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Computation of Streamlines associated with a low latitude Cyclone

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ABSTRACT. From available wind data at 500 mb, computations were made of the vorticity field associated with a cyclone, which struck the southern half of peninsular India in December 1964. By defining a stream function ψ, such that the Laplacian of ψ is the vorticity, we solved a Poisson equation for ψ by relaxation. It was assumed that ψ vanishes along the boundary of the region chosen for study. Comparisons were then made between (*i*) wind components computed from the gradient of ψ and (*ii*) the actual wind. Applying statistical tests, we find that there is a good agreement between the computed and the observed wind, suggesting that at 500 mb, the non-divergent part of the wind vector provides a good approximation to the actual wind.

1. Introduction

Synoptic meteorologists at low latitudes have been concerned about the lack of geostrophic balance in their region. To get over this difficulty, Riehl (1954) and Palmer *et al.* (1955) have suggested the kinematic method of analysis. The main difficulty, however, is that the streamlines drawn by this technique cannot be readily used in any prediction model. On the other hand, if we define a stream function such that the wind vector is measured by its gradient, we have a useful method of computing the non divergent part of the vector wind. It is generally accepted that the non-divergent component of the wind is about an order of magnitude larger than the irrotational component, and some support for this has been provided by Scott (1958). The method of delineating the wind field by a stream function has also been considered by Brown and Neilon (1961), Bedient and Vederman (1964) and Hawkins and Rosenthal (1965). In this paper we have computed the stream function associated with a cyclone which crossed the southern parts of India in December 1964. The purpose of this paper is to examine, on a statistical basis, how good an approximation to the actual wind was provided by the stream function at 500 mb.

We realise that the importance of the irrotational component of the wind vector, as compared to the non-divergent part is still a matter for further investigation at low latitudes. Thus, Hawkins and Rosenthal (1965) found that the stream function could not locate a cyclone at 1000 mb. But a part of this failure could be attributed to the choice of boundary conditions in the case studied by Hawkins and Rosenthal (*loc. cit.*). We have confined the present investigation to only 500 mb, where the irrotational component is likely to be relatively unimportant.

2. Theoretical considerations

If $\mathbf{V}$ represents the wind vector in a non-divergent wind field where $\nabla.\mathbf{V}=0$, then the stream function Ψ is related to $\mathbf{V}$ by

$$\mathbf{V} = \mathbf{k} \times \nabla \Psi \tag{2·1}$$

where $\mathbf{k}$ is the unit vector along the vertical, $\nabla = \mathbf{i}\,(\partial/\partial x) + \mathbf{j}\,(\partial/\partial y)$ and $\mathbf{i}$, $\mathbf{j}$ are unit vectors along the X, Y axes in a rectangular co-ordinate system.

If we represent the vertical component of vorticity by ζ, then from (2·1) it follows that —

$$\nabla^2\Psi = \zeta \tag{2·2}$$

Equation (2·2) can be solved as a boundary value problem for Ψ, if we know ζ. We can evaluate ζ from the actual wind field by computing the zonal u and the meridional v components of the observed wind. Using finite differences for derivatives, we have —

$$\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \simeq \frac{\triangle v}{\triangle x} - \frac{\triangle u}{\triangle y} \tag{2·3}$$

where $\triangle u$, $\triangle v$, $\triangle x$, $\triangle y$ represent finite increments in u, v, x and y respectively. By choosing sufficiently small increments, we may estimate the right hand side of (2·2) at each nodal point on a rectangular grid. Equation (2·2) can then be solved by relaxation. For boundary conditions we have assumed that Ψ vanishes along the boundary of the region under consideration. The boundaries were placed at a distance sufficiently away from the centre of the storm, so as to eliminate, as far as possible, the propagation of errors from the boundary into the interior of the region.

3. Numerical results

In Fig. 1, we have shown the 500-mb contours and upper winds at 1200 GMT on 23 December 1964. This chart corresponds to the period when the storm was centred over the extreme south of the Indian Peninsula. In Figs. 2 and 3 we show

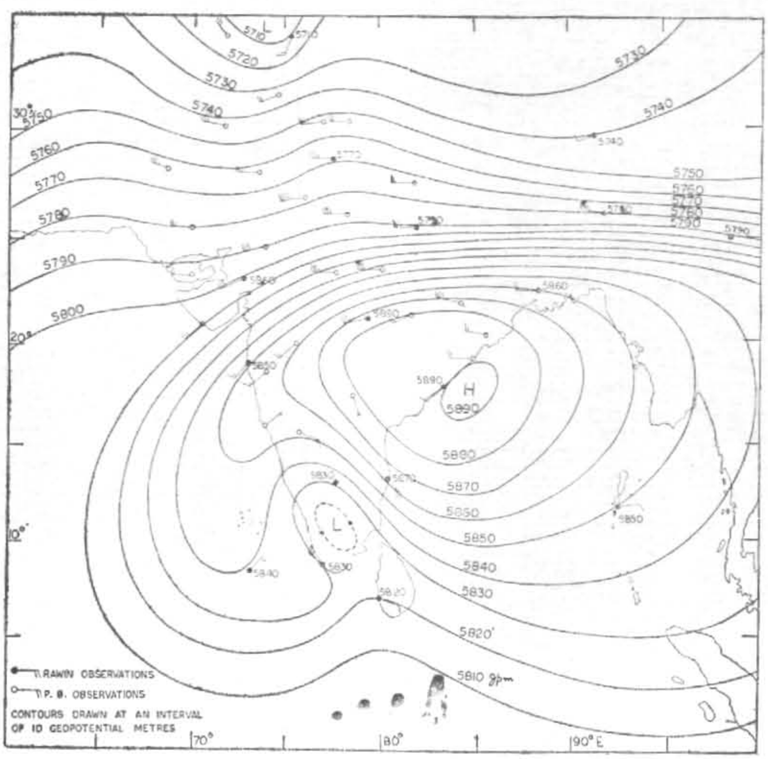

Fig. 1. 500-mb contours and upper winds at 1200 GMT of 23 December 1964

isopleths of u and v at 500 mb. These isopleths were drawn taking into consideration all the available upper wind data shown in Fig. 1. From the isopleths shown in Figs. 2 and 3, we computed ζ using (2·3). Our computations of ζ at each nodal point are shown in Fig. 4. In Fig. 4, we also show the size of the grid used by us.

If h represents the distance between any two nodal points, the equation for the residual ($R_{x,\,y}$) at any point x, y is —

$$R_{x,y} = \sum_{1}^{4} \Psi - 4\,\Psi_{x,y} - h^2\,\zeta_{x,y} \qquad (3\cdot1)$$

where $\sum_{1}^{4} \Psi$ represents the sum of Ψ at four adjacent points surrounding the point x, y.

As is usually done in obtaining a solution by relaxation, we liquidated all residuals by successive iteration. In the final result, care was taken to see that the sum of all the remaining residuals was negligibly small.

In our work we put

$$h = 2 \times 10^5 \text{ metres} \qquad (3\cdot2)$$

Trials made with a finer mesh indicate that truncation errors on account of the above choice of h was not likely to exceed 5 per cent. As mentioned earlier, it was assumed that $\Psi = 0$ along the boundaries, ABCD, of our rectangular grid.

The isopleths of $\Psi \times 10^1$ (m^2 sec^{-1}), as obtained by us, are shown in Fig. 5. It may be noted that there is a good agreement between the streamline pattern and the observed wind field

4. The observed and the computed wind fields

To test the agreement between the computed and the observed wind fields, we evaluated the zonal and the meridional components of the computed wind by measuring the gradient of the stream function Ψ. We have —

$$u_{\text{cal}} = -\,\partial\Psi/\partial y, \; v_{\text{cal}} = \partial\Psi/\partial x \qquad (4\cdot1)$$

The comparison between u_{cal}, v_{cal} and u_{obs}, v_{obs} was made in two sets. In the first set A, we considered twenty stations between 20° and 30°N, where the observed wind was mainly zonal. In the second set B, we considered eleven stations between 4° and 20°N, *i. e.*, we considered the region affected by the storm. The results of the comparison are shown in Table 1.

To provide a statistical basis for the comparison, we computed χ^2 between the calculated and the observed wind. The computed values of χ^2 are 27·26 and 20·86 for u and v components in set A. In set B, the computed values of χ^2 are 17·68 and 14·44. From tables of χ^2 distribution

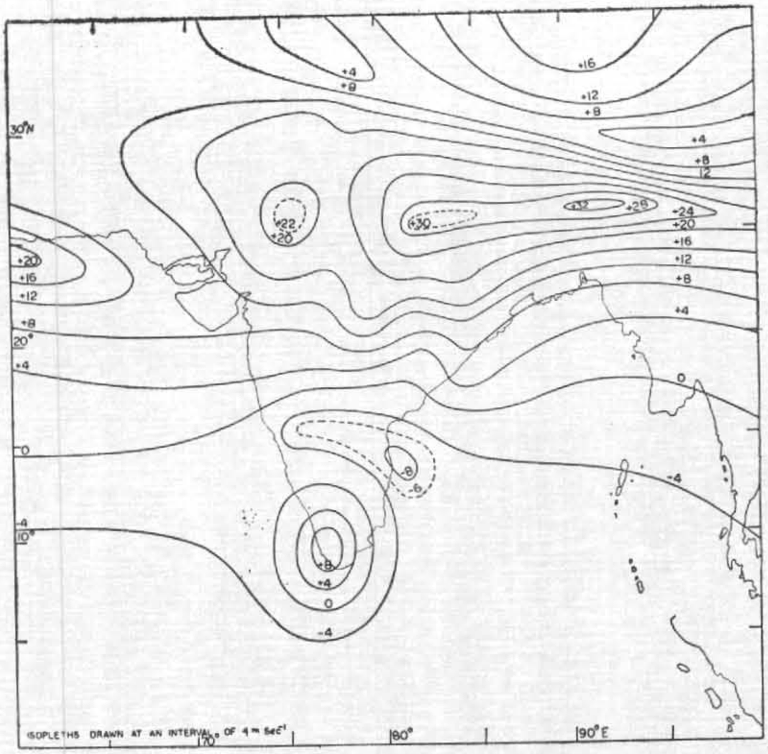

Fig. 2. Isopleths of zonal component (*u*)

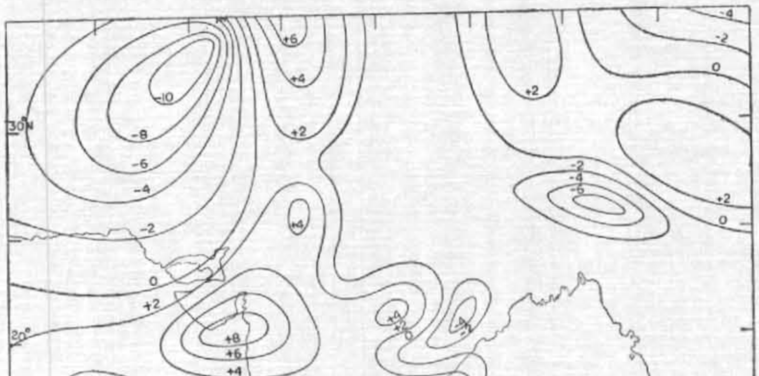

Fig. 3. Isopleths of meridional component (*v*)

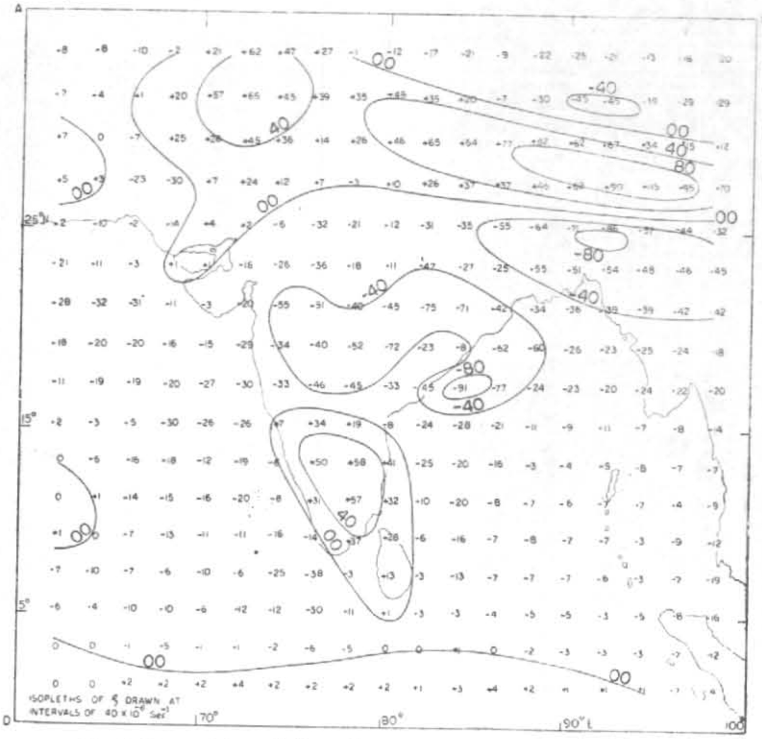

Fig. 4. Computed vorticity (ζ) in 10^{-6} sec^{-1} at each grid point

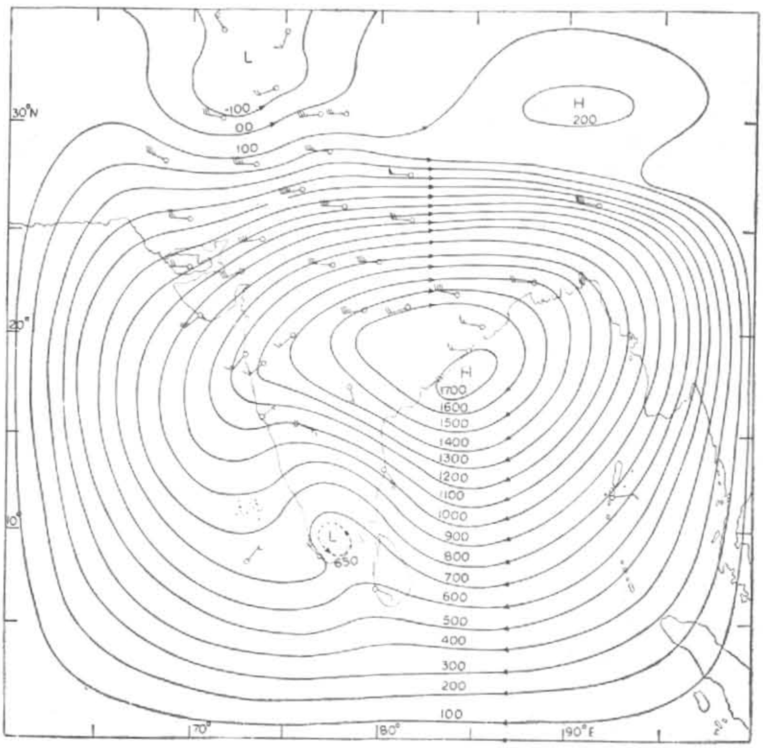

Fig. 5. Isopleths of stream function ($\psi \times 10^{4}$) in m^2 sec^{-1} at 1200 GMT of 23 December 1964

TABLE 1

Comparison between observed and calculated wind components (m. sec^{-1})

Station	Zonal component				Meridional component			
	obs.	cal.	Difference $=X$	$\frac{X^2}{\text{cal}}$	obs.	cal.	Difference $=Y$	$\frac{Y^2}{\text{cal}}$
			A : *Between Lat. 20° and 30°N*					
Gauhati	33	27	6	1·33	—6	—8	2	0·50
Calcutta	11	10	1	0·10	0	—2	2	2·00
Bhubaneshwar	8	4	4	4·00	—1	—1	0	0
Bahraich	25	20	5	1·25	0	0	0	0
Allahabad	31	25	6	1·44	0	0	0	0
Jharsuguda	19	13	6	2·77	—3	—2	1	0·50
Raipur	10	10	0	0	4	2	2	2·00
Delhi	18	13	5	1·92	0	2	2	2·00
Jaipur	22	17	5	1·47	4	4	0	0
Gwalior	20	16	4	1·00	0	0	0	0
Bhopal	16	13	3	0·69	0	2	2	2·00
Jabalpur	21	16	5	1·56	0	1	1	1·00
Nagpur	17	12	5	2·08	3	3	0	0
Bikaner	17	13	4	1·23	0	0	0	0
Udaipur	19	15	4	1·07	3	7	4	2·29
Ahmedabad	14	12	2	0·33	5	7	2	0·57
Veraval	9	6	3	1·50	7	7	0	0
Jacobabad	15	10	5	2·50	—6	—3	3	3·00
Chhor	13	10	3	0·90	0	1	1	1·00
Bhuj	9	8	1	0·12	0	4	4	4·00
				$\chi^2=27\cdot26$				$\chi^2=20\cdot86$
			B : *Between Lat. 4° and 20°N*					
Port Blair	—4	—7	3	1·29	—1	—7	6	5·14
Gopalpur	7	5	2	0·80	0	1	1	1·00
Madras	—7	—10	3	0·90	9	10	1	0·10
Colombo	—5	—6	1	0·17	2	2	0	0
Hyderabad	—1	—7	6	5·14	3	5	2	0·80
Aurangabad	7	6	1	0·17	5	8	3	1·12
Gadag	—7	—10	3	0·90	3	1	2	4·00
Trivandrum	10	7	3	1·29	—9	—7	2	0·57
Vengurla	—2	—9	7	5·44	—2	—1	1	1·00
Poona	5	3	2	1·33	5	7	2	0·57
Bombay	5	4	1	0·25	6	7	1	0·14
				$\chi^2=17\cdot68$				$\chi^2=14\cdot44$

TABLE 2

Comparison between observed wind and geostrophic wind (m/sec) between Lat. 20° and 30°N

Station	Observed wind	Geostrophic wind	Difference $=X$	$\frac{X^2}{\text{Geo. wind}}$
Gauhati	34	28	6	1·29
Calcutta	11	23	12	6·26
Bhubaneshwar	8	14	6	2·57
Allahabad	31	21	10	4·76
Jharsuguda	19	24	5	1·04
Raipur	11	24	13	7·04
Bahraich	25	12	13	14·08
Delhi	18	10	8	6·40
Jaipur	22	10	12	14·40
Gwalior	20	15	5	1·67
Bhopal	16	26	10	3·85
Jabalpur	21	30	9	2·70
Nagpur	17	25	8	2·56
Bikaner	17	11	6	3·27
Udaipur	19	13	6	2·77
Ahmedabad	15	15	0	0
Veraval	13	13	0	0
Jacobabad	17	9	8	7·11
Chhor	13	9	4	1·78
Bhuj	9	9	0	0
				χ^2=83·55

(Hoel 1958), it is found that the critical value of χ^2 at 5 per cent level is of the order of 30·14 for 19 degrees of freedom. We, therefore, conclude that the computed value of χ^2 for set A is not significant, *i.e.*, the agreement is good. For set B, the critical value of χ^2 is 18·31 for 10 degrees of freedom. Consequently, in this case also, the agreement between the observed and the calculated wind is good.

In Table 2, we made a similar comparison between the observed wind and the geostrophic wind at 20 stations between 20° and 30°N. This region was selected for comparison because the winds here were mainly zonal and the meridional component was negligible.

In this case, the computed value of χ^2 was found to be 83·55, whereas the critical value of χ^2 at 5 per cent level of significance for 19 degrees of freedom is only 30·14. We may, therefore, infer that difference is significant, and conclude that the geostrophic wind is a poor approximation to the actual wind. On the other hand, the results for set A (Table 1) indicate that the wind components computed from the Ψ field for the same region provide a better approximation.

5. Summary and conclusions

In this study, we have examined only one situation. While we cannot draw general conclusions from only one synoptic situation, it is possible to summarise the main results from this study as follows —

(*i*) Computations of the non-divergent part of the wind for the storm of December 1964 reveal good agreement between the calculated and the observed wind field at 500 mb. The non-divergent wind components were computed by measuring the gradient of the stream function Ψ.

(*ii*) At regions north of 20°N, the observed wind field was mainly zonal. Nevertheless, computations showed poor agreement between the geostrophic and the actual wind. On the other hand, agreement between the non-divergent wind and the actual wind was better.

(*iii*) There appears to be further need for research in the selection of the most appropriate boundary condition for solving equation (2·2). In the present study, we assumed that Ψ vanishes along the boundary, but more realistic boundary conditions can, perhaps, be considered.

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