$$\frac{\partial I}{\partial t} = K_0\frac{\partial^2 I}{\partial z^2} \quad (2)$$

Where $T(z,t)$ and $I(z,t)$ are the deviations of the air and soil temperatures respectively from thermal equilibrium, K_0 and $K(z)$ are the coefficient of thermal diffusivity of the soil and air, t is the time and $u(z,t)$ is the wind speed. The origin of the co-ordinate system is located at the earth's surface, the x-axis directed along the direction of wind and the z-axis directed vertically downward.

It is assumed that the air temperature at the height H of the meteorological instrument is known and denoted by—

$$T = T_H\ (t) \text{ at } z = -H \quad (3)$$

It is also assumed that the influx of direct and

*Formerly at the Division of Agricultural Meteorology, Poona scattered radiation to the earth's surface is known and its deviation from the climatic value is denoted by $S(t)$. The equation of the thermal balance at the earth's surface is—

$$-\lambda_0 \frac{\partial I}{\partial z}\bigg|_{z=0} + \lambda \frac{\partial T}{\partial z}\bigg|_{z=-z_0} + \mu I\bigg|_{z=0} = (1-\Gamma)\, S(t) \tag{4}$$

where Z_0 is the roughness parameter, μ is the coefficient characterising the radiation from the soil surface, Γ is the albedo of the bottom surface, $\lambda = KC\rho$ and $\lambda_0 = K_0 \rho_0 C_0$ are the coefficients of thermal conductivity of the air and soil respectively. C and C_0 and ρ and ρ_0 are the specific heat and density of the air and soil respectively.

At the boundary of the soil and air the following condition holds good—

$$I\bigg|_{z=0} = T\bigg|_{z=-z_0} = \theta(t) \quad \text{(say)} \tag{5}$$

Also the temperature of soil tends to equilibrium with the depth, *i.e.*,

$$I = 0 \text{ at } z = \infty \tag{6}$$

The physical characteristics of soil and turbulence of atmosphere are assumed to be constant in time. The influence of humidity of air or the moisture content of soil on the thermal processes have not been taken into account.

To study the mean daily soil and air temperatures, T, I and S will denote the deviations of the mean daily air temperatures, soil temperature and the thermal influx respectively from their monthly mean values. Introducing dimensionless quantities $\bar{t}$, $\bar{z}$, $\bar{x}$ and $\bar{u}$ by the following equations—

$$t = \tau\, \bar{t},\; z = Z\bar{z},\; x = u\tau\bar{x} \text{ and } u = U\bar{u} \tag{7}$$

where U, τ, and Z are the representative wind speed, time and height respectively.

It has been seen from a large number of observations of the air at the ground layers that —

$$K(z) = \nu\,|z| \tag{8}$$

where ν is an empirical constant.

Substituting from equations (7) and (8) into equation (1)—

$$\delta\left(\frac{\partial T}{\partial \bar{t}} + \bar{u}\frac{\partial T}{\partial \bar{x}}\right) = \frac{\partial}{\partial \bar{z}}\left[\bar{z}\frac{\partial T}{\partial \bar{z}}\right]$$

where $\delta = z/\nu\tau$ is the dimensionless parameter indicating the degree of steadiness of the process. The representative time τ in this case is not less than a day, the height Z is not greater than $H = 4$ metres and the observed value of ν is of the order of $0{\cdot}1$ m/sec.

$$\text{Hence } \delta < 7\times 10^{-4}$$

From this, it is clear that when the daily mean values of the temperature distribution along the height in the lowest surface layers are considered, the equation (1) may be written without serious error, in the form —

$$\frac{\partial}{\partial z}\left\{ K(z)\,\frac{\partial T}{\partial z}\right\} = 0 \tag{9}$$

Equation (9) shows that the daily mean temperature distribution of air in the surface layer is quasi-stationary. Though the equation (9) does not contain u, the wind speed effect of air advection has been taken into account by considering the air temperature at the level of the instrument shelter.

The solution of equation (9) satisfying the boundary conditions (3) and (5) is—

$$T = \theta + (T_H - \theta)\,\frac{\log(-z) - \log(z_0)}{\log(H) - \log(z_0)} \tag{10}$$

Substituting the value of T from (10) in (4),

$$\lambda_0 \frac{\partial I}{\partial z}\bigg|_{z=0} = \alpha\,\{\theta - F(t)\} \tag{11}$$

$$\text{where } \alpha = \mu\left(1 + \frac{1}{x}\right),\quad x = \frac{\mu \log(H/z_0)}{\nu c \rho}$$

$$\text{and } F(t) = \frac{1}{1+x}\, T_H(t) + \frac{1-\Gamma}{a}\, S(t) \tag{12}$$

Thus the problem of calculating soil temperature from the air temperature, reduces to solve the equation (2) with the boundary condition (11).

3. Derivation of computation formulae

Considering the instant $t=0$ as the time of one of the meteorological observations the following notation is introduced —

$$I(z, -n\tau) = I_n(z)$$

where τ denotes the time in days. According to this notation the index n will decrease with time. Similar notations have been used with T, S and θ. The value $F(t)$ on the nth day has been denoted by—

$$F_n = \frac{1}{1+x}\, T_n + \frac{1-\Gamma}{\alpha}\, S_n$$

The solution of the equations (2), (6) and (11) after taking the mean, over the intervals τ may be written in terms of several components.

$$\theta_{(0)} = \frac{F_0}{\tau}\int_0^{\tau}\left[1 - e^{a^2 t}\, \operatorname{erfc}(a\sqrt{t})\right] dt,$$

$$a = \frac{\alpha\sqrt{K_0}}{\lambda_0},\quad \tau = 1 \text{ day}$$

$$\theta_{(n)} = \frac{F_n}{\tau}\int_{n\tau}^{(n+1)\tau} \Big[e^{a^2(t-\tau)} \operatorname{erfc} \sqrt{t-\tau}\,) -$$

$$- e^{a^2t} \operatorname{erfc}(a\sqrt{t})\Big]\, dt, \quad n>0$$

Integrating by parts—

$$\theta_{(n)} = F_n\ \pi_n(\beta)\ , \qquad \beta = \frac{\lambda_0}{2\alpha\sqrt{K_0\tau}}$$

$$\text{where}\quad \pi_0 = 1 - \frac{4}{\sqrt{\pi}}\beta +$$

$$+ 4\beta^2 [\, 1 - e^{1/4\beta^2} \operatorname{erfc}(1/2\beta)\,]$$

$$\pi_{n+1} = -\frac{4}{\sqrt{\pi}}\ \beta \triangle^2 \sqrt{n} - 4\beta^2 \triangle^2 \times$$

$$\times \left(e^{\,n/4\beta^2} \right) \operatorname{erfc}\left(\frac{\sqrt{n}}{2\beta} \right)$$

where the formula of finite differences has been used, namely,

$\triangle^2 f(n) = f(n+2) - 2 f(n+1) + f(n)$

and θ_0 is given by

$$\theta_0 = \sum_{n=0}^{\infty} \theta_{(n)} = \sum_{n=0}^{\infty} F_n\ \pi_n(\beta) \qquad (13)$$

Similarly the daily mean soil temperature at a depth z can be found out if the mean surface soil temperature θ_n is known. The components of the soil temperature at the depth Z are given by—

$$I_{(0)} = \frac{\theta_0}{\tau}\int_0^{\tau} \operatorname{erfc}\left(\frac{z}{2\sqrt{K_0\tau}}\right) dt$$

$$I_{(n)} = \frac{\theta_n}{\tau}\int_{n\tau}^{(n+1)\tau} \Big[\operatorname{erfc}\left(\frac{z}{2\sqrt{K_0\tau}}\right) -$$

$$- \operatorname{erfc}\left(\frac{z}{2\sqrt{K_0(t-\tau)}}\right)\Big]\, dt, \quad n>0$$

After some simplification,

$$I_0(z) = \sum_{n=0}^{\infty} \theta_n\, M_n(\xi) \qquad (14)$$

$$\text{where,}\quad \xi = \frac{z}{2\sqrt{K_0\tau}}$$

$$M_0 = \frac{4}{\sqrt{\pi}}\int_{\xi} (\xi - \zeta)^2\, e^{\zeta^2}\, d\zeta \qquad (15)$$

$$\text{and}\quad M_{n+1}(\xi) = \triangle^2 [\, n M_0(\xi/\sqrt{n})], \quad n>0$$

Substituting

$$I_n = \sum_{p=0}^{n} \pi_p(\beta)\, F_{p+n},$$

analogus to equation (13) in equation (14) and changing the order of summation—

$$I_0(z) = \sum_{n=0}^{\infty} M_n^{(o)}(\xi,\beta)\ F_n \qquad (16)$$

$$\text{where}\quad M_n^{(o)}(\xi,\beta) = \sum_{p=0}^{n} M_p(\xi)\, \pi_{n-p}(\beta) \qquad (17)$$

Taking the values of the parameters (in C.G.S. systems)—

$\mu = 1{\cdot}3 \times 10^{-4}$, $\nu = 10$, $\rho c = 3 \times 10^{-4}$,

$H = 122$, $z_0 = 1$, $K_0 = 0{\cdot}0046$,

$\lambda_0 = 0{\cdot}0023$, $C_0 = 0{\cdot}2$ and $\rho_0 = 2{\cdot}5$,

the value of β comes out to be approximately $0{\cdot}1$.

Since $\beta << 1$

$$M_n^{(o)}(\xi,\beta) \approx M_n(\xi + \beta)$$

$$\text{i.e.,}\ I_0(z) = \sum_{0}^{\infty} M_n(\xi + \beta) F_n \qquad (18)$$

If it is assumed that the soil temperature at a depth has been calculated for a day, then

$$I_1(z) = \sum_{n=0}^{\infty} M_n(\xi + \beta)\, F_{n+1} \qquad (19)$$

Subtracting (19) from (18)—

$$I_0(z) = I_1(z) + \sum_{n=0}^{\infty} E_n(\xi + \beta)\, F_n \qquad (20)$$

where $E_0(\xi) = M_0(\xi)$

and $E_n(\xi) = M_n(\xi) - M_{n-1}(\xi)$, $n>0$

It may be shown that $E_n(\xi)$ decreases more rapidly with increase in n than $M_n(\xi)$. $E_n(\xi)$ for depths 10-15 cm practically vanishes for n=10–12. The values of $E_n(\xi)$ have been given in a table by Gutman.

Since the changes in S_n with increase in n are small, the contribution of $(1-\Gamma)S_n/\alpha$ to F_n is small in comparison to other factor of F_n. Hence the equation (20) may be written as—

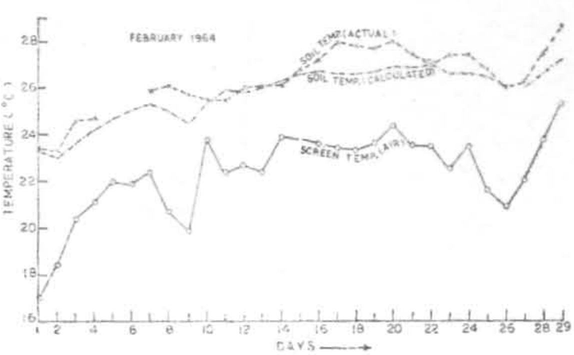

Fig. 1. Soil and air temperature — Poona

TABLE 1

New Delhi

Date	Screen temp. (°C)	Actual S.T. at 5 cm (°C)	Calculated S.T. (°C)	Screen temp. (°C)	Actual S.T. at 5 cm (°C)	Calculated S.T. (°C)
	January 1958			November 1958		
1	17·8	17·2	16·1	23·2	22·7	21·9
2	18·1	18·1	17·1	23·1	22·7	21·9
3	14·1	14·9	14·8	22·1	21·5	21·4
4	14·5	15·8	14·9	22·3	20·9	21·6
5	13·1	14·2	14·5	21·3	20·3	21·1
6	12·8	13·5	14·2	20·9	20·7	20·7
7	12·8	14·2	14·1	20·8	20·5	20·5
8	13·3	14·7	14·2	20·9	21·1	20·5
9	13·5	14·6	14·2	21·3	21·1	20·7
10	16·5	16·9	15·7	21·2	21·3	20·7
11	17·5	16·8	16·7	21·7	21·7	20·8
12	17·5	16·9	16·9	21·7	21·7	20·9
13	17·0	17·0	16·8	20·7	20·2	20·4
14	16·1	16·3	15·9	18·1	18·9	19·0
15	16·0	16·8	15·2	18·9	18·9	19·0
16	14·4	16·1	15·0	19·7	19·0	19·4
17	12·6	13·9	14·3	18·9	18·8	19·0
18	13·6	13·8	14·7	19·9	19·5	19·4
19	13·8	14·5	14·6	20·3	19·1	19·8
20	13·6	14·8	14·4	19·9	18·7	19·7
21	16·2	15 6	15·9	17·8	17·3	18·6
22	16·3	16·4	16·3	16·7	16·3	17·7
23	12·6	15·0	14·7	15·9	16·9	17·4
24	11·9	13·8	13·5	16·5	17·2	17·1
25	16·2	14.9	14·9	17·3	17·4	17·2
26	17·5	17·0	16·6	17·3	17·0	17·5
27	14·7	16·3	15·2	18·2	17·7	17·7
28	15·8	16·0	14·9	13·7	14·1	14·3
29	13·1	15·6	14·1	14·5	15·9	15·5
30	12·6	12·1	14·1	15·3	15·7	16·0
31	13·8	13·3	14·9	..	..	..

Actual S.T. on 31 December 1957 is 14·5° C

Actual S.T. on 31 October 1958 is 21·9°C

TABLE 2

Warangal

Date	Screen temp. (°C)	Actual S.T. at 5 cm (°C)	Calculated S.T. (°C)	Screen temp. (°C)	Actual S.T. at 5 cm (°C)	Calculated S.T. (°C)
		January 1958			May 1958	
1	24·3	28·7	29·2	32·2	38·4	39·7
2	23·7	29·8	29·0	33·7	39·5	40·3
3	24·5	29·0	29·4	34·1	40·2	40·8
4	23·7	29·3	29·1	31·3	33·7	39·4
5	23·1	29·1	28·7	32·5	37·1	39·6
6	24·5	29·7	29·3	33·1	36·7	40·0
7	25·1	29·5	29·8	33·3	40·1	40·2
8	24·3	29·3	29·5	34·5	40·7	41·1
9	24·9	29·7	29·8	35·9	40·9	41·9
10	24·3	29·3	29·5	35·9	41·3	42·1
11	24·3	28·7	29·5	35·7	41·4	42·0
12	24·9	29·6	29·8	35·7	41·3	42·1
13	25·0	29·5	29·9	34·6	41·3	41·5
14	24·6	29·0	29·7	34·2	40·7	41·1
15	25·1	29·7	29·9	34·7	41·3	41·3
16	25·7	30·0	30·0	35·1	41·8	41·6
17	24·7	29·8	29·9	35·1	40·3	41·6
18	22·3	27·5	28·5	35·9	41·4	42·0
19	21·1	27·1	27·5	36·8	42·5	42·6
20	21·9	27·5	27·7	35·7	42·7	42·2
21	22·5	28·0	28·0	35·1	41·3	41·8
22	22·3	26·9	28·0	33·9	40·6	41·1
23	23·2	28·7	28·4	33·9	41·3	40·9
24	23·0	29·1	28·4	34·7	41·9	41·2
25	23·4	29·3	28·6	35·5	41·7	41·7
26	24·5	30·1	29·3	35·1	42·1	41·7
27	24·9	30·1	29·7	34·9	42·5	41·6
28	25·8	30·5	30·2	35·5	43·0	41·9
29	26·7	32·1	30·9	36·0	42·7	42·3
30	25·9	32·1	30·7	37·6	42·9	43·2
31	24·7	31·6	30·0	37·7	43·5	43·5

$$I_0(z) = I_1(z) + \frac{1}{1+x} \sum_{n=0}^{\infty} T_n E_n(\xi+\beta) \quad (21)$$

And since $\sum_{n=0}^{\infty} E_n(\xi) = 0$, T_n may be taken as the actual value instead of deviation from the monthly mean.

4. Actual computations of some stations

(*a*) *Poona*—The Stevenson Screen temperature and soil temperature at 10 cm depth are recorded throughout the day by thermographs. Hourly temperatures are read from these thermograms and the mean of these values represent the daily mean temperature. The radiation data are also recorded at the Central Agrimet Observatory, Poona.

The values of the quantities C_0, ρ_0, K_0, calculated for the Black Cotton Soil (Poona Soil) by Dravid (1940) have been used.

The soil temperature at 10-cm depth has been calculated by equation (20) taking the values of the parameters as in C.G.S. unit.

$\lambda_0 = 0{\cdot}00067$, $K_0 = 0{\cdot}0015$, $C_0 = 0{\cdot}22$, $\rho_0 = 2{\cdot}00$
$\mu = 1{\cdot}3\times10^{-4}$, $\nu = 10$, $Z_0 = 1$, $H = 122$,
$\rho C = 3\times10^{-4}$, $\tau = 86400$, $\beta = 0{\cdot}039$,
and $\xi = 0{\cdot}439$ for $Z = 10$.

The soil temperature of 31 January 1964 has been used to calculate the soil temperatures for the month of February 1964. The month of February has been selected as it was practically only no-rainy month for that year. The screen temperature, actual and calculated soil temperature at 10-cm depth have been shown in Fig. 1. The correlation coefficient between the actual and calculated soil temperature are found to be 0·93, *i.e.*, the equation for the soil temperature accounts for 86 per cent of the variations in soil temperature at a depth of 10 cm.

(*b*) *New Delhi*—The Stevenson Screen temperature and soil temperature at 5-cm depth are not available here throughout the day. The only data available are at the minimum epoch (*i.e.*, 0700 hrs L.M.T.) and the maximum epoch (*i.e.*, 1400 hrs L.M.T.) collected at the Agromet Observatory in the Indian Agricultural Research Institute, New Delhi. The average of those two observations (*i.e.*, of minimum and maximum epoch) has been considered as the daily mean temperature and the hourly temperatures are not being recorded. The radiation data are collected under the I.G.Y. Scheme at the Meteorological Office, New Delhi.

Since the values of the constants K_0, λ_0, C_0 and ρ_0 for the soil at New Delhi are not available, the values of the constants for the mean soil given by Carslaw and Jaegar (1948) have been taken ($K_0=0\cdot0046$, $\lambda_0=0\cdot0023$, $C_0=0\cdot2$ and $\rho_0=2\cdot5$). The soil temperatures at 5-cm for the months of January and November 1958 have been calculated using the soil temperature value of 31 December 1957 and 31 October 1958 reespectively. The results are shown in Table 1. The correlation coefficients between the actual and calculated soil temperature are $0\cdot92$ and $0\cdot97$ in January and November respectively.

(c) *Warangal* — The daily mean soil temperature has been calculated in a similar way as was done in New Delhi. Since the radiation data are not recorded here, equation (21) has been used to calculate soil temperature at a depth of 5 cm for the month of January and May 1958. Since there was rain in November, May has been included instead of November in the calculation. The values of the soil constants have been taken as those used for New Delhi. The screen temperature, actual and calculated soil temperature are shown in Table 2. The correlation coefficients between the actual and calculated soil temperature come out to be $0\cdot86$ for both the months.

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