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A note on evaluation of the Richardson Number in relation to forecasting Clear Air Turbulence

P. S. N. SASTRY

Meteorological Office, Santacruz

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ABSTRACT. For forecasting clear air turbulence, Richardson's Number R_i is one of the widely used parameters. With increase in air traffic and aircraft operating at very high altitudes, the necessity for providing accurate information on turbulence in clear air is keenly felt. To facilitate the evaluation of R_i as soon as the upper air messages are received, nomograms and tables have been worked out and two simple methods are described for calculating R_i quickly.

1. Introduction

Turbulent conditions experienced by an aircraft in flight may be classified into four main categories —(*a*) Thermal, (*b*) Convective, (*c*) Mechanical and (*d*) Shear.

For aircraft cruising at high altitudes, the important types of turbulence are (*b*), (*c*) and (*d*) mentioned above. Type (*b*) is associated with clouds of great vertical development. Types (*c*) and (*d*) are termed as Clear Air Turbulence (CAT) or High Level Turbulence (HLT). This mechanical type turbulence is caused by mountain waves. Some theoretical and empirical rules have been obtained for forecasting this type of turbulence. Various attempts have been made to forecast turbulence caused by wind shear but no definite technique has been yet formulated. However, a certain amount of correlation has been found between low values of R_i and shear type of turbulence. Two simple methods for rapid calculation of R_i are, therefore, presented below.

2. Richardson Number and Clear Air Turbulence (CAT)

Clodman (1961) has summarised our knowledge about clear air turbulence and the various techniques for its prediction. However, it may be mentioned that the mechanism which causes clear air turbulence is not yet clearly understood.

The view has been often expressed that clear air turbulence is related to low values of Richardson's Number (R_i). For delineating synoptically favourable areas for the occurrence of CAT, we need a quick method for computing R_i . The nomograms and tables presented below will facilitate the calculation of R_i and the method does not involve many assumptions.

3. Evaluation of R_i

We define the Richardson Number R_i by

$$R_i = g\left(\frac{\partial u}{\partial z}\right) \Big/ \bar{\theta}\left[\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2\right] \quad (3\cdot 1)$$

where $\partial\theta/\partial z$ is the lapse rate of potential temperature θ , u and v are the east and north components of the wind and g is the acceleration due to gravity. Assuming that θ , u and v vary linearly with height in a layer of thickness of the order of 3 kilometres or less, the above expression may be written as —

$$R_i = g\,\frac{\triangle z}{(\triangle u)^2 + (\triangle v)^2}\,\frac{\triangle\theta}{\bar{\theta}} = g\,\frac{\triangle z}{K^2}\cdot\frac{\triangle\theta}{\bar{\theta}} \quad (3\cdot 2)$$

$$\text{where}\quad K^2 = (\triangle u)^2 + (\triangle v)^2$$

$\triangle z$ is the thickness between two layers whose mean potential temperature is $\bar{\theta}$, and $\triangle\theta$ is the difference in potential temperature between the two boundaries of the layer under consideration. Of the four variables on the right hand side of the equation,

(*i*) $\triangle z$ can be obtained directly from data in upper air messages,

(*ii*) $\triangle\theta$ and $\bar{\theta}$ can be found from a tephigram, and

(*iii*) K can be obtained with the help of a polar diagram (Fig. 1) using the principle of vector triangles.

Having obtained the values of the different variables, the two nomograms given in Figs. 2 and 3 can be used to get the values of $\triangle\theta / \bar{\theta}$ and $\triangle z / K^2$ respectively. Using these two values, R_i can be read off directly from Table 1.

A further simplification of the above process can be made, if a slight loss of accuracy is accepted. Values of R_i at any level vary considerably from place to place on all days. The variation generally covers a range from $0\cdot 1$ to 100. Thus, a small loss of accuracy of about 20 per cent will not affect the interpretation of the R_i distribution for CAT forecasting purposes. If this is accepted, a much faster calculation is possible.

TABLE 1

Richardson Number

$(\triangle\theta/\bar{\theta}) \times 10^2$	$\triangle z/K^2$																	
	1·0	1·5	2·0	2·5	3·0	3·5	4·0	4·5	5·0	5·5	6·0	6·5	7·0	7·5	8·0	8·5	9·0	9·5
0·5	·05	·07	·10	·12	·15	·17	·20	·22	·25	·27	·29	·32	·34	·37	·39	·42	·44	·47
1·0	·10	·15	·20	·25	·29	·34	·39	·44	·49	·54	·59	·64	·69	·73	·79	·83	·88	.93
1·5	·15	·22	·29	·37	·44	·51	·59	·66	·73	·81	·88	·95	·103	1·10	1·20	1·25	1·32	1·40
2·0	·20	·29	·39	·49	·59	·68	·79	·88	·98	1·08	1·18	1·27	1·37	1·47	1·57	1·66	1·77	1·86
2·5	·25	·37	·49	·61	·73	·85	·98	1·10	1·23	1·35	1·47	1·59	1·71	1·84	1·96	2·08	2·21	2·33
3·0	·29	·44	·59	·73	·88	1·03	1·18	1·32	1·47	1·62	1·77	1·91	2·06	2·21	2·35	2·50	2·65	2·79
3·5	·34	·51	·69	·86	1·03	1·20	1·37	1·44	1·72	1·93	2·06	2·23	2·40	2·57	2·75	2·91	3·09	3·26
4·0	·39	·59	·79	·98	1·17	1·37	1·57	1·76	1·96	2·16	2·35	2·55	2·75	2·94	3·14	3·33	3·53	3·72
4·5	·44	·66	·88	1·10	1·32	1·54	1·76	1·98	2·21	2·43	2·65	2·87	3·09	3·31	3·53	3·75	3·97	4·19
5·0	·49	·73	·98	1·23	1·47	1·71	1·96	2·21	2·45	2·70	2·94	3·19	3·43	3·67	3·92	4·17	4·41	4·65
5·5	·54	·81	1·08	1·35	1·62	1·88	2·16	2·43	2·70	2·96	3·23	3·50	3·77	4·04	4·31	4·58	4·85	5·12
6·0	·59	·88	1·18	1·47	1·77	2·05	2·35	2·65	2·94	3·23	3·53	3·82	4·09	4·41	4·71	5·00	5·30	5·59
6·5	·63	·95	1·27	1·59	1·91	2·22	2·55	2·87	3·19	3·50	3·82	4·14	4·46	4·77	5·10	5·41	5·73	6·05
7·0	·69	1·03	1·37	1·67	2·06	2·39	2·75	3·09	3·43	3·85	4·12	4·46	4·81	5·15	5·49	5·83	6·18	6·52
7·5	·73	1·10	1·47	1·84	2·21	2·57	2·94	3·31	3·67	4·04	4·41	4·78	5·15	5·51	5·88	6·25	6·61	6·98
8·0	·79	1·18	1·57	1·96	2·35	2·74	3·14	3·53	3·92	4·31	4·71	5·10	5·49	5·88	6·28	6·66	7·06	7·45
8·5	·93	1·40	1·86	2·33	2·79	3·25	3·72	4·19	4·65	5·12	5·59	6·05	6·52	6·98	7·45	7·91	8·38	8·85

To obtain Ri — Calculate K with the help of Fig. 1.
Read $(\triangle\theta/\bar{\theta}) \times 10^2$ from Fig. 2.
Read $\triangle z/K^2$ from Fig. ?, and read off from this table the Ri for the values of $\triangle z/K^2$ and $(\triangle\theta/\bar{\theta}) \times 10^2$ obtained

TABLE 2

Richardson Number (Ri_s)

K		$\triangle\theta$									
kt	mps	2	4	6	8	10	12	14	16	18	20
10	5	7·3	14·7	22·1	29·5	36·9	44·2	51·6	59·0	66·3	73 7
19	10	1·8	3·7	5·5	7·4	9·2	11·1	12·9	14·7	16·6	18·4
29	15	·8	1·6	2·5	3·3	4·1	4·9	5·7	6·5	7·4	8·2
39	20	·5	·9	1·4	1·8	2·3	2·7	3·2	3·7	4·1	4·6
58	30	·2	·4	·6	0·8	1·0	1·2	1·4	1·6	1·8	2·0
78	40	·10	·20	·30	·41	0·51	·61	·71	·81	·91	1·0
97	50	·07	·15	·22	·29	·37	·44	·51	·59	·66	·74
116	60	·05	·10	·15	·20	·26	·31	·36	·41	·46	·51
136	70	·04	·07	·11	·15	·19	·23	·26	·30	·34	·38
155	80	·03	·05	·08	·11	·14	·17	·20	·22	·25	·28
175	90	·02	·05	·07	·09	·11	·14	·16	·18	·20	·23
194	100	·02	·04	·05	·07	·09	·11	·13	·14	·16	·18

$Ri = (\triangle z/\bar{\theta}).(\triangle\theta/K^2).\,g$, $\triangle z/\bar{\theta} = 9{\cdot}4$ (in metres), K—as obtained from Fig. 1 (in metres /sec)

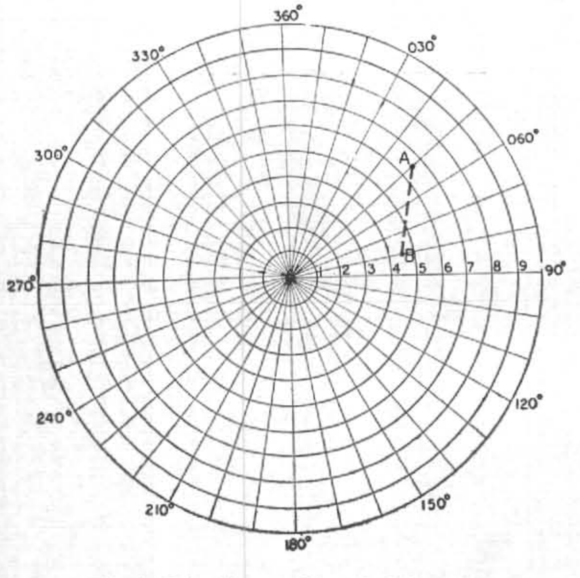

Fig. 1. Polar diagram for calculating K
A represents 050°/65 kt, B represents 080°/45 kt
Distance AB on the same scale gives K

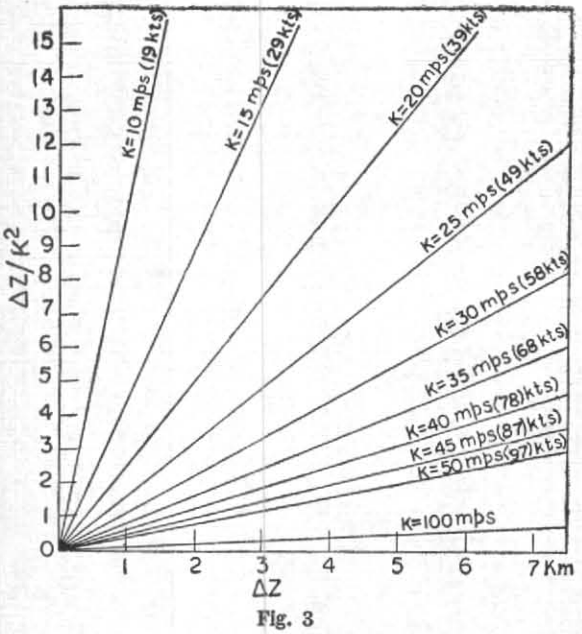

Fig. 3

Even though the individual values of $\triangle z$ and $\bar{\theta}$ for standard isobaric intervals vary fairly largely from level to level and from day to day, the variation in the ratio $\triangle z / \bar{\theta}$ is very small. We find from an examination of widely different upper air conditions that the ratio remains within about 20 per cent of 9·4. Assuming a constant value of 9·4 for $\triangle z / \bar{\theta}$, R_i can be determined directly in terms of $\triangle\theta$ and K from Table 2, after obtaining $\triangle\theta$ and K.

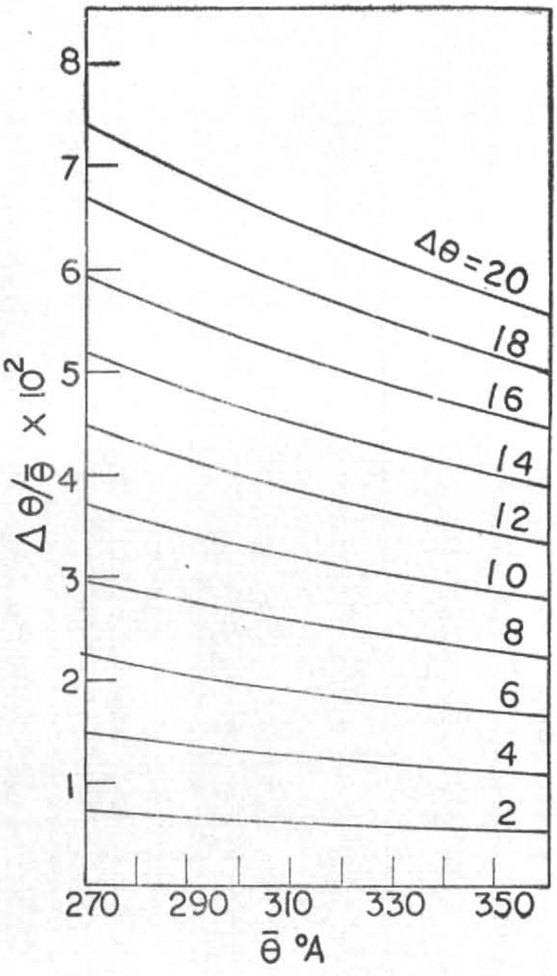

Fig. 2

For obtaining R_i for layers between the jet/maximum wind level and a standard isobaric level, a correction has to be applied. Usually the level of maximum wind is around 10·5 km (250 mb) where the normal value of $\bar{\theta}$ is 340°A. z for this value of $\bar{\theta}$ is 3250 metres. Hence, R_i for any layer at such a level can be evaluated from the expression—

$$R_i' = (R_{i\,s} \times \triangle z') / 3250 \qquad (3\cdot3)$$

where $R_{i\,s}$ is the value given in Table 2 and $\triangle z'$ is the thickness of the layer between the level of maximum wind and 300/200 mb. Table 3 gives us R_i values for different values of $\triangle z'$ and $R_{i\,s}$.

4. Summary and Conclusions

The only assumption we have made is the linear variations of θ , u and v with height.

By making the additional assumption that $\triangle z/\bar{\theta}$ is 9·4 for the layers 1000 to 700, 700 to 500, 500 to 300 and 300 to 200 mb; a quicker method can be used for evaluating $R_{i\,s}$.

TABLE 3

Ri_s	$\triangle z'$ (metres)									
	100	200	300	400	500	600	700	800	900	1000
0·5	·01	·03	·05	·06	·08	·09	·11	·12	·14	·15
1·0	·03	·06	·09	·12	·15	·19	·21	·25	·28	·31
2·0	·06	·12	·19	·25	·31	·37	·43	·49	·55	·62
3·0	·09	·19	·28	·37	·46	·55	·65	·74	·83	·92
4·0	·12	·25	·37	·43	·61	·74	·86	·98	1·10	1·20
5·0	·15	·31	·46	·61	·77	·92	1·00	1·20	1·40	1·50
6·0	·18	·37	·55	·74	·92	1·10	1·300	1·500	1·700	1·800
7·0	·21	·43	·65	·86	1·10	1·30	1·50	1·70	1·90	2·10
8·0	·25	·49	·73	·98	1·20	1·50	1·70	2·00	2·20	2·50
9·0	·28	·55	·83	1·10	1·40	1·70	1·90	2·20	2·50	2·80

To use this table — Calculate K from Fig. 1 between the level of maximum wind/jet and a standard isobar value 200 mb or 300 mb etc. Note $\triangle\theta$ and $\triangle z'-\triangle z'$ is the thickness in metres between the level of maximum wind/jet and a standard isobar value 200 mb or 300 mb etc

From Table 2 find out Ri_s. Read against Ri_s and $\triangle z'$ to obtain the Richardson Number for the layer

R_i can be calculated for any layer of the atmosphere, *i.e.*, between the mandatory levels reported in upper air code, or between any intermediate level, such as, significant levels, level of maximum wind etc, and a standard isobaric level, provided the height and temperature and wind data of the levels are known.

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Clodman, J. 1961 *W.M.O. Tech. Note*, 39.