551·509 : 551·506

Forecasting five-day mean contours of 700 mb using empirical influence coefficients

J. SHUKLA and R. SURYANARAYANA

Institute of Tropical Meteorology, Poona

(*Received 16 January 1967*)

ABSTRACT. With the help of 5-day mean data for the ten-year period (1955-64) for July and August, empirical influence coefficients have been worked out. With these coefficients we derive a linear prediction formula for 700-mb contours at 12 radiosonde stations in India. We assume that the predicted 700-mb contour height is a linear function of the contours in the past pentad. The coefficients in our prediction formula are evaluated by the method of least squares using past data.

The usefulness of the method was tested with data of 2 years (1965-66) and we find good success in forecasting broad features on 5-day mean charts.

1. Introduction

Medium range prediction techniques are of great value to forecasters in India. The prediction of 5-day mean rainfall anomalies was attempted by several workers with the help of 700-mb contour heights of the previous pentad (5-day period) as selected points. Studies (Shukla 1967 and Mooley 1967) have been made of a possible relationship between rainfall anomalies and 700-mb height anomalies. Pant *et al.* (1965) made an attempt to set up linear regression equations between the mean pentad height and the height on the first day of the pentad for five grid points over India. Although, this was a pioneer attempt in this line, the method cannot withstand the operational demands of the forecaster on account of the inherent limitations of the method. The aim of the present study is to devise a more general and operationally convenient method of forecasting 5-day mean contour heights for 12 radiosonde stations in India.

2. Let $H_1, H_2 \ldots\ldots H_{12}$ represent the 700-mb height values at 12 radiosonde stations in the current pentad. The 700-mb height in the next pentad at any one station (say station 1) may be represented by ϕ_1. We assume ϕ_1 is related to $H_1, H_2 \ldots\ldots H_{12}$ by —

$$\phi_1 - (\bar{\phi_1}) = \sum_{1}^{12} A_n H_n + A_{13} \qquad (2\cdot1)$$

The bar (—) denotes the mean value (3000 gpm in the present case). Our aim is to find out A_1 through A_{13} (hereafter called empirical influence coefficients or predictor operators) for all the twelve stations with the help of past meteorological data. A linear relation between the predictors and the predictand is an assumption, but we propose to consider non-linear aspects in a later investigation.

The method adapted to find these coefficients is based on the principle of least squares, which has the property of minimizing the errors of forecasts.

In matrix notation, equation (2·1) may be expressed as,

$$\phi = AH + \epsilon \qquad (2\cdot2)$$

where ϕ is a $n \times 1$ vector of forecast heights,

H is a $n \times K$ vector of observed heights,

A is a $K \times 1$ vector of coefficients, and

ϵ is a $n \times 1$ vector of errors.

In our studies, $n = 120$, $K = 13$. For convenience in computation, H_{13} was fed into the computer memory as unity so as to have a constant term.

The sum of squares of the error is —

$$S = (\phi - AH).\ (\phi - AH) \qquad (2\cdot3)$$

A necessary condition for minimizing S is —

$$\frac{\partial S}{\partial A} = 0$$

$$\text{or } 2H\,(\phi - AH) = 0$$

$$\text{Whence } A = (H - H)^{-1}\ H.\,\phi \qquad (2\cdot4)$$

where $(H—H)$, is the matrix of the sums of squares and products of the elements of the column vectors composing H. The matrix is non-singular and may be inverted.

TABLE 1

Forecast (F) and Observed (O) values (+ 3000 gpm)

Period (1965)	F	O	F	O	F	O	F	O	F	O	F	O
	JODHPUR		ALLAHABAD		CALCUTTA		GAUHATI		BOMBAY		DELHI	
4—8 Jul	97·67	112	84·07	83	75·95	90	78·37	71	88·87	126	91·22	92
9—13 Jul	101·69	111	88·18	82	86·04	53	81·64	56	102·73	108	96·24	69
14—18 Jul	93·06	106	66·62	75	59·88	76	72·53	87	82·86	75	74·46	73
19—23 Jul	77·28	112	70·93	94	71·71	100	83·50	102	83·55	78	79·10	79
24—28 Jul	74·38	74	80·52	53	85·01	42	88·46	86	86·58	97	86·72	65
29 Jul—2 Aug	82·89	72	61·91	43	60·09	71	78·80	73	99·47	94	69·55	37
3—7 Aug	75·03	118	62·50	90	73·24	91	81·49	69	98·17	110	73·57	85
8—12 Aug	101·35	127	86·81	113	82·00	118	81·05	94	94·74	110	95·54	109
13—17 Aug	106·45	139	91·77	110	99·92	110	94·31	95	88·75	119	104·68	113
18—22 Aug	107·63	132	90·34	98	89·87	93	86·78	112	99·35	104	102·50	107
23—27 Aug	101·78	97	82·21	68	77·70	85	88·07	111	85·30	109	92·05	86
	PORT BLAIR		VERAVAL		VISAKHAPATNAM		MADRAS		TRIVANDRUM		NAGPUR	
4—8 Jul	119·77	137	77·86	109	79·06	119	106·59	134	123·09	144	71·46	106
9—13 Jul	123·80	115	93·71	109	92·57	75	113·90	114	126·34	144	86·55	104
14—18 Jul	107·75	120	72·36	76	55·50	74	98·31	105	119·20	132	58·52	71
19—23 Jul	120·08	138	75·86	73	78·70	91	105·91	116	127·97	146	70·64	88
24—28 Jul	132·29	115	80·18	64	86·35	66	114·03	123	133·85	134	77·34	93
29 Jul—2 Aug	122·70	119	74·41	81	74·33	85	115·43	119	122·90	141	68·06	85
3—7 Aug	121·93	122	96·61	105	94·09	102	115·34	127	131·14	128	86·51	104
8—12 Aug	119·60	139	83·05	130	84·83	124	106·22	116	121·53	112	81·65	125
13—17 Aug	114·76	129	94·34	117	84·05	123	101·95	119	118·49	128	99·76	127
18—22 Aug	117·17	112	85·95	110	81·60	94	102·21	103	117·49	117	84·80	99
23—27 Aug	109·21	129	80·07	101	72·16	94	97·01	121	113·39	135	76·62	110

TABLE 2

Rank correlations between observed and forecast values on 22 occasions

Period (pentad dates)	Rank correlations for years 1965	1966
4 Jul—8 Jul	0·74	0·74
9 Jul—13 Jul	0·87	0·60
14 Jul—18 Jul	0·80	0·07
19 Jul—23 Jul	0·54	0·62
24 Jul—28 Jul	0·60	0·88
29 Jul—2 Aug	0·81	0·62
3 Aug—7 Aug	0·80	0·95
8 Aug—12 Aug	0·12	0·87
13 Aug—17 Aug	0·60	0·59
18 Aug—22 Aug	0·79	0·43
23 Aug—27 Aug	0·72	0·37

The major computational part of the problem is to invert a 13 × 13 matrix, which has been carried out numerically on an electronic computer.

3. Practical considerations

In such problems, one is confronted with the problem of selection of stations and the selection of data. In our study, we considered data from all radiosonde stations in India. As far as the selection of data is concerned, keeping in view the stability and accuracy of the predictor parameters, we decided to classify the data on a seasonal basis on the assumption that broad physical processes have less variability during the course of a season. In the present study only the data for July and August for 700 mb were used. It is proposed to extend the study to other seasons later.

Five-day mean contour heights of 700 mb for all the 12 radiosonde stations have been used for the month of July and August for the ten-year period (1955-64)*. As a month consists of six pentads, 120 is our sample size, and we have a system of 120 simultaneous linear algebraic equations in 13 unknowns. As discussed earlier, the 13 unknowns are evaluated from 120 equations by the method of least squares.

It may be recalled that the basic computational problem is the inversion of a 13 × 13 matrix and the multiplication of this matrix by a column vector ϕ (Eq. 2) will give us the column vector of coefficients. The inverted matrix remains the same in all the operations and only the column vector ϕ changes for all the stations, giving different sets of coefficients for different stations.

For convenience in computation only contour heights in excess of 3000 gpm were taken into consideration for all the stations and all the pentads. It may be pointed out that because we assume a linear relationship, this simplification does not decrease the accuracy of the method.

After the coefficients have been evaluated, the sum of constant term, the products of coefficients and heights of 700 mb in excess of 3000 gpm for the corresponding stations will give us the height of the 700 mb surface in the next pentad for the particular stations under considerations.

The order of stations for the purpose of multiplication with the coefficients has been indicated in Table 1 and same order is maintained for the computation for other stations.

We have also evaluated the difference between the observed values and the values calculated with the use of regression coefficients. It was observed that the difference was less than 20 gpm in nearly 85 per cent of 120 cases for each station.

4. Results

Complete set of 12 coefficients and constant terms have been evaluated for all 12 stations. They are given in Appendix I.

The twelve regression equations, corresponding to 12 stations, are presented in Appendix II.

The letter S stands for 700-mb contour height (in excess of 3000 gpm) of a station and suffixes 1 through 12 are for the 12 stations as indicated under the the foot note of Table 1. A prime denotes the predicted 700-mb contour height in the next pentad for the corresponding station. As an example, S_1 and S'_1 denotes the 700 mb contour height (in excess of 3000 gpm) for Jodhpur during the current and next subsequent pentad.

Thus, using observed values of 700-mb contours for 12 radiosonde stations during the current pentad, a forecaster may prepare the forecast pentad heights with the help of the equations in Appendix II and a small hand computing machine.

The method for calculating the forecast heights has been illustrated in Appendix III by preparing a sample forecast for one station.

5. Verification

To test this technique a series of 22 forecasts were prepared for July and August with the data of 1965 and 1966. The observed and forecast values are presented in Table 1 for a few pentads. In order to test the success of the method in forecasting the occurrence or non-occurrence of chief features (like troughs and lows etc) on the 5-day mean chart, rank correlations have been worked out between the observed and forecast contour heights for 22 occasions. The values of the rank correlations are given in Table 2. It is seen that the average values of the rank correlations is 0·64 but if we exclude two unusual cases (one in each year, 3 August—7 August 1965 and 14 July—18 July 1966), the average is 0·70.

The observed and forecast contour height charts are shown in Figs. 1 and 2 for two occasions.

*After 1961, Ahmedabad data has been taken for Veraval

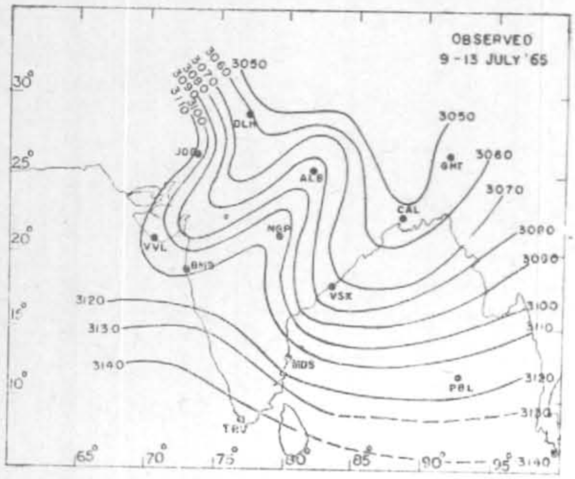

Fig. 1(a)

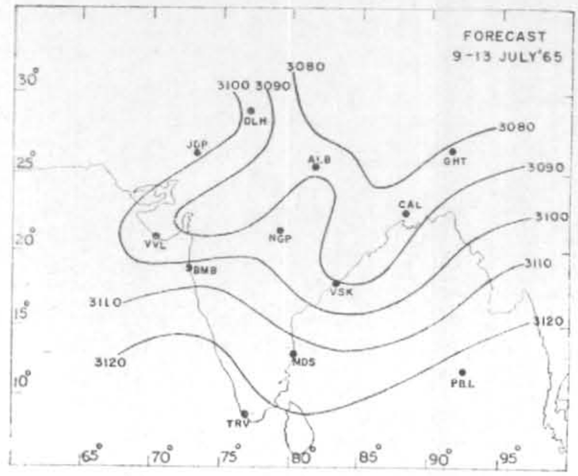

Fig. 1(b)

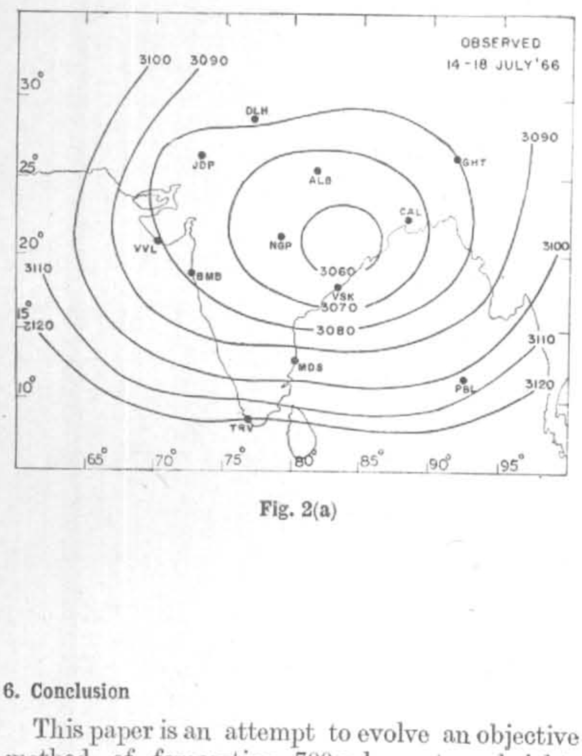

Fig. 2(a)

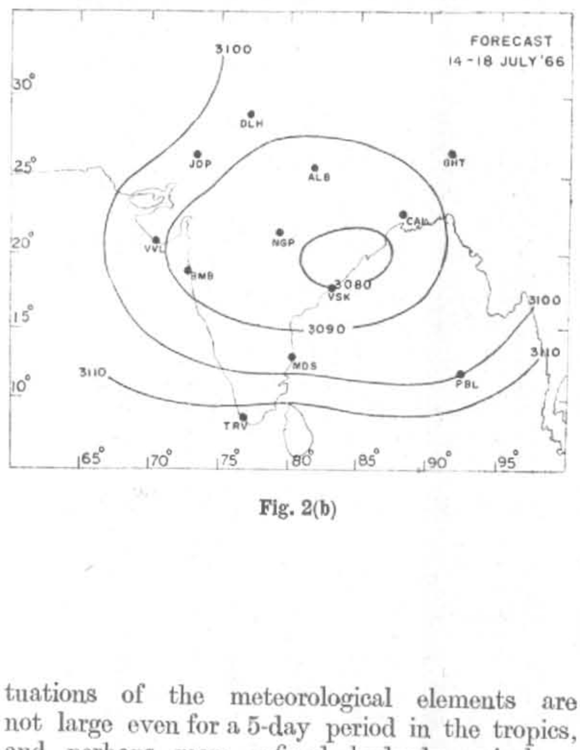

Fig. 2(b)

6. Conclusion

This paper is an attempt to evolve an objective method of forecasting 700-mb contour heights over 12 radiosonde stations. It is proposed to extend the study using data for other levels also. We find that the method shows good success in forecasting the broad synoptic patterns on 5-day mean charts.

The stability of the coefficients may be tested and modified, if needed, with more independent data which is likely to become available in future.

It may, however, be pointed out that unlike the extra-tropical latitudes the temporal fluctuations of the meteorological elements are not large even for a 5-day period in the tropics, and perhaps more refined hydrodynamical or statistical models may be needed for objective medium range forecasting purposes.

7. Acknowledgement

We are grateful to Dr. P. R. Pisharoty for very useful discussions, we had in the course of the work. We are also thankful to Dr. K. R. Saha for going through the manuscript and providing invaluable suggestions. Our thanks are also due to all the members of M.R.F. Unit for their help and co-operation.

REFERENCES

Mooley, D. A.	1967	*Indian J. Met. Geophys.*, **18**, 4, p. 477.
Pant, P. S., Das, S. K. and Natarajan, T. R.	1965	*Ibid.*, **16**, 3, p. 351.
Shukla, Jagdish	1967	*Ibid.*, **18**, 3, p. 369.

APPENDIX I

Regression coefficients and constant terms

	JDP	ALB	CAL	GHT	BMB	DLH	PBL	VVL	VZG	MDS	TRV	NGP	Const
JDP	+·22	+·134	—·118	—·015	+·401	—·019	—·114	—·014	+·188	+·137	—·385	—·034	70·008
ALB	+·014	+·335	+·182	—·114	+·192	+·061	—·141	—·204	+·260	—·171	—·097	—·205	44·121
CAL	—·118	—·011	+·383	—·034	+·066	+·038	+·262	+·041	+·115	+·106	—·142	—·014	26·539
GHT	—·146	+·011	+·117	+·149	—·044	+·045	+·185	+·037	—·024	—·030	—·150	+·016	75·766
BMB	—·084	—·004	+·027	+·076	+·538	—·119	+·138	—·265	+·068	+·258	—·256	—·163	69·244
DLH	+·157	+·092	+·210	—·089	+·078	+·023	—·040	—·094	+·208	+·255	—·243	—·096	48·551
PBL	+·043	—·030	+·147	+·036	—·013	—·021	+·240	—·285	—·045	+·376	+·025	—·016	60·541
VVL	—·176	—·099	+·027	+·206	+·167	—·288	+·217	+·203	+·388	—·017	—·145	—·173	53·485
VZG	—·126	—·134	+·260	+·042	+·010	—·125	—·043	—·180	+·544	+·374	—·181	—·340	74·523
MDS	—·140	—·077	+·068	+·060	+·077	+·026	+·235	—·152	—·007	+·285	+·013	—·083	67·905
TRV	—·044	—·076	+·155	—·065	—·104	—·026	+·241	—·031	—·023	+·044	+·184	—·106	95·559
NGP	—·175	—·081	+·350	+·041	+·237	—·164	+·026	—·098	+·302	+·031	—·262	—·118	81·137

(1) JDP—Jodhpur, (2) ALB—Allahabad, (3) CAL—Calcutta, (4) GHT—Gauhati, (5) BMB—Bombay, (6) DLH—Delhi, (7) PBL—Port Blair, (8) VVL—Veraval, (9) VZG—Visakhapatnam, (10) MDS—Madras, (11) TRV—Trivandrum and (12) NGP—Nagpur. Const—Constant

APPENDIX II

		S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}	S_{11}	S_{12}	
S'_1	=	+·22	+·134	—·181	—·015	+·401	—·019	—·114	—·014	+·188	+·137	—·385	—·034	+70·003
S'_2	=	+·014	+·335	+·182	—·114	+·192	+·061	—·141	—·204	+·260	+·171	—·097	—·205	+44·121
S'_3	=	—·118	—·011	+·383	—·034	—·066	+·038	+·262	+·041	+·115	+·106	—·142	—·014	+26·539
S'_4	=	—·146	+·001	+·117	+·149	—·044	+·045	+·185	+·037	—·024	—·030	—·150	+·016	+75·766
S'_5	=	—·084	—·004	+·027	+·076	+·538	—·119	+·138	—·265	+·068	+·258	—·256	—·163	+69·244
S'_6	=	+·157	+·092	+·210	—·089	+·078	+·023	—·040	—·094	+·028	+·255	—·243	—·096	+48·551
S'_7	=	+·043	—·030	+·147	+·036	—·013	—·021	+·240	—·285	—·045	+·376	+·025	—·016	+60·541
S'_8	=	—·176	—·099	+·027	+·206	+·167	—·288	+·217	+·203	+·388	—·017	—·145	—·173	+53·485
S'_9	=	—·126	—·134	+·260	+·042	+·010	—·125	—·043	—·180	+·544	+·374	—·181	—·340	+74·523
S'_{10}	=	—·140	—·077	+·068	+·060	+·077	+·026	+·235	—·158	—·007	+·285	+·013	—·083	+67·905
S'_{11}	=	—·044	—·076	+·155	—·065	—·104	—·026	+·241	—·031	—·023	+·044	+·184	—·106	+95·559
S'_{12}	=	—·175	—·081	+·350	+·041	+·237	—·164	+·026	—·098	+·302	+·031	—·262	—·118	+81·137

APPENDIX III

A sample forecast calculation

(*i*) To compute the (forcast) 700-mb contour height over Delhi for pentad 4–8 July 1965

(*ii*) The 700-mb contour height in the pentad 29 June –3 July 1965 is known and is as follows —

	JDP	ALB	CAL	GHT	BMB	DLH	PBL	VVL	VZG	MDS	TRV	NGP
700 mb contour height 3000+ in gpm	127	87	79	77	110	105	128	98	111	122	147	105

(*iii*) Therefore, in agreement with the equation in Appendix II the 700-mb contour height over Delhi in the next pentad is—

$$=3000+127\times0\cdot157+87\times0\cdot092+79\times0\cdot210-77\times0\cdot089$$

$$+110\times0\cdot078+105\times0\cdot023-128\times0\cdot040-93\times0\cdot094$$

$$+111\times0\cdot208+122\times0\cdot255-147\times0\cdot243-105\times0\cdot096$$

$$+48\cdot55$$

$$=3091 \text{ gpm}$$

The actual observed value of 700-mb contour height over Delhi in the pentad 4–8 July 1935 was 3092 gpm