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Vector Mean Wind and Standard Vector Deviations for selected Rawin stations in India

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ABSTRACT. A statistical investigation of all available Rawin data at 3·0, 6·0, 9·0, 12·0, 14·1 and 16·2 km above sea level, for five Indian stations, covering the years 1961 and 1962 is reported in this paper. The stations are Bombay, Calcutta, Madras, New Delhi and Trivandrum. The tables showing the mean meridional and zonal components together with the standard vector deviations for the two seasons, December-February and July-August are included. The vector mean winds and the standard vector deviations for each station are given. The distribution of winds at the different levels mentioned above are tested for circularity and found to be mostly noncircular for the stations south of Lat. 20°N and mostly circular for stations north of it.

1. Introduction

A subject which has found important applications in meteorology is the application of modern statistical methods to vector quantities like winds. In the Tropics where the geostrophic relation does not hold, a better understanding of upper winds can be had from the statistical parameters of wind vectors. Winds, the magnitude and direction of which vary simultaneously, are usually resolved into two components, north and east. Wind components along a specified axis tend to be normally distributed. But this does not imply that the resultant wind vector is normally circularly distributed.

If V is the speed of the wind at an instant of time and its direction measured clockwise from north, the north and east components V_N and V_E are given by

$$V_N = V \cos \alpha \, , \; V_E = V \sin \alpha$$

The vector mean wind $\mathbf{V}_R$ is given by

$$\mathbf{V}_R = \frac{1}{n} \sqrt{\Sigma \, V_E^{\;2} + \Sigma \, V_N^{\;2}}$$

and its direction α by

$$\tan \alpha = \Sigma V_E \, / \, \Sigma \, V_N$$

where n is the number of observations.

The standard vector deviation σ, is defined (Brooks and Carruthers 1953) as a standard length taken as a measure of the dispersion (in all directions) about the vector mean. This is similar to the standard deviation of a scalar quantity, which is a standard length giving a measure of dispersion along a line about the scalar mean.

The standard vector deviation may be represented by a circle drawn with the vector mean as centre and standard vector deviation as radius. This circle will enclose, under the assumption of a circular normal distribution of winds, 63 per cent of the heads of all wind vectors plotted from a common origin.

If the distribution is noncircular and if the two components are independent, the standard vector deviation as defined above is given by

$$\sigma^2 = \sigma_E^{\;2} + \sigma_N^{\;2} \qquad (1)$$

where σ_N and σ_E are the standard deviations of the meridional and zonal components respectively.

$$\sigma_N^{\;2} = \frac{1}{n} \, \Sigma \, v_N^{\;2} \qquad \sigma_E^{\;2} = \frac{1}{n} \, \Sigma \, v_E^{\;2}$$

v_N and v_E being the departures of the components from their respective mean values.

If the two components are correlated the standard vector deviation is represented by (Hovmoller, see ref.)

$$\sigma_\theta^{\;2} = \sigma_E^{\;2} \cos^2\theta + \sigma_N^{\;2} \sin^2\theta + r_{NE} \, \sigma_N \, \sigma_E \, \sin 2\theta \qquad (2)$$

θ being the angle which the direction in which σ_θ (standard vector deviation) is determined makes with the East axis and r_{NE} represents the correlation coefficient between the north and east components.

2. Analysis of the data

Rawin observations available at 00 and 12 GMT for Bombay, Calcutta, Madras, New Delhi and Trivandrum for the two years 1961 and 1962 have been made use of. Daily values of V_N and V_E were made available to me by the Deputy Director General of Observatories (Climatology and Geophysics), Poona. From these the seasonal mean values $\overline{V}_N$ and $\overline{V}_E$ are calculated. The vector mean wind $\mathbf{V}_R$ is then obtained. The standard deviations σ_N and σ_E have been worked out from the meridional and zonal components of the winds. The standard vector

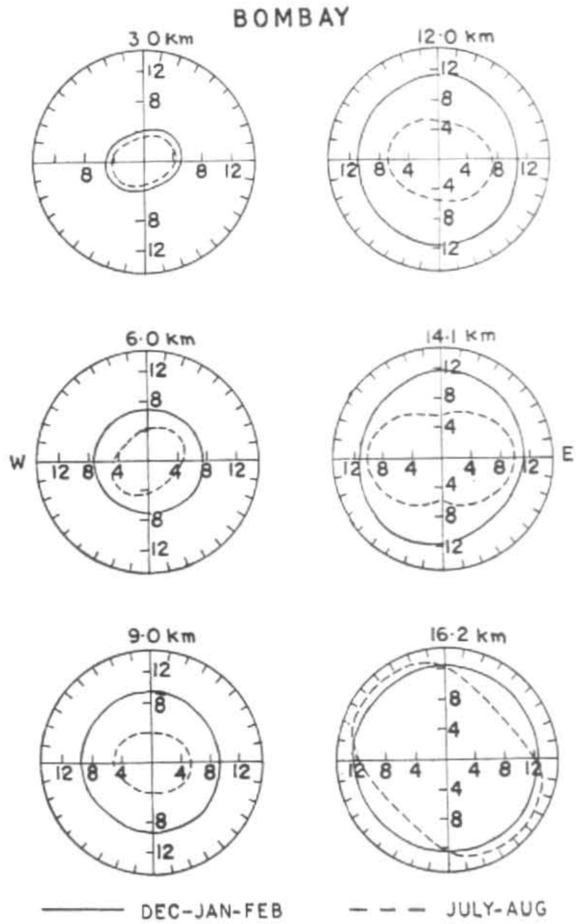


Fig. 1

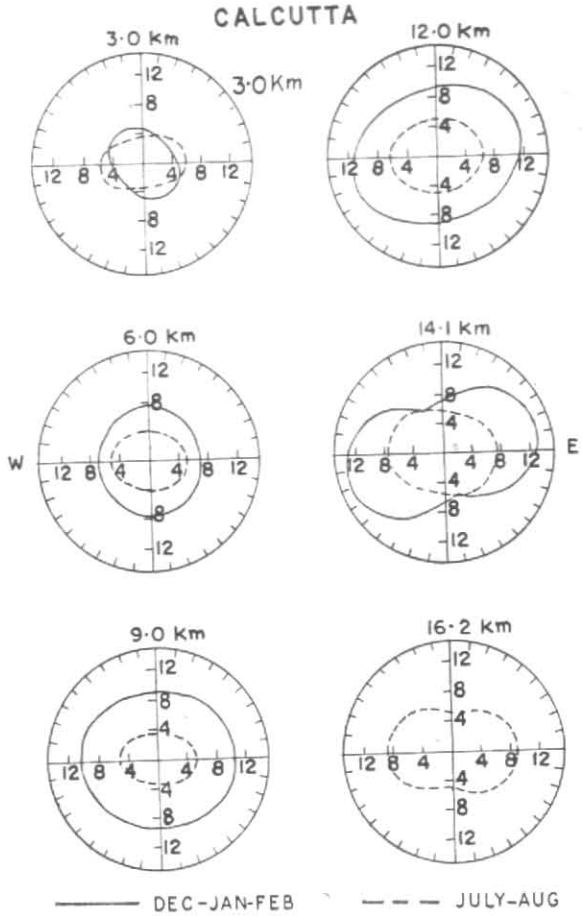


Fig. 2

Figs. 1-2. Standard vector deviations in different directions. The dotted and full line curves refer to different seasons. A line from the centre of the diagram terminating on the line of the curve represents the magnitude of the standard vector deviations in that direction

deviation of Brooks and Carruthers is then obtained from σ_N and σ_E. The results of these computations are given in Tables 1 to 5.

There is no exact test for testing the circularity, ellipticity, or noncircularity of the distribution. A circular distribution is a special case of the general distribution for which $\sigma_N = \sigma_E$ and $r_{NE} = 0$. This is a necessary and sufficient condition for circularity. Uptil now it had been assumed that the noncircular distributions were elliptical (Clarkson 1956 and Crutcher 1962). But when the actual values of standard vector deviation were calculated for each 10° interval of θ (from 0° to 360°) from the equation (2) making use of σ_N, σ_E and r_{NE} and then plotting the values of σ *vs* θ on a polar graph it was found that many of them were noncircular but not elliptical. In many cases $r_{NE} \neq 0$. It is even as high as ·52 at New Delhi and ·73 at Madras. Even when $r_{NE} = 0$ the distribution may not be elliptical due to the ratio of σ_N to σ_E being far from one. So ellipticity like circularity is also a special case. It is difficult to lay down limits within which a distribution can be said to be elliptical. The decision depends upon σ_N, σ_E and r_{NE}. Figs. 1 to 5 show the distributions of winds for the five stations.

3. Results obtained

Tables 1 to 5 present the results of the study.

3.1. *Mean winds*

Bombay — At 3·0 km mean vector wind is higher (8·4 mps) in July–August than in December–February (2·9 mps). From 6·0 to 12·0 km speeds are higher in winter than in July–August, when the northerly component is replaced by the southerly. On the other hand, in July–August, at levels above 12·0 km the speed increases steadily when the westerly components are replaced by easterlies. It reaches a value of about 30 mps at 16·2 km

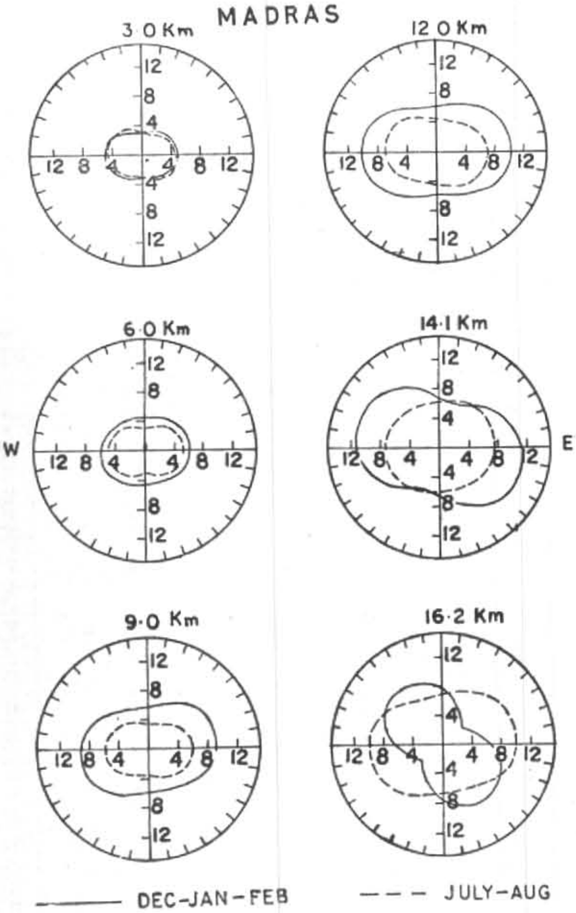


Fig. 3

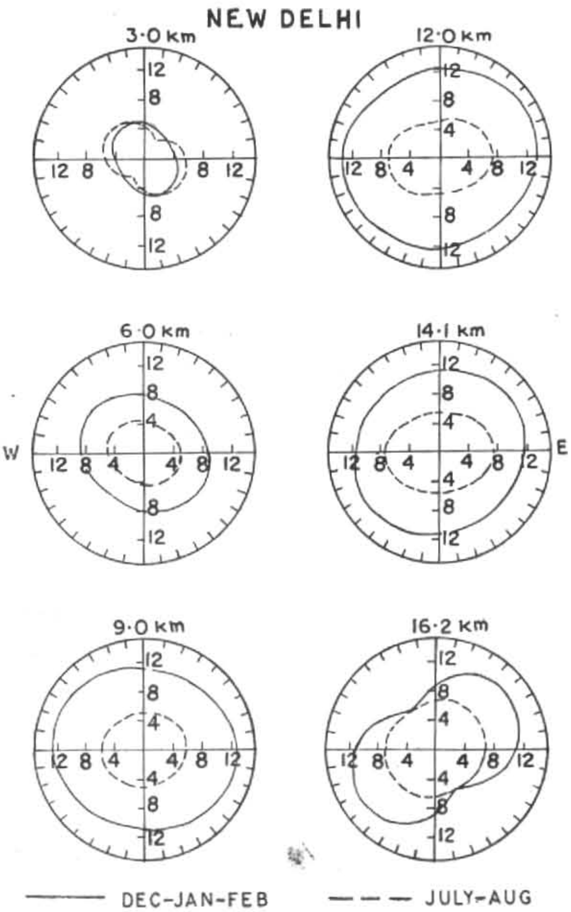


Fig. 4

Figs. 3–4. Standard vector deviations in different directions. The dotted and full line curves refer to different seasons. A line from the centre of the diagram terminating on the line of the curve represents the magnitude of the standard vector deviations in that direction

after which it is not possible to say anything definite due to the paucity of the data.

Calcutta — At all altitudes winter is the season of stronger winds reaching a maximum value of 33 mps at 12·0 km after which the speed decreases slowly. July–August is the period of lighter winds steadily increasing from 3·0 km up to 12·0 km. In winter winds are westerlies and in monsoon they are easterlies.

Madras — July–August is the period of strong winds, though the speed does not increase with altitude. From 3·0 to 9·0 km winds are westerlies and decrease with height from 10·1 to 5·4 mps. Above 9·0 km the westerlies are replaced by easterlies and the speed increases from 4·6 mps to a maximum value of 31 mps at 14·1 km after which it decreases. Winter is the season of lighter winds reaching a maximum of only 8·9 mps at 12·0 km.

New Delhi — Winter is the season of strong westerlies which increase with altitude reaching a maximum at 12·0 km of the order of 40-41 mps after which it decreases with height. In July-August winds are light easterlies which are practically constant in speed (1-2 mps) upto 9·0 km with an increase in speed upto 16·2 km (9·4 mps).

Trivandrum — July–August is the period of stronger winds. From 3·0 to 9·0 km they are light westerlies decreasing with altitude. At 9·0 km they become easterly, the speed increasing with altitude, reaching a maximum value of 33 mps at 14·1 km. Throughout winter winds are light easterlies with the maximum speed of 6 to 7 mps occurring at 12·0 km.

On the whole there is a definite latitudinal change of the vector mean wind. At lower latitudes it is stronger in monsoon than in the winter, reaching a maximum speed in the neighbourhood of the tropopause, while at higher latitudes it is stronger in winter than in the monsoon, reaching a maximum speed at about 12·0 km.

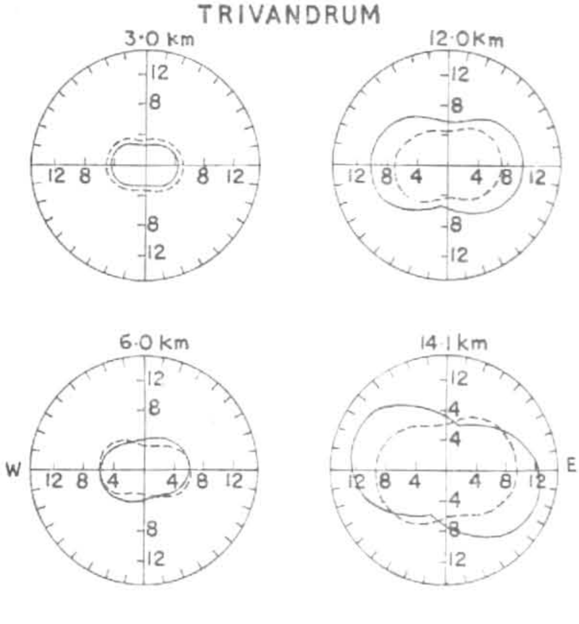


Fig. 5

Fig. 5. Standard vector deviations in different directions. The dotted and full line curves refer to different seasons. A line from the centre of the diagram terminating on the line of the curve represents the magnitude of the standard vector deviations in that direction

3.2. Standard vector deviation

For all stations the value of the standard vector deviation is higher in winter than in the monsoon. The maximum value of the standard vector deviation in winter occurs between 12·0 km and 16·2 km, being generally at a higher altitude (14.1 km) at the lower latitudes (Madras and Trivandrum). During winter, standard vector deviations in the troposphere are greatest in the northern area (New Delhi) than in the southern area (Madras and Trivandrum) and greater over Bombay than over Calcutta. In mid-monsoon Bombay has the greatest standard vector deviation. It decreases rapidly towards north and east and slowly towards the south.

Altitudinal and Latitudinal variation

(*a*) *Altitudinal* — Figs. 6 to to 10 show the variation of σ with height. In each of the graphs the abscissa gives the value of σ in mps and the ordinate gives the ratio of the mean temperature in degrees Celcius to the mean pressure in millibars. In other words the ordinate is very nearly proportional to the reciprocal of the air density ρ.

Only in July-August, for all stations except Trivandrum the curve σ *vs* T/P is nearly a straight line, which implies that $\sigma\rho$ is constant. For Trivandrum, a smooth curve is obtained as in winter. The curves for December–February for all stations are smooth up to the maximum value of σ (which is denoted by σ_m by Brooks *et al.*, 1950). The curves are power curves of the form $Y = CX^{\alpha}$. Above the level of σ_m, σ decreases through the remaining range of positions of the tropopause.

(*b*) *Latitudinal* Figs. (11–13)—For December-February, the variation of σ with latitude is not much at 3·0 km. At 6·0 and 9·0 km σ goes on increasing slowly with latitude reaching a maximum of 11·8 mps at 6·0 km and 16·9 mps at 9·0 km at New Delhi. At 12·0, 14·1 and 16·2 km σ is highest over Bombay (15·9, 16·2 and 18·0 mps respectively) and lowest at Madras (11·9, 13·2 and 9·8 mps respectively). A secondary maximum is indicated at New Delhi. In July–August at 3·0 km σ increases slowly with latitude from 5·6 mps at Madras to 6·9 mps at New Delhi. At 6·0 and 12·0 km it is practically constant with latitude. At 9·0 km it is highest over New Delhi (7·8 mps) and lowest over Trivandrum (6·9 mps). At 14·1 and 16·2 km the curves are wavy reaching a peak value at Bombay.

4. Conclusions

As we go southwards towards the equator the distribution of winds become more and more noncircular. During the monsoon the distribution of winds is slightly noncircular in northern parts also.

The vector mean wind undergoes a latitudinal and seasonal change. At lower latitudes it is stronger in monsoon than in winter, while at higher latitudes it is stronger in winter than in monsoon.

The standard vector deviation is more in winter than in monsoon at all five stations.

The latitudinal change of the standard vector deviation is remarkable at higher altitudes.

The altitudinal change differs with the seasons. In monsoon the law, $\sigma\rho$ = constant is valid while in winter it is not.

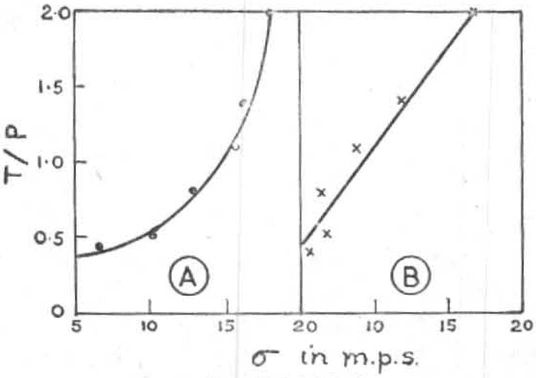


FIG. 6. BOMBAY

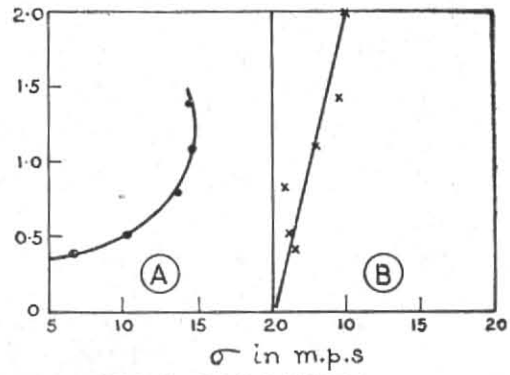


FIG. 7. CALCUTTA

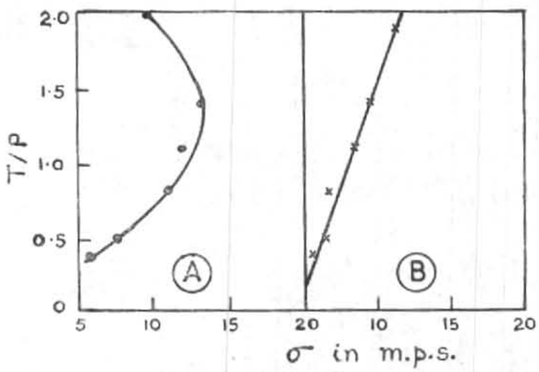


FIG. 8. MADRAS

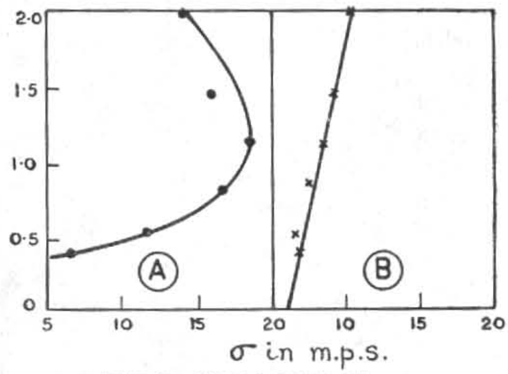


FIG. 9. NEW DELHI

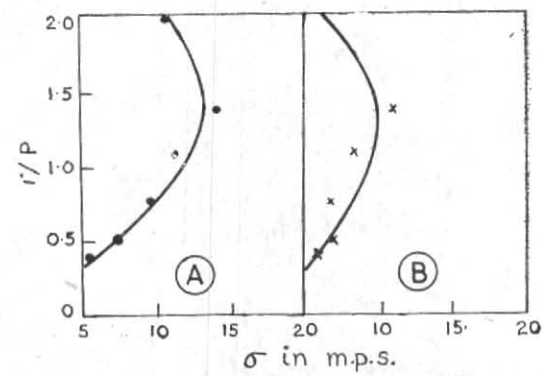
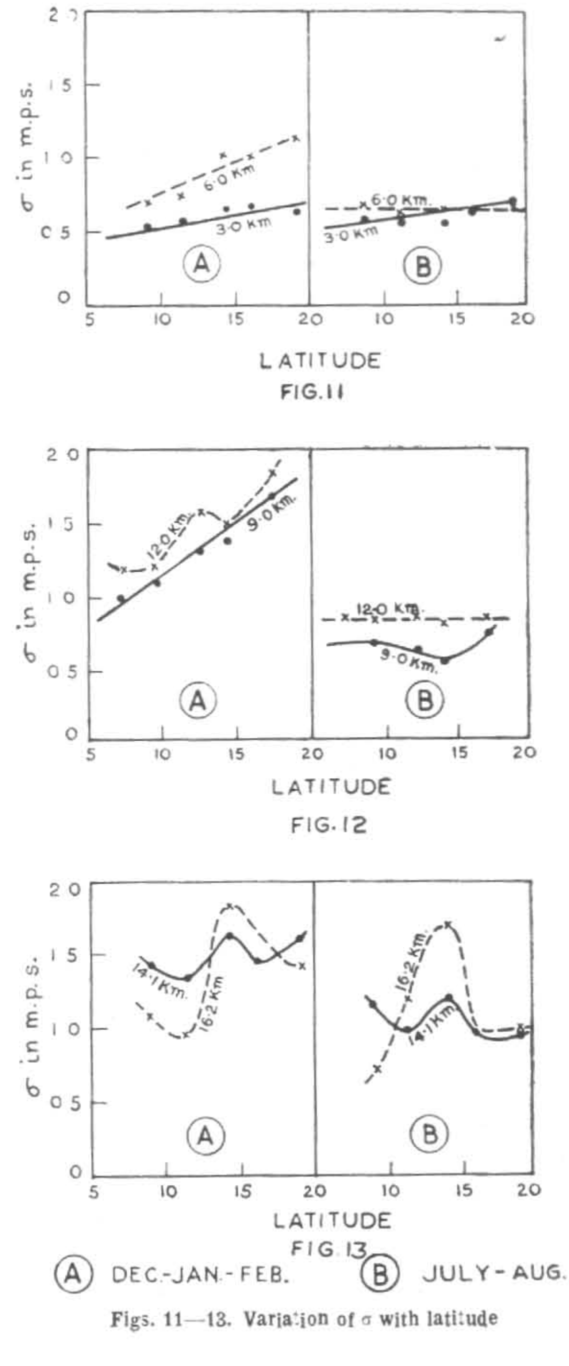


FIG. 10. TRIVANDRUM

(A) DEC-JAN-FEB (B) JUL-AUG

Figs. 6—10. Variations of σ with altitude



Figs. 11—13. Variation of σ with latitude

TABLE 1

BOMBAY (Lat. 19° 05′ N, Long. 72° 53′E)

Height (km)	No. of Obs. (a)	(b)	V_N (mps) (a)	(b)	V_E (mps) (a)	(b)	D_R (deg) (a)	(b)	$\mathbf{V}_R$ (mps) (a)	(b)	σ_N (mps) (a)	(b)	σ_E (mps) (a)	(b)	r_{NE} (a)	(b)	σ (mps) (a)	(b)
3·0	359	231	0·3	—2·2	—2·9	—8·1	276	255	2·9	8·4	4·1	3·5	5·3	4·6	·27	·10	6·7	5·8
6·0	347	224	3·1	1·1	—11·9	—0·4	284	340	12·3	1·1	7·4	4·2	7·1	5·1	·01	·38	10·3	6·6
9·0	304	189	3·0	1·1	—20·0	7·1	276	81	20·1	7·2	9·5	4·0	9·0	5·0	·01	—·08	13·1	6·4
12·0	193	152	—3·4	—0·2	—23·2	16·6	262	91	23·5	16·6	11·9	5·7	10·5	6·9	·01	—·16	15·9	9·0
14·1	89	84	—4·4	—0·3	—21·6	26·5	258	90	22·2	26·5	12·1	6·4	10·9	10·0	·14	·02	16·2	11·9
16·2	33	31	—2·4	—2·6	—14·1	30·4	260	95	14·3	30·5	12·7	12·1	12·7	11·7	00	—·36	18·0	16·9

TABLE 2

CALCUTTA (Lat. 22° 39′ N, Long. 88° 27′ E)

Height (km)	No. of Obs. (a)	(b)	V_N (mps) (a)	(b)	V_E (mps) (a)	(b)	D_R (deg) (a)	(b)	$\mathbf{V}_R$ (mps) (a)	(b)	σ_N (mps) (a)	(b)	σ_E (mps) (a)	(b)	r_{NE} (a)	(b)	σ (mps) (a)	(b)
3·0	350	241	2·4	—2·2	—9·9	1·3	283	149	10·1	2·5	4·8	3·7	4·6	5·3	—·30	·12	6·7	6·5
6·0	328	236	0·1	—1·4	—20·4	4·8	273	106	20·4	5·0	7·5	3·6	6·9	5·3	·04	—·14	10·3	6·4
9·0	246	220	—2·4	—0·2	—30·9	8·3	265	103	31·0	8·3	9·3	3·3	10·1	4·9	—·04	—·03	13·8	5·9
12·0	144	188	—8·6	2·6	—32·3	15·8	255	80	33·4	16·1	9·3	5·1	11·5	6·2	·15	—·02	14·8	8·1
14·1	48	133	—8·6	5·4	—30·7	19·9	254	75	31·9	20·6	7·1	6·2	12·8	7·5	·35	—·11	14·6	9·7
16·2	6	63	—	5·9	—	26·6	—	77	—	27·2	—	4·9	—	8·6	—	—·11	—	9·9

TABLE 3

MADRAS (Lat. 13° 00′ N, Long. 80° 11′ E)

Height (km)	No. of Obs. (a)	(b)	V_N (mps) (a)	(b)	V_E (mps) (a)	(b)	D_R (deg) (a)	(b)	$\mathbf{V}_R$ (mps) (a)	(b)	σ_N (mps) (a)	(b)	σ_E (mps) (a)	(b)	r_{NE} (a)	(b)	σ (mps) (a)	(b)
3·0	356	247	1·1	1·3	1·9	—10·1	60	278	2·2	10·2	3·2	3·0	4·9	4·8	·04	—·19	5·9	5·6
6·0	351	239	1·4	—0·5	0·5	—5·4	19	265	1·5	5·4	4·5	3·2	6·0	5·4	·05	·01	7·5	6·3
9·0	335	225	0·2	0·5	—4·2	4·6	273	84	4·2	4·6	6·1	3·8	9·1	5·7	·14	·14	10·9	6·9
12·0	274	194	—6·5	3·7	—6·0	19·7	223	79	8·8	20·0	6·2	4·7	10·2	7·0	·07	·14	11·9	8·5
14·1	105	159	—7·0	3·5	—5·2	30·9	216	83	8·7	31·1	7·2	6·4	11·1	7·4	—·34	·06	13·2	9·8
16·2	135	117	—3·3	1·3	—2·4	30·3	216	87	4·1	30·3	7·5	6·7	6·3	9·5	—·73	·12	9·8	11·6

D_R — Direction of the Vector Mean Wind in degrees

(a) December-February (b) July-August

TABLE 4

NEW DELHI (Lat. 28° 35′ N, Long. 77° 12′ E)

Height (km)	No. of Obs. (a)	No. of Obs. (b)	V_N (mps) (a)	V_N (mps) (b)	V_E (mps) (a)	V_E (mps) (b)	D_R (deg) (a)	D_R (deg) (b)	$\mathbf{V}_R$ (mps) (a)	$\mathbf{V}_R$ (mps) (b)	σ_R (mps) (a)	σ_R (mps) (b)	σ_E (mps) (a)	σ_E (mps) (b)	r_{NE} (a)	r_{NE} (b)	σ (mps) (a)	σ (mps) (b)
3·0	358	243	1·4	1·6	— 6·5	0·1	282	3	6·7	1·6	5·2	4·5	4·3	5·2	—·26	—·52	6·7	6·9
6·0	353	232	1·3	0·0	—18·3	0·7	274	0	18·3	0·7	8·2	4·4	8·5	5·1	—·20	—·22	11·8	6·8
9·0	323	227	0·5	—0·9	—33·2	1·2	271	127	33·2	1·5	11·3	5·1	12·6	5·9	—·07	·02	16·9	7·8
12·0	191	213	—1·7	—2·3	—40·9	2·9	267	128	40·9	3·7	12·4	5·0	13·3	7·1	·09	·11	18·6	8·7
14·1	106	200	—1·0	—1·5	—38·0	5·2	268	106	38·0	5·4	11·3	5·8	11·4	7·4	·10	·06	16·0	9·4
16·2	57	167	0·8	0·6	—26·1	9·4	287	86	26·1	9·4	8·6	6·6	11·0	6·8	·52	·09	14·0	9·5

TABLE 5

TRIVANDRUM (Lat. 8° 27′ N, Long. 76° 57′ E)

Height (km)	No. of Obs. (a)	No. of Obs. (b)	V_N (mps) (a)	V_N (mps) (b)	V_E (mps) (a)	V_E (mps) (b)	D_R (deg) (a)	D_R (deg) (b)	$\mathbf{V}_R$ (mps) (a)	$\mathbf{V}_R$ (mps) (b)	σ_N (mps) (a)	σ_N (mps) (b)	σ_R (mps) (a)	σ_R (mps) (b)	r_{NE} (a)	r_{NE} (b)	σ (mps) (a)	σ (mps) (b)
3·0	348	246	0·8	2·7	2·4	—11·3	71	283	2·5	11·6	2·8	3·1	4·5	5·1	—·20	—·20	5·4	6·1
6·0	343	243	1·9	0·5	4·6	— 3·8	67	277	5·0	3·9	4·0	3·4	5·9	6·1	·09	—·17	7·2	7·0
9·0	314	230	—0·6	1·0	2·6	7·0	103	82	2·7	7·1	5·5	4·5	8·1	5·2	·23	—·03	9·8	6·9
12·0	279	188	—6·4	4·9	2·3	23·2	160	78	6·8	23·7	5·6	4·9	10·0	7·2	—·08	·11	11·5	8·7
14·1	258	153	—4·5	5·2	3·4	32·8	143	81	5·6	33·2	7·3	6·4	12·2	9·1	—·38	·15	14·2	11·1
16·2	198	101	—1·7	0·7	2·7	24·5	122	88	3·2	24·5	5·8	5·5	9·3	3·5	—·03	·05	10·9	6·5

D_R — Direction of the Vector Mean Wind in degrees

(a) December–February (b) July–August

5. Acknowledgements

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