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Reflection of electromagnetic waves in inhomogeneous medium

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ABSTRACT. Reflection coefficient and impedence of electromagnetic waves in a stratified layered medium following mathematical laws compatible with the propagation conditons associated with the diurnal and seasonal variations, have been deduced.

1. Introduction

Reflection of electromagnetic waves from inhomogeneous medium has been studied extensively in the case of special profiles of conductivity. Although a simple expression for reflection coefficient may result by assuming exponential variation of the conductivity with height, yet in practice because of the influence of diurnal and seasonal changes, the conductivity profile may be governed by different mathematical functions in layered medium. The reflection coefficient for a plane wave incident at the tropospheric layer has been studied when four different analytical forms for the derivatives of the refractive index are assumed (Wait 1962). In view of this, it appears necessary to investigate the reflection coefficient in layered medium governed by hyperbolic as well as exponential profiles which are decreasing with height. The object of this paper is, therefore, to find the reflection coefficient in layered medium assuming a hyperbolic profile for $z<H$ and exponential profile for $z>H$. The study has been confined to the lower atmosphere only where the diurnal and seasonal influence on the conductivity profiles are maximum for tropospheric radio wave propagation.

2. Maxwell equation in an inhomogeneous medium

Assuming the field is time dependent as exp. $(i\omega t)$, Maxwell's equations in an inhomogeneous field, free from a source with a tensor dielectric (ϵ), a scalar magnetic permeability (μ) and conductivity (σ) are given by—

$$\left.\begin{array}{ll} \left[\sigma(z)+i(\epsilon)\,\omega\right]\vec{E} = \text{rot}\,\vec{H} \\ -i\mu\omega\,\vec{H} = \text{rot}\,\vec{E} \end{array}\right\} \quad i=\sqrt{-1} \qquad (2\cdot1)$$

where ω is the angular frequency; $\vec{H}$ and $\vec{E}$ are the magnetic field intensity and electric vector respectively. Since the present investigation is confined only to the perpendicular polarisation wherein electric vector is always parallel to the plane of stratification and the field is assumed to vary as exp. $(-i\lambda x)$ every where, the equations $(2\cdot1)$ under these assumptions, take the form—

$$i\mu\omega H_x = \partial E_y/\partial z \qquad (2.2)$$

$$i\mu\omega H_z = i\lambda E_y \qquad (2.3)$$

$$[\sigma(z)+\epsilon(z)\omega]\,E_y = \partial H_x/\partial z + i\lambda H_z \qquad (2.4)$$

z being, perpendicular to earth's surface. Here E_y is the tangential component along Y-axis of the electrical field. H_x and H_z are the tangential and normal components of the magnetic field intensity. Combining $(2\cdot2)$ to $(2\cdot4)$, we get,

$$d^2E_y/dz^2+[k^2(z)-\lambda^2]E_y=0 \qquad (2.5)$$

where,

$$\lambda = k_o \sin\theta$$

θ being the angle of incidence and

$$k(z) = k_0 \quad \text{for } z<0$$

$$k^2(z) = -\gamma^2 = i\mu\omega[\sigma(z)+i\epsilon\omega] \qquad (2.6)$$

where, γ is the propagation constant.

3. Variation of conductivity profiles

Let the surface of the earth, assumed to be horizontal, be separated from the atmosphere by the plane $z=0$ where Z-axis is taken vertically upwards. Let the atmosphere be divided into two parts —a layer and a half-space separated from each other by the plane $z=H$ as shown in Fig. 1.

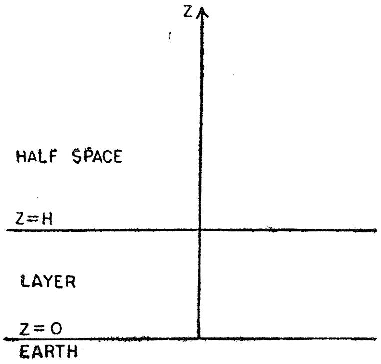

Fig. 1

Let us represent the propagation constants compatible with the possible diurnal as well a seasonal variation as —

$$-\gamma^2 = k^2(z) = k_1^2 \operatorname{sech}^2(pz+\theta_0) \quad \text{for } z \leqslant H \quad (3.1\,a)$$
$$= (k_3^2 - k_2^2)e^{-\beta z} + k_2^2 \quad \text{for } z \geqslant H \quad (3.1\,b)$$

where, p, θ_0, k_2, k_3, and β are all constants depending upon $\gamma(z)$ in the layer and half space.

for $z \geqslant H \quad R_e \beta > 0$

$$\left.\begin{aligned} &\text{and} \quad \lim_{z\to 0} k(z) = k_1 \operatorname{sech} \theta_0 \\ &\lim_{z\to\infty} k(z) = k_2 \end{aligned}\right\} \quad (3.2)$$

4. Solution

On substitution from (3·1 a) into (2·5), it follows —

$$\frac{d^2E_y}{d\xi^2} - \frac{2\xi}{1-\xi^2}\frac{dE_y}{d\xi} + \frac{\nu(\nu+1) - a^2 - \nu(\nu+1)\xi^2}{(1-\xi^2)^2} E_y = 0 \quad (4.1)$$

where,

$$\xi = \tanh(pz + \theta_o) \quad (4.2)$$
$$\nu(\nu+1) = k_1^2/p^2 \quad (4.3)$$
$$a^2 = \lambda^2/p^2 = k_o \sin^2\theta/p^2 \quad (4.4)$$

Equation (4·1) is the associated Legendre's differential equation whose general solution is given by (Bateman 1953) —

$$E_y = AP_\nu^{\,a}(\xi) + B\,Q_\nu^{\,a}(\xi) \quad \text{for } z \leqslant H \quad (4.5)$$

where, $P_\nu^a(\xi)$ and $Q_\nu^a(\xi)$ are associated Legendre's functions of the first and second kind respectively. Substituting from (3·1 b) into (2·5) the governing equation in E_y becomes for $z > H$ —

$$\frac{d^2E_y}{dz^2} + \left[(k_3^2 - k_2^2)e^{-\beta z} + (k_2^2 - k_o^2\sin^2\theta)\right]E_y = 0 \quad (4.6)$$

Remembering that $E_y \to 0$ as $z \to \infty$, the solution of (4·6) is given by (Wait 1962) —

$$E_y = C\,J_\delta\left\{\frac{2}{\beta}\sqrt{k_3^2 - k_2^2}\,\exp.(-\beta z/2)\right\} \quad \text{for } z \geqslant H \quad (4.7)$$

J being the Bessel function of the first kind. In the above —

$$\delta^2 = -(k_2^2 - k_o^2\sin^2\theta)(2/\beta)^2 \quad (4.8)$$

and C is a constant.

The boundary conditions at $z = H$ are —

(i) Tangential component of the electric field vector, *viz.*, E_y must be continuous at $z = H$

(ii) Tangential component of magnetic field intensity, *viz.*, H_x must be continuous at $z = H$

These conditions lead to the following equations —

$$\left.\begin{aligned} E_y^{(1)} &= E_y^{(2)} \\ \frac{1}{i\mu_1\omega}\frac{\partial E_y^{(1)}}{\partial z} &= \frac{1}{i\mu_2\omega}\frac{\partial E_y^{(2)}}{\partial z} \end{aligned}\right\} \text{at } z = H \quad (4.9)$$

where $E_y^{(1)}$ and $E_y^{(2)}$ denote the values of E_y in layer and half space respectively.

From (4·9) we can find the constants A, B of Eq. (4·5) in terms of C of Eq. (4·7).

The impedence in the layer $0 \leqslant Z \leqslant H$ can be expressed after simplification as —

$$Z = -E_y/H_x = -\frac{i\mu_1\omega\{\phi_1 P\nu^a(\tanh z) + \psi_1 Q\nu^a(\tanh z)\}}{\phi_1 f_1(\tanh z) + \psi_1 f_2(\tanh z)} \quad (4\cdot 10)$$

and the reflection coefficient for perpendicular incident is —

$$R_1 = -\frac{\eta_o/\cos\theta - Z}{\eta_o/\cos\theta + Z}$$
$$= -[\eta_o\{\phi_1 f_1(\tanh z) + \psi_1 f_2(\tanh z)\} + i\mu_1\omega\cos\theta\{\phi_1 P\nu^\alpha(\tanh z) + \psi_1 Q\nu^\alpha(\tanh z)\}] / [\eta_o\{\phi_1 f_1(\tanh z) + \psi_1 f_2(\tanh z)\} - i\mu_1\omega\cos\theta\{\phi_1 P\nu^\alpha(\tanh z) + \psi_1 Q\nu^\alpha(\tanh z)\}] \quad (4\cdot 11)$$

where, $\eta_o = \sqrt{\eta_o/\epsilon_o}$

ϵ_0, μ_0, are the dielectric constant and permeability in the homogeneous medium $z < 0$ and —

$$\phi_1 = [\{f_2(\tanh H) + \mu_1/\mu_2 . \delta\beta/2 . Q\nu^{\alpha}(\tanh H)\} \times \times J_{\delta}(u) - \beta u/2 . Q\nu^{\alpha}(\tanh H)\, J_{\delta+1}(u)] / /[\{P\nu^{\alpha}(\tanh H) f_2(\tanh H) - - Q\nu^{\alpha}(\tanh H) f_1(\tanh H)\}] \quad (4.12)$$

$$\psi_1 = [\{f_1(\tanh H) + \mu_1/\mu_2 . \delta\beta/2 . P\nu^{\alpha}(\tanh H)\} \times J_{\delta}(u) - \beta u/2\, P\nu^{\alpha}(\tanh H) . J_{\delta+1}(u)] / /[Q\nu^{\alpha}(\tanh H) f_1(\tanh H) - P\nu^{\alpha}(\tanh H) \times \times f_2(\tanh H)\}] \quad (4.13)$$

$$u = (2/\beta)\sqrt{k_3^2 - k_2^2)}\ \exp.(-\beta H/2)$$

$$f_1(\tanh H) = \left[\frac{d}{dz} P\nu^{\alpha}\ (\tanh z)\right]_{z=H} = -\nu \tanh H .\ P\nu^{\alpha}(\tanh H) + + (\nu + \alpha)\, P^{\alpha}{}_{\nu-1}(\tanh H) \quad (4.14)$$

$$f_2(\tanh H) = \left[\frac{d}{dz} Q\nu^{\alpha}\ (\tanh H)\right]_{z=H} = -\nu \tanh H .\ Q\nu^{\alpha}(\tanh H) + + (\nu+\alpha)\, Q^{\alpha}{}_{\nu-1}(\tanh H) \quad (4\cdot 15)$$

REFERENCES

Bateman, H. 1953 *Higher transcendental functions* , 1, pp. 154-161, McGraw Hill Book Co.

Wait, J. R. 1962 *Electromagnetic waves in stratified media,* p.372, Pergamon Press.