Nomograms for evaluating Richardson Number for forecasting Clear Air Turbulence

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ABSTRACT. Research in the last decade has shown that one of the meteorological parameters which is associated with clear air turbulence (CAT) is Richardson's Number. To make Richardson number Ri useful for forecasting the location and intensity of CAT, we made a study under Indian conditions of the association between CAT and different ranges in the value of Ri . In this paper a set of nomograms is presented by which Ri may be estimated without elaborate calculations and with a fair degree of accuracy for the layer between 300 and 200 mb. Similar nomograms can be prepared for other isobaric levels for the use of different types of aircraft in operation in India. If a singularity, like a wind maximum, is found between 300 and 200 mb, Ri may be also computed separately for the layer between 300 mb and the level of maximum wind and for the layer between the level of maximum wind and 200 mb.

1. Introduction

The phenomenon of clear air turbulence is a major hazard for high level aviation. All turbulence causes varying degrees of discomfort. But occasionally aircraft experience, without visible indication or warning, severe jolts or bumps, which are referred to as clear air turbulence (CAT). While discussing the subject of CAT, Reiter (1963) observed "Dimensions of turbulent areas are usually small. Occasionally one may, however, run into turbulence extending over several hundreds of kilometres. As an average value one may take 900 metres in the vertical and 75 to 100 kilometres in the horizontal". Although CAT has been observed and studied for well over a decade, much remains to be known. There is no satisfactory method at present for forecasting areas of CAT at aviation meteorological offices.

A parameter which has been found, useful by many workers in this field, is Richardson's number R_i . This expresses the ratio of the static stability of the atmosphere to the square of the vertical wind shear at a given point. Under ideal conditions, values of R_i greater than unity are associated with no turbulence, while values less than unity are associated with turbulent conditions. It has to be recognised that the phenomenon of CAT occurs in various dimensions, ranging from small scale motion to meso-scale phenomena. Opinion is rather divided on the practical utility of Richardson's criterion for CAT forecasting. But, considering the fact that most of the other criteria are rather qualitative, and that there is a practical need for objective methods for forecasting turbulence, it is felt that careful estimates of R_i are likely to provide a useful practical tool for prediction of turbulence.

Pinus (1957) found that 85 per cent of high level turbulence was associated with R_i values less than 4. Berenger and Heissat (1959) indicate that with $R_i \leqslant 1$, 70 per cent or more flights report turbulence and with $R_i > 8$, less than 10 per cent do so. Rustenback (1963) studying CAT over USA finds that R_i is a useful parameter under certain meteorological and geographical conditions. He finds that the best correlation between low R_i and CAT occurred in the eastern half of USA below both the level of maximum wind and the tropopause. Briggs (1961) studied 105 cases of severe CAT at heights above 10,000 ft in the vicinity of the British Isles, and found that about 70 per cent of CAT were associated with R_i below 5. As regards forecasting, Briggs finds that the mean values of $R_i < 5$ and of horizontal shear $> 0 \cdot 3$ hr^{-1} cover more than 80 per cent of the occurrence of CAT. Rao (1964) observed that the lower the R_i , the greater is the probability of turbulence and the more severe it is likely to be. Theoretically, it has been shown by Sutton (1953) that turbulence will decay when $R_i > (1—a)$, where a is a positive number whose value has not been determined in general. Following this many authors have assumed that CAT occurs in regions of $R_i < 1$.

2. Nomograms for calculating Richardson Number

In this paper a set of nomograms is presented, which may be used for a quick calculation of R_i from upper air teletype messages, for the layer 300—200 mb. Jet aircraft usually cruise in this layer. Mazumdar and Jamil (1962) have briefly discussed the essential details of Richardson's theory of atmospheric turbulence and have cited several interesting cases of turbulence. In a

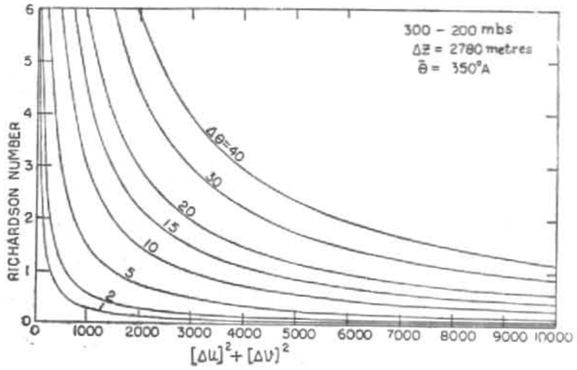

Fig. 1

simplified form, Richardson's number is given by the formula —

$$R_i = g\left(-\frac{\partial\theta}{\partial z}\right)\Big/\bar{\theta}\ \left[\left(\frac{\partial u}{\partial z}\right)^2+\left(\frac{\partial v}{\partial z}\right)^2\right] \quad (2\cdot1)$$

where $(\partial\theta/\partial z)$ is the lapse rate of potential temperature θ, and u and v are the components of the horizontal wind in the west-east and south-north directions respectively. Now, considering a layer bounded by two pressure surfaces 300 mb and 200 mb, and assuming that θ, u and v vary linearly with height z in the layer, R_i may be obtained from the relation —

$$R_i = g\left(\frac{\triangle\theta}{\triangle z}\right)\Big/\bar{\theta}\ \left[\left(\frac{\triangle u}{\triangle z}\right)^2+\left(\frac{\triangle v}{\triangle z}\right)^2\right] \quad (2\cdot2)$$

where $\triangle u$ and $\triangle v$ represent differences between the upper and lower surfaces of the layer in u and v and $\triangle z$ is the thickness of the layer. $\triangle\theta$ is the difference in potential temperature between the upper and lower surface. The sign of $\triangle\theta$ is important as it makes R_i positive or negative. The expression may be simplified to the form —

$$R_i = \left(\frac{g\,\triangle z}{\bar{\theta}}\right)\left[\frac{\triangle\theta}{(\triangle u)^2+(\triangle v)^2}\right] \quad (2\cdot3)$$

where $\bar{\theta}$ is the mean potential temperature of the layer. To a first approximation $\triangle z$ and $\bar{\theta}$ may be assumed to remain constant. Mean values of $\triangle z$ and $\bar{\theta}$ have been worked out using the evening radiosonde data of winter and summer months for the 5-year period between 1956—1960. This has been computed for 12 Indian radiosonde stations. Using these values, the curves of Fig. 1 are drawn for different values of $\triangle\theta$, with $[(\triangle u)^2 + (\triangle v)^2]$ along the x-axis and R_i along the y-axis. From the large number of R_i values determined using this nomogram for Indian and extra Indian stations it is found that as $\triangle z$ increases $\bar{\theta}$ also increases, so that $(\triangle z/\bar{\theta})$ remains fairly close to a constant value. If greater accuracy is needed with the

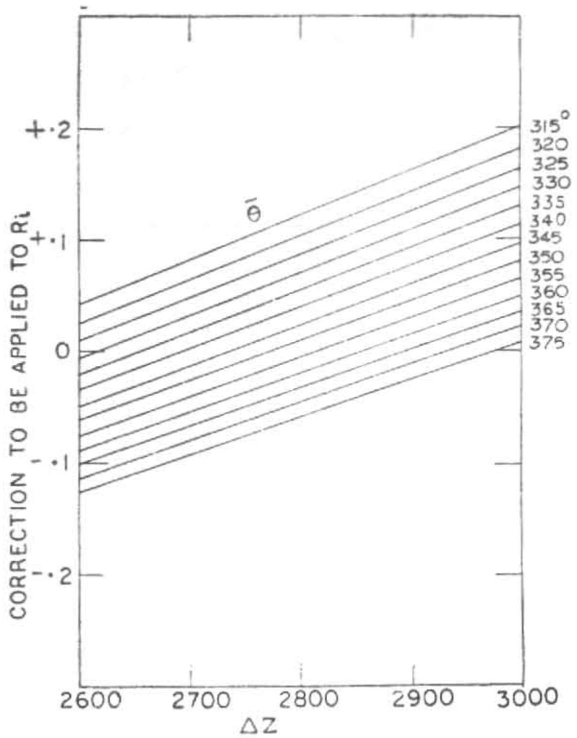

Fig. 2. Fractional correction to be applied to the value of R_i using the constant values of $\triangle z$ as 2780 m and $\bar{\theta}$ as 350° A

available data, the actual values of $\triangle z$ and $\bar{\theta}$ may be taken from upper air messages and a suitable correction may be applied to the value of R_i obtained from Fig. 1. The value of the correlation may be determined by using Fig. 2 but, in general, this is small.

In this paper derivatives are replaced by finite differences. Thus $\partial\theta/\partial z$ is taken as approximately equal to $\triangle\theta/\triangle z$. The upper air teletype messages give wind and temperature values only for standard isobaric levels like 300 mb and 200 mb. Therefore, only by this approximation can the available data be made use of, if one takes the wind and temperature as varying linearly with height in the layer under consideration. However, if the wind reaches a maximum value in the layer or if the temperature lapse rate changes abruptly within the layer, then this will not be a good approximation. Taking the case of a wind maximum between 300 and 200 mb, values of R_i for (*i*) the layer between 300 mb and the level of maximum wind, and (*ii*) the layer between the level of maximum wind and 200 mb can be separately computed by using Fig. 3. The method is to take the thickness of the 300—200 mb layer as approximately constant (2780 m). Then, from 300 mb to the level of maximum wind, let us represent the change in the components of wind by $\triangle u'$ and $\triangle v'$. The corresponding values at the same rate of increase or decrease for the entire layer of thickness 2780 m are found by

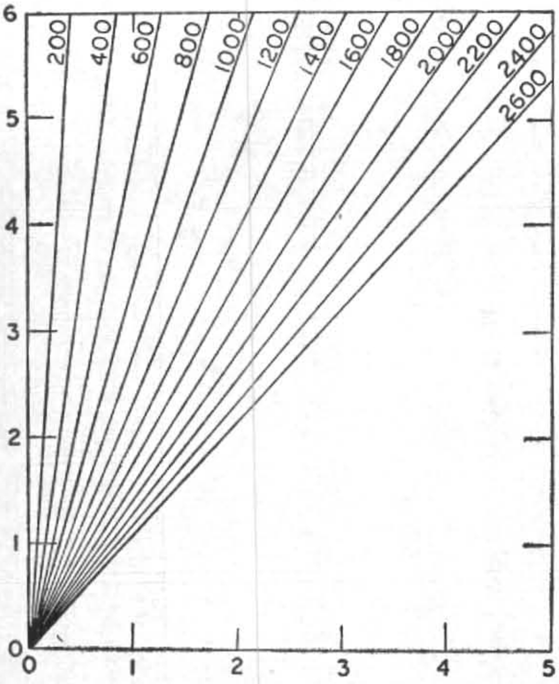

Fig. 3

using Fig. 3. In Fig. 3 these values are given along the y-axis. The $\triangle u'$, $\triangle v'$ or $\triangle \theta'$ values are given along the x-axis. The inclined lines are for different values of $\triangle z'$. Knowing the values of $\triangle u$, $\triangle v$ and $\triangle \theta$, R_i can be found by using Fig. 1.

3. Method of using the Nomograms

To illustrate the method of using the nomograms we present two examples.

Case (1)

The following data are extracted from a teletype message giving the 0000 Z radiosonde-rawin ascent for Port Blair on 1 August 1964.

	Wind (Direction and speed)	Temperature (°C)	Height (gpm)
300 mb	080°/06 kt	−30·2	9650
200 mb	100°/46 kt	−54·0	12390
Maximum wind	100°/66 kt	—	17060

As the maximum wind level is above 200 mb, and as no change in the temperature lapse rate is reported in the layer, it is assumed that the wind and temperature vary linearly with height within the layer. We have —

$\triangle\theta = 347—343 = +4°$, $\triangle u = 39$ kt and $\triangle v = 9$ kt. So $[(\triangle u)^2 + (\triangle v)^2] = 1602$. From Fig. 1 R_i is read off as $0{\cdot}72$.

$\triangle z = 12390 - 9650 = 2740$ m and $\bar{\theta} = (343 + 347)/2 = 345°$A

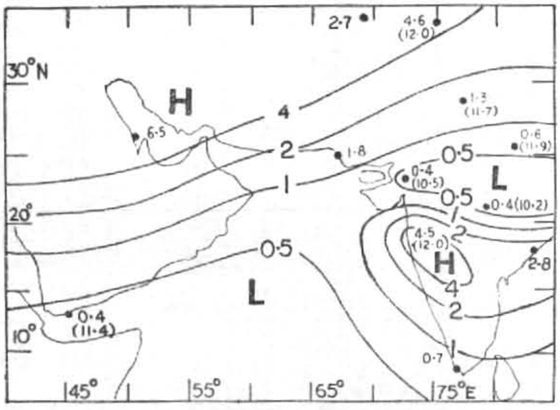

Fig. 4. Synoptic R_i Chart 000Z, 24 December 1963, for 300-200 mb layer

From Fig. 2, the fractional correction to be applied to $0{\cdot}72$ is $+0{\cdot}001$. Hence the corrected value of $R_i = 0{\cdot}72$, as the correction is negligible.

Case (2)

The data given below are from the 0000 Z upper air observations at Ahmedabad on 31 December 1964.

	Wind (Direction and speed)	Temperature (°C)	Height (gpm)
300 mb	260°/87 kt	—35·8	9480
200 mb	260°/108 kt	—49·5	12220
Maximum wind	250°/136 kt	—	10870

As there is no change in temperature lapse rate reported in the layer, the potential temperature is taken as increasing linearly with height from 300 to 200 mb. $\triangle\theta = 19°$. For the layer 300 mb to the level of maximum wind, $\triangle z' = 1390$ m, $\triangle u' = 42$ kt and $\triangle v' = 32$ kt. Using Fig. 3, the corresponding values of $\triangle u$ and $\triangle v$ for 300—200 mb (*i.e.*, for 2780 m) are determined. Taking $\triangle u' = 42$ kts along x-axis and finding out the inclined line for 1390 m, the $\triangle u$ value is read off along the y-axis as 85 kts. Similarly $\triangle v$ is found to be 64 kts. Hence, $[(\triangle u)^2 + (\triangle v)^2] = 11321$. Then R_i is found for the layer 300 mb to the level of maximum wind using Fig. 1 as $0{\cdot}49$ (R_i for $11321/2 = 5660$ m and $\triangle\theta = 19°$ is found as $0{\cdot}98$. Half this gives the required value).

4. Conclusion

Using these nomograms R_i values can be computed fairly quickly. Quickness and simplicity are useful for operational use. Daily computations of R_i along the principal air routes may lead to valuable results. R_i can be also studied with other parameters, like, horizontal wind shear, curvature of stream lines etc to develop reliable guides for CAT prediction.

One of the authors (Joseph) is trying to compute synoptic R_i charts for forecasting areas of high probability for clear air turbulence occurrence. A synoptic R_i chart for the 300—200 mb layer for 0000 Z of 24 December 1963 is given in Fig. 4. Areas of low and high values of R_i are marked in the figure. It may be of interest to note that on this day a jet aircraft at about 10·5 km, flying from Aden to Bombay at about 0000 Z, experienced almost continuous moderate clear air turbulence from Aden to 60°E and severe clear air turbulence from 60° to 65°E, while the rest of the flight was smooth.

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