

551-501-7 : 551-506-7

## Divergence, vorticity and vertical motion in the fields of winter and monsoon circulations over India

P. P. SAJNANI

*Meteorological Office, Poona*

*(Received 19 January 1968)*

**ABSTRACT.** Using upper wind data of 69 pilot balloon and rawin stations in India, Pakistan, Burma and Ceylon, multi-level fields of divergence, vorticity and vertical motion have been computed on five consecutive days in the months of January and July 1958 to study the winter and monsoon circulations over the Indian region. Computations have been made at 37 grid points 2.5 degrees apart, with the help of a digital computer. A filtering technique was used to smoothen the observed winds so as to filter out noise.

The study provides a convenient method of computing divergence directly from observed winds instead of doing so indirectly through the vorticity equation. The computed values of divergence and vorticity show that the two are of comparable magnitude. Examination of the divergence fields does not reveal narrow bands of convergence, characteristic of well-defined frontal systems. The distribution of vertical velocities in the monsoon field indicates the existence of a source region over the eastern half of India and a sink over the western half.

### 1. Introduction

Divergence, vorticity and vertical motion are three important parameters of atmospheric circulation. It is generally recognised that vorticity is the predominant factor in atmospheric motion compared to divergence. While this may be true for extra-tropical latitudes, the question has to be carefully examined for lower latitudes. The present study was undertaken with special reference to conditions over India.

Multi-level fields of divergence, vorticity and vertical motion have been computed on five consecutive days for the months of January and July 1958, representing the winter and monsoon circulations over the Indian region. The salient features brought out by an examination of these fields are discussed.

### 2. Data used

Upper wind data of 69 pilot balloon and rawin stations in India, Pakistan, Burma and Ceylon have been used. Observations of the morning hour taken at 00 GMT were preferred as they are not vitiated by the effects of insolation and convective turbulence. The periods selected were 18 to 22 January 1958 and 12 to 16 July 1958.

Care was taken to exclude days marked by monsoon depressions/storms and active western

disturbances, which would have disturbed the general flow conditions over the area.

Computations were carried out at 37 grid points 2.5 degrees apart, covering the Indian land area from 72.5°E to 90°E and 27.5°N to 10°N excluding regions close to the foot of the Himalayas. The location of these points is shown in Figs. 3 and 4.

Computations were confined upto 500 mb in January and 700 mb in July, as adequate number of observations were not available above these levels in the respective months.

### 3. Method of analysis and computation

The observed winds to the nearest knot at each of the 69 stations were resolved into their  $u$  and  $v$  components, level by level.  $u$  was considered positive for wind blowing from the west and, similarly,  $v$  was taken positive for winds blowing from the south. The values of  $u$  and  $v$  were then plotted on maps and isopleths drawn at intervals of 4 knots. This was the first stage of smoothing. Separate maps were prepared for each level and for each component.

From these analysed maps, values of  $u$  and  $v$  were picked up at 73 grid points, 2.5 degrees apart covering the area extending one grid distance around the 37 grid points at which computations

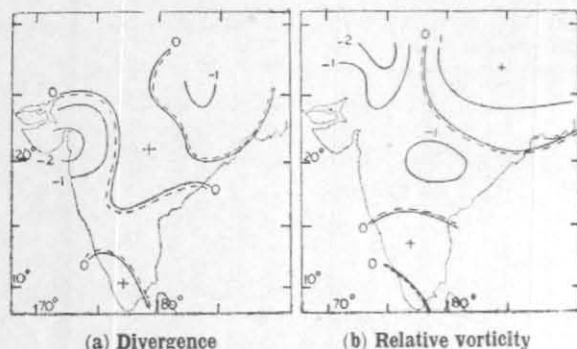


Fig. 1. 20 January 1958

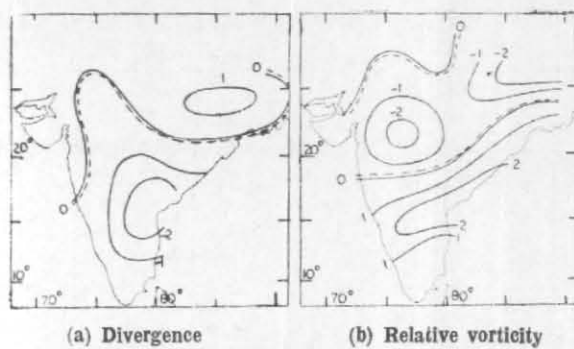


Fig. 2. 14 July 1958

Figs. 1 and 2. Divergence and vorticity fields at 700 mb on 20 January and 14 July 1958  
(Unit:  $10^{-5}$  sec $^{-1}$ )

were to be made. These grid point values were then further smoothed by fitting polynomials to the data.

A grid-mesh of nine points was taken, the point at which the smoothed values were to be obtained, being at the centre. Let this be called the Central Point. A quadratic surface given by the equation—

$$V = ax^2 + bxy + cy^2 + dx + ey + f \quad (1)$$

was fitted to the values of  $u$  and  $v$  at these nine points. In the above equation,  $V$  is the smoothed wind ( $u$  or  $v$ ) at any point whose co-ordinates with reference to the central point as origin are given by  $x$  and  $y$ . The values of the six coefficients  $a$ ,  $b$ ,  $c$ ,  $d$ ,  $e$  and  $f$  are determined from the analysed values of  $u$  and  $v$  at the nine grid points using the principle of least squares.

If  $V_{ij}$  is the analysed wind at a point  $(x_i, y_j)$  and  $V'_{ij}$  is the smoothed wind at the same point, then the difference between these two ( $V_{ij} - V'_{ij}$ ), is the error in estimation. This may be denoted by  $\Delta_{ij}$ . The best fit for the quadratic surface is obtained when the sum of the squares of the individual errors at all the nine points, is a minimum. In effect, we minimise the expression,

$$\sum_{i=1}^{i=3} \sum_{j=1}^{j=3} (\Delta_{ij})^2 = H \quad (2)$$

This condition is satisfied when  $\partial H/\partial a$ ,  $\partial H/\partial b$ ,  $\partial H/\partial c$ ,  $\partial H/\partial d$ ,  $\partial H/\partial e$  and  $\partial H/\partial f$  vanish. The solution of these six simultaneous equations gives the values of  $a$ ,  $b$ ,  $c$ ,  $d$ ,  $e$  and  $f$  for the particular grid-mesh in question. Substituting these values and also the appropriate values of  $x$  and  $y$  in equation (1), one can obtain the smoothed values of wind at any point located within the grid-mesh. By putting  $x=0$  and  $y=0$ , equation (1) becomes—

$$V_0 = f \quad (3)$$

which gives the smoothed wind at the central point.

Partial differentiation of equation (1) with respect to  $x$  and  $y$  gives—

$$\frac{\partial v}{\partial x} = \frac{\partial}{\partial x} (ax^2 + bxy + cy^2 + dx + ey + f) \\ = 2ax + by + d \quad (4)$$

$$\frac{\partial v}{\partial y} = \frac{\partial}{\partial y} (ax^2 + bxy + cy^2 + dx + ey + f) \\ = bx + 2cy + e \quad (5)$$

By putting  $x = 0$ ,  $y = 0$ , the above equations become—

$$\frac{\partial v}{\partial x} = d \quad \text{and} \quad \frac{\partial v}{\partial y} = e \quad (6)$$

These give the smoothed values of the first order partial horizontal derivatives of  $u$  and  $v$ , i.e.,  $\partial u/\partial x$ ,  $\partial u/\partial y$ ,  $\partial v/\partial x$  and  $\partial v/\partial y$ , at the Central Point.

The above process described by equations (1) to (6) was repeated for each grid-point in succession, till the entire wind field encompassed by the 73 grid points was smoothed.

The smoothed values of  $u$  and  $v$  and horizontal derivatives  $\partial u/\partial x$ ,  $\partial u/\partial y$ ,  $\partial v/\partial x$  and  $\partial v/\partial y$  derived as above were utilised for the computation of horizontal velocity divergence ( $\text{div}_H \mathbf{V}$ ), vertical component of vorticity ( $\zeta$ ), the vertical  $p$ -velocity ( $\omega$ ) and other terms occurring in the equations of motion. In the present paper, however, we shall confine ourselves to the study of the first three quantities only.

#### 4. Divergence and vorticity

Figs. 1 and 2 show the divergence and vorticity fields on one day each in January and July at 700-mb level. These show well-marked patterns exhibiting systematic changes from one level to another and also continuity in time.

It was interesting to find that these could not be inferred from an inspection of the wind flow charts.

TABLE 1

Average magnitude of divergence and relative vorticity (unit:  $10^{-5} \text{ sec}^{-1}$ ) for different latitudes at different pressure levels (mb) along meridian  $77.5^\circ \text{E}$ 

Lat. (°N)	January (18-22) 1958					July (12-16) 1958				
	900	800	700	600	500 mb	900	800	700	600	500 mb
(a) Divergence										
27.5	0.4	0.6	0.6	0.7	0.9	0.7	0.7	0.4	Data coverage not adequate	
25	0.4	0.3	0.8	0.7	0.3	0.4	0.7	0.6		
22.5	0.5	0.5	0.4	0.8	0.8	0.7	0.6	0.4		
20	0.6	0.7	0.4	0.4	0.6	0.8	0.6	0.6		
17.5	0.3	0.8	0.6	0.6	0.7	0.7	0.4	1.1		
15	0.2	0.5	0.6	0.7	0.6	0.6	0.6	1.2		
12.5	0.3	0.4	0.5	0.8	0.5	0.5	0.8	1.6		
10	0.9	0.5	0.4	0.7	0.4	0.9	0.6	1.1		
(b) Relative Vorticity										
27.5	0.4	0.1	0.6	0.8	1.3	1.2	1.4	0.7	Data coverage not adequate	
25	1.8	1.1	0.6	0.7	0.3	0.7	0.5	1.0		
22.5	1.5	1.7	1.1	1.2	0.8	0.7	0.5	1.1		
20	1.5	0.9	1.1	1.5	1.2	1.0	0.6	0.8		
17.5	0.7	1.0	0.8	1.8	1.1	0.5	0.5	0.5		
15	0.4	0.8	0.8	1.0	1.2	0.5	1.1	1.1		
12.5	0.5	0.2	1.1	1.1	2.1	1.1	1.2	0.4		
10	0.7	0.9	0.4	1.5	2.0	1.0	1.0	0.8		

This would seem to indicate the desirability of actually computing divergence and vorticity fields for any diagnostic study, instead of trying to visualise them from the wind patterns. Table 1 gives the average value of divergence and vorticity (the absolute values averaged over five days irrespective of sign) for different latitudes at different pressure levels, along a central meridian  $77.5^\circ \text{E}$  in the months of January and July. It is seen that vorticity values are two to three times higher than the divergence values in January but in July the two are almost of equal magnitude.

The overall average magnitude of divergence computed for the Indian region is  $0.6 \times 10^{-5} \text{ sec}^{-1}$  in January and  $0.9 \times 10^{-5} \text{ sec}^{-1}$  in July. Perhaps a higher value might have resulted, if the winds had not been smoothed (Landers 1955).

##### 5. Vertical velocity

Vertical  $p$ -velocity was computed from the equation of continuity in pressure co-ordinates—

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial \omega}{\partial p} = 0 \quad (7)$$

by stepwise integration of the divergence field from surface to 900 mb and thereafter at intervals

of 100 mb. Positive values of  $\omega$  indicate sinking motion and negative values rising motion.

The unit adopted is  $10^{-5} \text{ mb sec}^{-1}$  ( $100 \times 10^{-5} \text{ mb sec}^{-1} \approx 1 \text{ cm sec}^{-1}$ ). For obtaining the divergence profile in the layers below 900 mb, it was assumed that in the layer extending from 900 mb down to about 300 m a.g.l., the divergence had the same sign and magnitude as at 900-mb level and, as a consequence, it changed linearly becoming zero at the surface, which was considered to be a level surface. We have, in effect, ignored the effects of topography and friction.

Figs. 3 and 4 depict the multi-level vertical velocity fields on a day in January and July. An important feature of the distribution of vertical velocity in winter is the existence of two well-defined updraft areas one over the northern parts of India and the other over south Peninsula. These may perhaps be attributed to the passage of westerly waves across north India and the easterly waves across south Peninsula. In the monsoon field one finds practically the entire eastern half of India dominated by an updraft area with an area of downdraft lying over the western half. This is in agreement with the findings of Das (1962), Krishnamurti (1966) and Saha (1966).

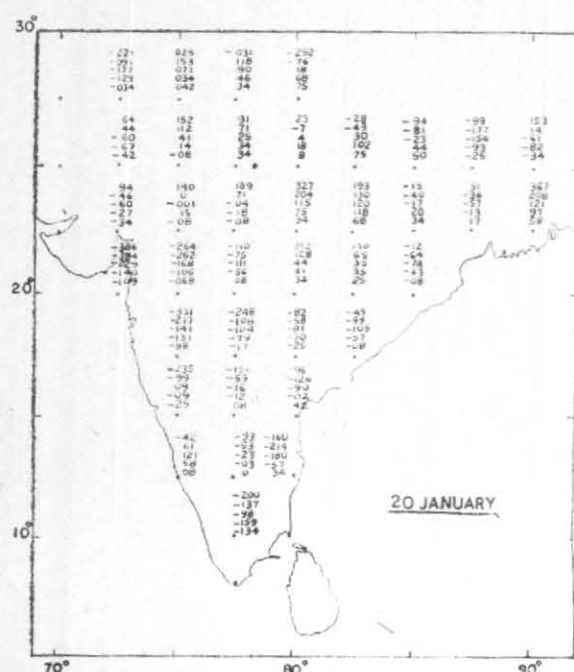


Fig. 3

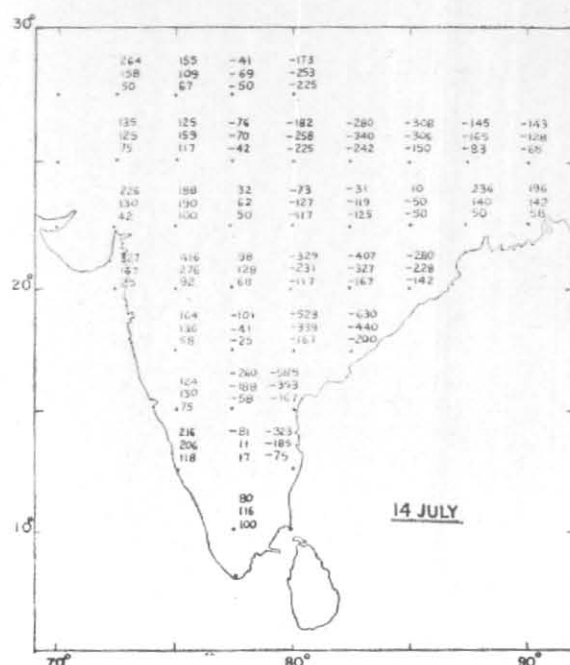


Fig. 4

Figs. 3 and 4. Vertical  $p$ -velocity ( $\omega$ ) at successive isobaric levels at interval of 100 mb, the bottom level being 900 mb

Unit:  $10^{-5}$  mb sec $^{-1}$ . Negative values indicate upward motion

## 6. Conclusion

Horizontal divergence is usually computed indirectly from the vorticity equation. The present study shows that divergence can be computed directly from the observed winds after they are suitably smoothed to eliminate probable observational errors and 'noise' due to unimportant small-scale disturbances.

The computed values of divergence and vorticity show that the two are of comparable magnitude. This suggests that divergence cannot be neglected in formulating dynamical models for our region. Moreover, the divergence values being significant the streamlines tend to be of a spiral shape (and not closed), the angle of the spiral depending upon the ratio of divergence to vorticity. This has an

important bearing on the techniques to be adopted for analysis of wind fields in the tropics.

Examination of the divergence fields does not reveal narrow bands of convergence characteristic of well-defined frontal systems.

The distribution of vertical velocities in the monsoon field indicates the existence of a source region over the eastern half of India and a sink over the western half.

## 7. Acknowledgement

The author is indebted to the authorities of the Tata Institute of Fundamental Research, Bombay for permission to use their computer CDC-3600 to carry out the computational work for this study.

## REFERENCES

- |                     |      |  |
|---------------------|------|--|
| Charney, J. G.      | 1948 | On the scale of atmospheric motion, <i>Geophys. Publ.</i> , 17, 2.   |
| Das, P. K.          | 1962 | Mean vertical motion and non-adiabatic heat sources over India during the monsoon, <i>Tellus</i> , 14, pp. 212-220.  |
| Krishnamurti, T. N. | 1966 | Numerical studies of organised circulation in subtropical latitudes. Final Rep. to ESSA under Grant CWF 10877.   |
| Landers, H.         | 1955 | A three dimensional study of the horizontal velocity divergence, <i>J. Met.</i> , 12, 5.   |
| Saha, K. R.         | 1966 | On the distribution of vertical velocity in the monsoon field and the structure of the monsoon circulation, Pre-published Rep., Inst. Tropical Meteorology, Poona. |
| Scott, J. R.        | 1958 | An essay on low latitude wind analysis, <i>Quart. J.R. met. Soc.</i> , 84, 360, p.166.   |