Distribution of energy in *P* and *S* Seismic bodily waves

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ABSTRACT. It is generally assumed that the energy in seismic bodily *S* waves is double that in the seismic bodily *P* waves. A theoretical explanation based on Debye's theory of specific heats of solids is suggested. The deduced result could also explain the observed fact that the wave frequency of *P* waves is generally higher as compared to the wave frequency of *S* waves.

Gutenberg and Richter (1942) in calculating the energy of earthquakes used only the maximum waves, which at short distances are *S* waves. The energy in *P* waves was neglected. Subsequently Gutenberg and Richter (1956) have, however, taken the energy of *P* waves as half that in *S* waves. The authors did not, however, indicate any theoretical basis for this type of partition of energy in seismic bodily waves. In this communication, an attempt will be made to establish some theoretical basis for the partition of energy in *P* and *S* type seismic waves. It is also generally observed that the periods of *P* waves are comparatively shorter than the periods of *S* group of waves. This will also follow from the deduction.

Debye (1912) in his celebrated theory on the specific heats of solids has shown that the number of modes of longitudinal vibrations contained between frequency limits ν and $\nu+d\nu$ enclosed in volume V is

$$= \frac{4\,\pi\,\nu^2\,d\nu}{\alpha^3}\,.\,V \tag{1}$$

where α is the velocity of longitudinal waves in isotropic solid medium, and the number of modes for transverse vibrations between the same limits

$$= \frac{8\,\pi\,\nu^2\,d\nu}{\beta^3}\,.\,V \tag{2}$$

where β is the velocity of transverse waves in the same medium. In the second case the expression is doubled as each transverse waves is two sided or is equivalent to two waves polarised at right angles to each other. These are commonly known as *SH* and *SV* types in seismological literature. Debye then adds above two expressions to obtain total number of independent modes of vibrations and integrates the expression to obtain limiting maximum frequency, ν_m.

The above theory has been modified by Slater and also by Born and Karman (1912) and Born (1916). The objection raised by Slater is for the dispersion of elastic waves. This is not of much importance because bodily seismic waves are practically non-dispersive. Born, however, pointed out that the highest overtone is not limited by identical ν_m for both longitudinal and transverse vibrations as assumed by Debye but by an idential λ for both types of vibrations. Since longitudinal wave velocity is greater than the velocity of transverse waves, the limiting ν_m for longitudinal waves (*viz.*, ν_{ml}) is greater than ν_{mt} for transverse waves. Integrating (1) and (2) for longitudinal and transverse vibrations, we have —

$$\int_0^{\nu_{ml}} \frac{4\,\pi\,\nu^2\,d\nu}{\alpha^3}\,.\,V = \frac{4\pi\,\nu_{ml}^3}{3\alpha^3}\cdot V$$

and
$$\int_0^{\nu_{mt}} \frac{8\,\pi\,\nu^2\,d\nu}{\beta^3}\,.\,V = \frac{8\,\pi\,\nu_{mt}^3}{3\beta^3}\cdot V$$

The relation between ν_{ml} and ν_{mt} is

$$\frac{\nu_{ml}}{\alpha} = \frac{\nu_{mt}}{\beta} \tag{3}$$

If we now assume that the average energy $U\nu$ for each oscillator is same both for longitudinal and transverse vibrations of frequency ν, then E_P the energy for *P* waves is given by

$$E_P = \frac{4\,\pi\,\nu_{ml}^3\,V}{3\,\alpha^3}\,.\,U\nu$$

and
$$E_S = \frac{8\,\pi\,\nu_{mt}^3\,V}{3\,\beta^3}\,.\,U\nu$$

where E_S is the energy in *S*-waves.

$$\text{or}\quad \frac{E_P}{E_S} = \tfrac{1}{2}\,\frac{\nu_{ml}^3}{\alpha^3}\cdot\frac{\beta^3}{\nu_{mt}^3} \tag{4}$$

On making the substitution from (3) we have,

$$E_P \,/\, E_S = \tfrac{1}{2} \qquad (5)$$

or in other words, the energy in P waves is exactly half of the energy in S waves. This is in perfect accord with the assumption of Gutenberg (1942) and Richter (1956). For a number of shallow focus earthquakes Bath (1958) has calculated the energy of P and S waves assuming spherically symmetrical energy radiation from the focus and neglecting attenuation with distance. The mean value with standard error from thirteen observations is—

$$E_S \,/\, E_P = 1{\cdot}5 \pm 0{\cdot}4$$

According to Bath (1958), the large scatter of his observations is due to above assumption of spherical symmetry of the energy radiation. Bath himself has pointed out that the above assumption is not justified due to focal mechanism of earthquakes. He has further suggested that it is necessary to use the records at a number of stations in all directions from the epicentre to determine the energy ratio $E_S \,/\, E_P$ for a given earthquake. Moreover, the theoretical relation holds good near the focus. This is on account of differential attenuation for P and S waves with distance and at a distance from the focus the surface waves predominate. These surface waves derive their energy from body waves in a complicated process, the mechanism of which is not known.

From (3) we have,

$$\frac{\nu_{ml}}{\nu_{mt}} = \frac{\alpha}{\beta} = \sqrt{3} = 1{\cdot}73 \qquad (6)$$

in the case of solids having Poisson's ratio $\sigma = \frac{1}{4}$. The above equation at once suggests that the limiting frequency ν_{ml} is always greater than the limiting frequency ν_{mt} [3]. This is in accord with the general observation that the wave periods in the S group of waves is always greater than wave periods in the P group of waves.

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