



Best fitting of probability distributions for monthly and annual maximum rainfall prediction in Junagadh region (Gujarat-India)

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सार — शुष्क और अर्ध-शुष्क क्षेत्र में वर्षा एक अल्प और महत्वपूर्ण जलविज्ञानिक परिवर्ती है। जूनागढ़ (गुजरात-भारत) मानसून वर्षा की अनिश्चितताओं से जूझता है और फलस्वरूप कृषि और अन्य जल संसाधन प्रबंधन गतिविधियों को नुकसान होता है। इसलिए, कम या अधिक वर्षा के कारण जल संसाधन संरक्षण और फसल क्षति के मुद्दों पर तत्काल ध्यान देने की आवश्यकता है। किसी भी क्षेत्र का जल संसाधन विकास अपवाह और प्राप्त वर्षा की मात्रा पर निर्भर करता है। वर्षा के बेहतर आवृत्ति विश्लेषण और पूर्वानुमान के लिए उपयुक्त संभावित वितरण का चयन किया जाना चाहिए और वर्षा की ऐतिहासिक समय श्रृंखला में फिट किया जाना चाहिए। दैनिक वर्षा डेटा 1984 से 2021 तक की 38 वर्षों की अवधि के लिए एकत्र किया गया। यह शोध अधिकतम वर्षा के बेहतर पूर्वानुमान हेतु सर्वश्रेष्ठ के चयन के लिए लगातार एक से पांच दिनों तक की मासिक और वार्षिक अधिकतम वर्षा के आठ अलग-अलग सैद्धांतिक संभाव्यता वितरणों को शामिल करने का प्रयास करता है। अधिकतम वर्षा का पूर्वानुमान ची-स्क्वायर और नैश-सटक्लिफ दक्षता के समंजन के निर्धारण के लिए प्रेक्षित मानों के साथ अपेक्षित मानों की तुलना करके किया गया। प्राप्त परिणामों से पता चला कि जूनागढ़ क्षेत्र की मासिक और वार्षिक अधिकतम वर्षा के पूर्वानुमान के लिए गम्बेल वितरण सबसे बेहतर रहा।

ABSTRACT. Rain is a meager and crucial hydrological variable in arid and semi-arid region. Junagadh (Gujarat-India) reels under monsoon rainfall uncertainties and thereby the agriculture and other water resources management activities suffer. Therefore, urgent attention is needed to address water resources conservation and crop damage issues due to deficits or excess rainfall. Water resources development of any locality depends on amount of runoff generated and rainfall received. Appropriate probability distributions need to be selected and fitted to the historical time series of rainfall for better frequency analysis and forecasting of the rainfall. The daily rainfall data was collected for a period of 38 years, *i.e.*, from 1984 to 2021. This research attempts to fit eight different theoretical probability distributions to the monthly and annual maximum rainfall for one to five consecutive days to select the best one for the better prediction of maximum rainfall. For determination of goodness of fit Chi-Square and Nash-Sutcliffe Efficiency were carried out by comparing the expected values with the observed values. The results indicated that the Gumbel distribution emerged to be the best fit for the prediction of monthly and annual maximum rainfall of Junagadh Region.

Key words – Rainfall, Frequency analysis, Probability distributions, Gumbel distribution, Goodness of fit, Chi-Square, Nash-Sutcliffe Efficiency.

1. Introduction

India receives approximately 75 to 80 percent annual precipitation of total about 4000 km³ annual precipitation during rainy season under the influence of South-West monsoon (Kumar *et al.*, 2005). The farmers of arid and semi-arid region are always vulnerable to yield losses due to extreme climate fluctuations. Due to its erratic nature and characteristic spatiotemporal variation, rainfall becomes the predominant key risk factor that has a direct or indirect effect on agriculture. Hence, the design and

management of hydraulic structures, irrigation water supply, planning of soil conservation, flood control systems and optimal crop planning are not based on the long-term average of rainfall records but on rainfall depths that can be expected for a specific probability. Hydrological events have numerous and unpredictable sources of uncertainties about the physical processes (Hosking and Wallis, 1997). However, the stochastic model (hydrological frequency analysis) can be used as a tool to estimate how often a specified event will occur on average in a region from the available data (Bhakar *et al.*,

2006). In this method the magnitudes of events for design return periods are determined beyond the recorded range. Because of the strong temporal variability of rainfall, the design and management of irrigation water supply, flood control systems and hydraulic structures are based on rainfall depths that can be expected for a specific probability.

Rainfall analysis using probability distribution models has already been investigated by several researchers. Kumar (2000) and Singh (2001) found that the Log Normal (LN) distribution is the best-fit probability distribution for annual maximum daily rainfall in India. Amin *et al.* (2016) found that the Log-Pearson Type-III (LP-III) distribution was the best-fit distribution to estimate annual maximum rainfall in the northern regions of Pakistan. Eslamian *et al.* (2007) suggested that the Generalized Extreme Value (GEV) and LP-III distributions provided the best fit to estimate maximum monthly rainfall as an extreme event in Iran. Lee (2005) and Ogunlela (2001) evaluated that LP-III distribution fitted best to the rainfall distribution of the Chia-Nan plain area of Taiwan and Nigeria respectively. The LN-II distribution was found the best-fit probability distribution for one to five consecutive days' maximum rainfall for Accra, Ghana (Kwaku and Duke, 2007). Olofintoye *et al.*, (2009) identified that 50% of stations follow LP-III distributions and 40% follow Pearson Type-III distributions for peak daily rainfall in Nigeria. The Gamma probability distribution provided the best fit to monthly maxima rainfall in arid regions of Libya (Sen and Eljadid, 1999). LP-III distribution recommended by the U.S. Water Resources Council (USWRC) in 1967 and was found the best method of flood frequency analysis in the United States (Arora & Singh, 1989). Zalina *et al.* (2002) concluded that the GEV distribution is the most appropriate distribution for describing the annual maximum rainfall series in Malaysia. Hanson and Vogel (2008) reported that Pearson Type-III distribution fitted the best to the daily rainfall in the United States. According to Bhakar *et al.* (2008) the Gumbel distribution was the best fit for monthly maximum rainfall in India. Sharma and Singh (2010) evaluated that LN and Gamma distribution were the best fit probability distribution for the annual and seasonal time scale while GEV distribution was observed for weekly time scale as best fit probability distribution in Pantnagar (India).

The present work focuses to find the best-fit models for determining the frequency of extreme rainfall events and to extract expected maximum monthly as well as annual rainfall over return periods of 3, 4, 5, 10, 15, 20, 25, 30 and 35 years in the Junagadh (Gujarat-India) region. These can be used to develop plans and policies for better management of water resources and agricultural

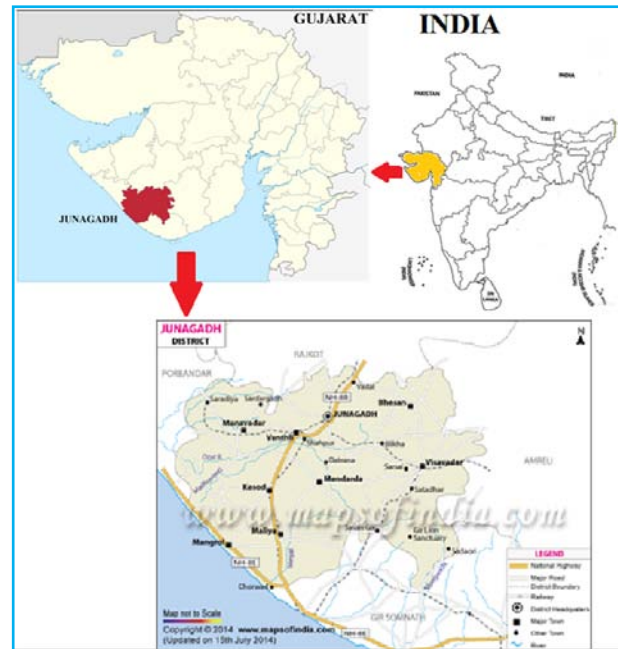


Fig. 1. Junagadh (Gujarat-India) region
(Source : www.mapsofindia.com)

issues. The findings would be useful for agriculturists, hydrologists, designers of hydraulic structures, irrigation engineers, environmental managers, and planners of water resources to develop better plans and policies.

2. Study area and data sets

2.1. Study area

Junagadh (Gujarat-India) is located in western Gujarat and is surrounded by Arabian Sea and forest area. It is located at a longitude of 20.47° N to 21.45° N and a latitude of 70.15° E to 70.55° E (<https://junagadh.nic.in/map-of-district>). Fig. 1 shows the location of the study region. Population of Junagadh district is 2,742,291 as per Census 2011 and will be reached to 2,891,917 in 2022 (estimates as per aadhar uidai.gov.in Dec 2020 data). Total geographical area of Junagadh district is 8846.1 Sq. km. It is 7th largest district in Gujarat and 75th largest in India in terms of total area. Population density of the district is 327 persons per km² (<https://www.indiamapia.com/Junagadh.html>). The region is semi-arid region, and it has mean annual rainfall of 1186 mm based on the dataset used in the study. The maximum mean of monthly rainfall was received as 93.79 mm in the month of July while minimum mean of monthly rainfall was received as 53.43 mm in the month of August.

Periodic uncertain and inadequate rainfall pattern, over-exploitation and mismanagement of ground water,

TABLE 1

Description of selected probability distributions

Distribution	PDF	CDF	Quantile Function	Parameter	Parameter Estimates
Gumbel	$f(x) = \frac{1}{\sigma} \exp[-z - \exp(-z)]$ $z = \frac{x - \xi}{\sigma}$	$F(x) = \exp[-\exp(-z)]$	$X_T = \bar{X} + K\sigma_{n-1}$	Location ξ , Scale σ , \bar{Y}_n and S_n from table	$K = \frac{(Y_T - \bar{Y}_n)}{S_n}$ $Y_T = -\ln \ln \left(\frac{T}{T-1} \right)$ $\xi = \mu$
V. T. Chow	$f(x) = \frac{1}{\sigma} \exp[-z - \exp(-z)]$ $z = \frac{x - \xi}{\sigma}, Z = \log \log \left(\frac{T}{T-1} \right)$	$F(x) = \exp[-\exp(-z)]$	$X_T = A + BZ$	Constants A and B	$A = \left[\sum_{i=1}^N \frac{Z_i}{N} \right] - \left[B \sum_{i=1}^N \frac{Z_i}{N} \right]$ $B = \frac{\sum_{i=1}^N Z_i X_i - \sum_{i=1}^N Z_i \sum_{i=1}^N \frac{Z_i}{N}}{\sum_{i=1}^N Z_i^2 - \frac{\sum_{i=1}^N Z_i^2}{N}}$
LP-III	$f(x) = \frac{1}{x \alpha \Gamma(k) \left(\frac{Z - \xi}{\alpha} \right)^{k-1}} \exp\left(-\frac{Z - \xi}{\alpha}\right), Z = \log(x)$	$F(x) = \begin{cases} G\left(k, \frac{Z - \xi}{\alpha}\right), \alpha > 0 \\ 1 - G\left(k, \frac{Z - \xi}{\alpha}\right), \alpha < 0 \end{cases}$	$Z_T = \bar{Z} + K_z \sigma_z$ $X_T = \text{Anti log } Z_T$	K_z Coefficient of Skewness C_s	K_z Corresponding to C_s and T
LN	$f(x) = \frac{1}{(x - \xi)\sigma\sqrt{2\pi}} \exp\left\{-\frac{1}{2} \left[\frac{\log(x - \xi) - \mu}{\sigma} \right]^2\right\}$ $Z = \log(x)$	$F(x) = \Phi\left[\frac{\log(x - \xi)}{\sigma}\right]$	$Z_T = \bar{Z} + K_z \sigma_z$ $X_T = \text{Anti log } Z_T$	K_z Coefficient of Skewness $C_s = 0$	K_z Corresponding to C_s and T
EXP	$f(x) = \frac{1}{\alpha} \exp\left(-\frac{x - \xi}{\alpha}\right)$	$F(x) = 1 - \exp\left(-\frac{x - \xi}{\alpha}\right)$	$X_T = \xi - \alpha \ln\left(\frac{T}{T-1}\right)$	Location ξ , Scale α ,	$\xi = \mu - \alpha$ $\sigma^2 = \alpha^2$
GEV	$f(x) = \frac{1}{\alpha} (1 - kZ)^{\frac{1}{k-1}} \exp[-(1 - kZ)^{1/k}]$ $z = \frac{x - \xi}{\sigma}$	$f(x) = \exp[-(1 - kZ)^{1/k}]$	$X_T = \xi + \frac{\alpha}{k} \left[-\left(1 - \ln \frac{1}{T}\right)^{\frac{1}{k}} \right]$	Location ξ , Scale α , Shape k	$k = 7.817740c + 2.930462c^2 + 13.641492c^3 + 17.206675c^4$ $c = \frac{2}{3 + \tau_3} - \frac{\ln 2}{\ln 3}$
GP	$f(x) = \frac{1}{\alpha} \left(1 - k \frac{x - \xi}{\alpha}\right)^{\frac{1}{k-1}}$	$F(x) = 1 - \left[1 - k \frac{x - \xi}{\alpha}\right]^{\frac{1}{k}}$	$X_T = \xi + \frac{\alpha}{k} \left[1 - \left(1 - \frac{1}{T}\right)^k\right]$	Location ξ , Scale α , Shape k	$\lambda_1 = \xi + \frac{\alpha}{1 + k}$ $\lambda_2 = \frac{\alpha}{(1 + k)(2 + k)}$ $\tau_3 = \frac{(1 - k)}{(3 + k)}$ $\alpha = \frac{\lambda_2 k}{\Gamma(1 + k)\Gamma(1 - 2^{-k})}$ $= \lambda_1 + \frac{\alpha[\Gamma(1 + k) - 1]}{k}$
Gamma	$f(x) = \frac{1}{\alpha^\beta \Gamma(\beta)} (x - \varepsilon)^{\beta-1} e^{-\frac{(x - \varepsilon)}{\alpha}}$	$f(x) = \frac{\Gamma\left(\frac{x - \varepsilon}{\alpha}\right)(\beta)}{\Gamma(\beta)}$	$X_T = \varepsilon + \alpha \log(T)$	Location ε , Scale α , Shape β	$\lambda_1 = \varepsilon + \alpha$ $\lambda_2 = \frac{\alpha}{2}$

limited aquifer water storage capacity, and insufficient natural water conservation are all critical challenges for this region. Water availability is a major issue in this area and therefore rainfall is largely influenced on the water resource management, cropping patterns and crop water requirements, irrigation scheduling and environmental assessment.

2.2. Data sets

The daily rainfall data for 38 years (1984-2021) were obtained from the website (<http://www.jau.in/index.php/annual-weather-reports--weather-data>) of Junagadh Agro-meteorological Cell which is located at between latitude of 21°31' N and longitude of 70°33' E with an altitude of 61 m. The weather parameters are recorded at the Agro-meteorological observatory; Department of Agronomy affiliated with Junagadh Agricultural University, Junagadh and published annually. The daily rainfall data for years 1984-2021 were considered and analyzed for extreme rainfall events. The monthly and annual series of extreme rainfall datasets are derived from the daily rainfall data and used in frequency analysis. The relation between the magnitudes of is obtained by arranging the sample data in descending order of magnitude. Then each data is assigned with rank $m=1$, for the first entry and so on till $m=N$, for the last event. The probability P of an event to or exceeded is given by the Weibull formula $P=m/(N+1)$.

3. Methodology

3.1. Selection of probability distributions

It is important but difficult to select the best-fit probability distribution to make effective analysis of rainfall for any location. In this study total eight probability distributions (Gumbel, Van Te chow (V. T. Chow), (LP-III), Log Normal (LN), Exponential (EXP), GEV, Generalized Pareto distribution (GP) and Gamma distributions) which are the most frequently used or recommended in extreme rainfall analyses are presented. Table 1 shows the description of selected probability distribution functions, viz., probability density function (PDF), cumulative density function (CDF), quantile function and parameter estimates. The method of moments (MOM) and the L-moments are used to estimate the parameters of the selected distributions.

3.2. Goodness of fit criteria

3.2.1. Chi-square test

The validity of selected probability distribution models is checked using goodness-of-fit test statistics,

TABLE 2

NSE Goodness of fit Ratings for Model calibration

NSE Range	Calibration Rating	Model Application
0.5 to 1.0	Excellent	Planning, Preliminary design, Final Design
0.4 to 0.49	Very Good	Planning, Preliminary design, Final Design
0.3 to 0.39	Good	Planning, Preliminary design
0.2 to 0.29	Fair	Planning
<0.2	Poor	Screening

which quantify the compatibility of a random sample with the theoretical probability distribution. The Chi-square test is used to test the degree of agreement between observed data and those expected upon a given null and alternative hypothesis. The null hypothesis is usually set up as the uninteresting hypothesis and hence, we wish to reject it. The Chi-square test is applied for testing the null hypothesis: the rainfall data follows the selected probability distribution adequately. If the calculated values of the Chi-square test statistic are lower than those of the critical values at the chosen significance level then the null hypothesis is accepted and the selected distribution is taken to be acceptable for rainfall estimation. For this study the critical value of Chi-Square distribution is obtained 15.5073 at significant level of 0.05 and 8 degree of freedom.

The Chi-Square values, χ^2 can be calculated as Equation (1):

$$\chi^2 = \sum_{i=1}^n \frac{(O_i - E_i)^2}{E_i} \tag{1}$$

where

O_i = Observed frequency

E_i = Expected frequency

i = number of observations (1, 2,n)

3.2.2. Nash-Sutcliffe efficiency (NSE)

NSE is normalized statistic used to determine the relative magnitude of the residual variance (Moriassi *et al.*, 2015 and Son *et al.*, 2019). It is widely accepted and provided a better choice for a dimensionless goodness of fit (Green and Stephenson, 1986; Pretorious *et al.*, 2013). NSE quantifies how well a model simulation can predict

the outcome variable. It lies between 1.0 (perfect fit) and $-\infty$. NSE value of zero has the same predictive power as the mean while NSE value less than zero indicate that the mean value of the observed time series would have been a better predictor than the model. NSE goodness of fit ratings for model calibration is presented in Table 2. NSE is calculated using Equation (2):

$$NSE = 1 - \frac{\sum_{i=1}^n (E_i - O_i)^2}{\sum_{i=1}^n (O_i - \bar{O})^2} \quad (2)$$

where:

\bar{O} = Mean of observed values

4. Results and discussion

In this section, the eight selected probability distributions were used to compute the annual maximum rainfall for one day to five consecutive days. The results obtained from the fitting of the eight probability distributions are presented from Appendix 1 to 25. The objective is to show which distribution provides the most appropriate fit for the month as well as for the year to the daily precipitation data, extracted from 1984 to 2021. The Chi-Square test results, NSE values, rating and ranking based on the NSE values and estimated parameters values of all the distributions are presented from Appendix 1 through 25.

4.1. Annual maximum rainfall

To assess the quality of the fit of the distributions mentioned before, it is necessary to assess the value of the Chi-Square test statistic and NSE coefficient. From Appendix 1 and 3, it can be seen that the Chi-Square test values obtained for all the distributions were more than the critical value of Chi-Square distribution (15.5073 at significant level of 0.05 and 8 degree of freedom) which indicated that they have not passed the test and none of them are considered to be a good one. However, according to NSE ranking V T Chow, LN and GEV distributions performed better to estimate one day annual maximum rainfall while Gumbel, LN and V T Chow distributions performed better to estimate three days annual maximum rainfall. It is observed from Appendix 2 and 4 that only Gumbel distribution has passed the Chi-Square test. However, Gumbel, LN and V T Chow distributions performed better with rank 1, 2 and 3 respectively based on NSE statistics to estimate two days annual maximum rainfall whereas LN, Gumbel and GEV distributions performed better with rank 1, 2 and 3 respectively based on NSE statistics to estimate four days annual maximum

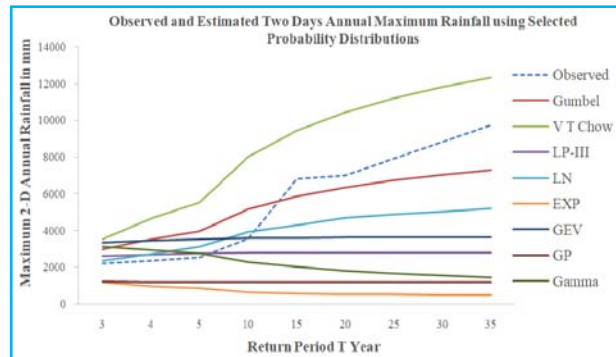


Fig. 2. Observed and estimated two days annual maximum rainfall using selected probability distributions

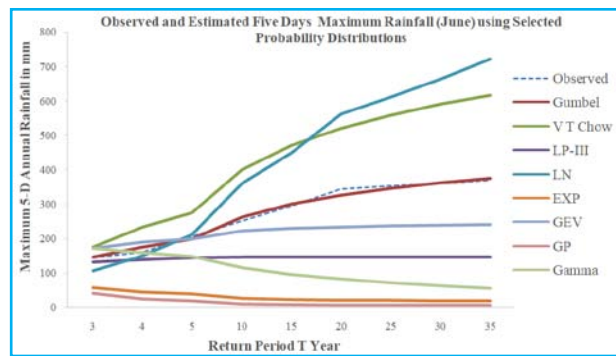


Fig. 3. Observed and estimated five days maximum rainfall (June) using selected probability distributions

rainfall. For five days annual maximum rainfall (Appendix 5) only Gumbel distribution has passed the Chi-Square test. LN, Gumbel and GEV were identified as good performance distribution with 1, 2 and 3 rank based on NSE statistics. Overall, the best performance was found by Gumbel distribution to estimate two days annual maximum rainfall (passed the Chi-Square test, NSE value 0.9166 with rank 1). The worst result obtained by EXP distribution with NSE value -0.2447 and poor rating for three days annual maximum rainfall. Fig. 2 was created to facilitate the spatial visualization of the selected distribution for estimation of two days annual maximum rainfall. It is expressed see that the Gumbel distribution was the best fit to the data; it underestimated the two days consecutive rainfall up to 15 years return period. After that it overestimated the two days consecutive rainfall.

4.2. Monthly maximum rainfall (June)

The selected probability distributions were adjudged by comparing the average of Chi-Square and NSE values obtained for these distributions corresponding to return period 3, 4, 5, 10, 15, 20, 25, 30 and 35 years respectively for the month of June as shown in Appendix 6 through 10.

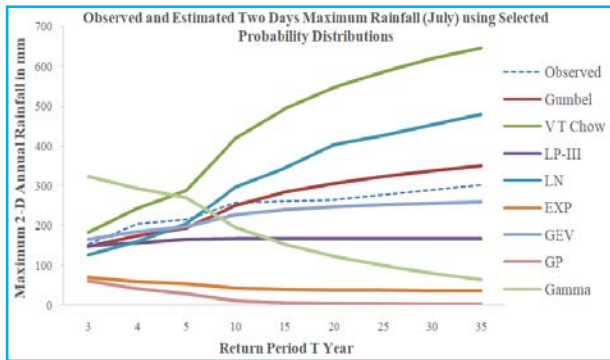


Fig. 4. Observed and estimated two days maximum rainfall (July) using selected probability distributions

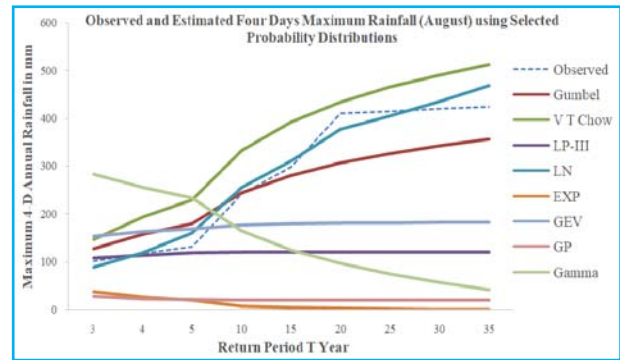


Fig. 5. Observed and estimated four days maximum rainfall (August) using selected probability distributions

From the Chi-Square test and NSE values it was concluded that one day to five days consecutive maximum rainfall of June was best fitted by Gumbel and GEV distributions as compared to other distributions. Except three days maximum rainfall of June Gumbel distribution stood on 2nd rank based on NSE values; rest of the cases Gumbel distribution attained 1st rank. The best result was obtained for five days consecutive maximum rainfall of June by Gumbel distribution with NSE value 0.9933. The worst result obtained by GP distribution with NSE value -8.0486 and poor rating for one day annual maximum rainfall. By looking at Fig. 3 it is possible to identify that Gumbel distribution had the highest intimacy of best fit among all for five days maximum rainfall of June.

4.3. Monthly maximum rainfall (July)

For July, Gumbel and GEV were passed Chi-Square test for all one to five days maximum rainfall while LN was passed Chi-Square test for only three days maximum rainfall with NSE value 0.8461 and attained 3rd rank (Appendix 11 to 15). Gumbel distribution was attained 1st rank four times and attained rank 4 in two-day maximum rainfall of July. The GEV distribution was reached to 2nd rank four times and 1st rank once in two days maximum rainfall of July. The Gumbel distribution consistently performed better for three to five days maximum rainfall of July but the GEV distribution performed best for two days maximum rainfall of July with NSE value 0.9899. The GP distribution had shown nastiest performance with NSE value -13.2470 and poor rating for one day maximum rainfall of July. Fig. 4 shows the best performance of the GEV distribution for two days maximum rainfall of July. It can be seen that the GEV distribution mostly under estimated while the Gumbel distribution slightly overestimated the rainfall after 15 years of return period.

4.4. Monthly maximum rainfall (August)

The goodness of fit test and the overall ranking based on NSE of the selected distributions for one to five days maximum rainfall of August are presented from Appendix 16 to 20. From the values that are shaded in the above tables, it is clear that the LN and Gumbel distributions consistently passed goodness of fit test for all one to five days maximum rainfall of August. Based on the comparison of the NSE values, LN distribution was attained 1st rank for all cases except for two days maximum rainfall of August. Gumbel distribution was attained 1st rank for two days, 2nd rank for three days, 3rd rank for four and five days and 5th rank for one day maximum rainfall of August. V T Chow distribution received 2nd rank for one- and four-days maximum rainfall of August. Exponential distribution was found unreliable as it was not passed goodness of fit test and shown poor rating with NSE value -1.2754 for two days maximum rainfall of August. LN distribution was emerged as the best fit distribution with NSE value 0.9946 for four days maximum rainfall of August (Fig. 5).

4.5. Monthly maximum rainfall (September)

It is noticed from Appendix 21 through 25 that the selection of the Gumbel distribution showed an interesting behaviour. Only Gumbel distribution was passed Chi-Square test for all five cases followed by LN distribution which was passed all the cases except for five days maximum rainfall of September. Gumbel distribution was showed good fitting potential with 2nd rank for one day, 3rd rank for two days and 1st rank for three to five days maximum rainfall of September with NSE values 0.8740, 0.9126, 0.9284, 0.9531 and 0.9603 respectively. LN distribution was showed reasonable fitting potential with 1st rank for one and two days, 2nd rank for three days and

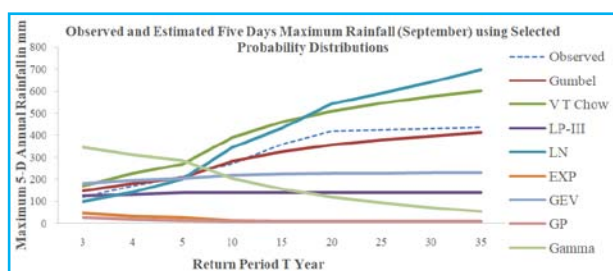


Fig. 6. Observed and estimated five days maximum rainfall (September) using selected probability distributions

3rd rank for four days maximum rainfall of September with NSE values 0.9176, 0.9492, 0.9188 and 0.8998 respectively. The Gumbel distribution gave the best fit for five days maximum rainfall of September and the EXP distribution gave the poorest rating with NSE value - 0.4871 for three days maximum rainfall of September. Comparative plot of selected distributions showing observed and estimated five days maximum rainfall of September is presented in Fig. 6.

5. Conclusions and recommendations

Regional rainfall analysis was made by eight different probability distributions for one to five days maximum monthly and annual rainfall of Junagadh region. Daily rainfall data from 1984 to 2021 were used to determine the best fit probability distribution for the study region. The findings of this study allowed us to draw the following conclusions and recommendations.

(i) Gumbel distribution showed the best fits for two days, four days and five days consecutive annual maximum rainfall while others were not passed goodness of fit test for any cases.

(ii) The best fit performance of Gumbel distribution was found for two days consecutive annual maximum rainfall with NSE value 0.9166 in the Junagadh region.

(iii) Gumbel and GEV distributions were fitted well to the rainfall data as compared to other distributions for one to five days consecutive maximum rainfall of June and July. However, Gumbel distribution gave the best result with NSE value 0.9933 for five days consecutive maximum rainfall of June and the GEV distribution performed the best with NSE value 0.9899 for two days maximum rainfall of July.

(iv) LN and Gumbel distributions presented the overall best fits to the data for all one to five days maximum

rainfall of August. LN distribution was appeared as the best fit distribution with NSE value 0.9946 for four days maximum rainfall of August

(v) Only Gumbel distribution was passed Chi-Square test for all one to five days maximum rainfall of September. Gumbel distribution showed the best fit with NSE values 0.9603 for five days maximum rainfall of September.

It is recommended that Gumbel distribution should be considered in the final selection of optimum probability distribution for one to five days maximum monthly and annual rainfall in Junagadh (Gujarat-India) region.

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References

- Amin, M. T., Rizwan, M. And Alazba, A. A., 2016, "A best-fit probability distribution for the estimation of rainfall in northern regions of Pakistan", *Open Life Sci.*, **11**, 432-440.
- Arora, K. and Singh, V. P., 1989, "A comparative evaluation of the estimators of the log Pearson type 3 distributions", *J. Hydrol.*, **105**, 19-37.
- Bhakar, S. R., Bansal, A. N., Chhajed, N. and Purohit, R. C., 2006, "Frequency analysis of consecutive day's maximum rainfall at Banswara, Rajasthan, India", *ARPN Journal of Engineering and Applied Sciences*, **1**, 3, 64-67.
- Bhakar, S. R., Iqbal, M., Devanda, M., Chhajed, N. and Bansal, A. K., 2008, "Probability analysis of rainfall at Kota", *Ind. J. Agric. Res.*, **42**, 201-206.
- Eslamian, S. S. And Feizi, H., 2007, "Maximum Monthly Rainfall Analysis Using L-Moments for an Arid Region in Isfahan Province, Iran", *J. Appl. Meteorol. Climatol.*, **46**, 494-503.
- Green, I. and D. Stephenson, 1986, "Criteria for Comparison of Single Event Models", *Hydrological Sciences Journal*, **31**, 3, 395-411.
- Hanson, L. S. and Vogel, R., 2008, "The probability distribution of daily rainfall in the United States", In *World Environmental and Water Resources Congress*, Ahupua'A (1-10).
- Hosking, J. R. M. and Wallis, J. R., 1997, "Regional frequency analysis: an approach based on L-moments", Cambridge: Cambridge University Press, England.

Kumar, A., 2000, "Prediction of annual maximum daily rainfall of Ranichauri (Tehri Garhwal) based on probability Analysis", *Ind. J. Soil Conserv.*, **28**, 178-180.

Kumar, R., Singh, R. D. and Sharma, K. D., 2005, "Water resources of India", *Curr. Sci.*, **89**, 5, 794-811.

Kwaku, X. S. and Duke, O., 2007, "Characterization and frequency analysis of one day annual maximum and two to five consecutive days maximum rainfall of Accra, Ghana", *ARPN J. Eng. Appl. Sci.*, **2**, 27-31.

Lee, C., 2005, "Application of rainfall frequency analysis on studying rainfall distribution characteristics of Chia-Nan plain area in Southern Taiwan", *Crop Environ. Bioinf*, **2**, 31-38.

Moriassi, D. N., Gitau, M. W., Pai, N. and Daggupati, P., 2015, "Hydrologic and Water Quality Models: Performance Measures and Evaluation Criteria", *Transactions of the American Society of Agricultural and Biological Engineers*, **58**, 6, 1763-1785.

Ogunlela, A. O., 2001, "Stochastic Analysis of Rainfall Evens in Ilorin, Nigeria", *J. Agric. Res. Dev.*, 39-50.

Olofintoye, O. O., Sule, B. F. and Salami, A. W., 2009, "Best-fit probability distribution model for peak daily rainfall of selected cities in Nigeria", *New York Science Journal*, **2**, 3, 1-12.

Pretorius, H., W. James and J. Smit., 2013, "A Strategy for Managing Deficiencies of SWMM Modeling for Large Undeveloped Semi-Arid Watersheds", *Journal of Water Management Modeling*, 21: R246-01. doi : 10.14796/JWMM.R246-01.

Sen, Z. and Eljadid, A. G., 1999, "Rainfall distribution functions for Libya and Rainfall Prediction", *Hydrol. Sci. J.*, **44**, 665-680.

Sharma, M. A. and Singh, J. B., 2010, "Use of probability distribution in rainfall analysis", *New York Science Journal*, **3**, 9, 40-49.

Singh, R. K., 2001, "Probability analysis for prediction of annual maximum rainfall of Eastern Himalaya (Sikkim mid hills)", *Ind. J. Soil Conserv.*, **29**, 263-265.

Son, K., Lin, L., Band, L. and Owens, E. M., 2019, "Modelling the interaction of climate, forest ecosystem and hydrology to estimate catchment dissolved organic carbon export", *Hydrological processes*, **33**, 1448-1464. <https://doi.org/10.1002/hyp.13412>.

Zalina, M. D., Desa, M. N. M., Nguyen, V. T. A. and Kassim, A. H. M., 2002, "Selecting a probability distribution for extreme rainfall series in Malaysia", *Water science and technology*, **45**, 2, 63-68.



APPENDIX 1

Estimation of one day annual maximum rainfall and goodness of fit for theoretical probability distributions

Return Period (T), Years	Observed Rainfall, mm	Gumbel	V T Chow	LP-III	LN	EXP	GEV	GP	Gamma
3	1187	1516	1805	1338	1205	569	1773	598	1624
4	1282	1830	2394	1391	1403	447	1854	572	1520
5	1394	2062	2830	1446	1634	380	1894	565	1438
10	1698	2748	4117	1460	2075	257	1960	560	1187
15	2858	3135	4844	1461	2290	220	1978	560	1039
20	5225	3406	5352	1462	2527	201	1985	560	935
25	5232	3615	5744	1463	2622	190	1989	560	854
30	5240	3785	6063	1463	2721	183	1992	560	788
35	5248	3928	6332	1463	2824	178	1994	560	732
Chi-Square value		47.67	287.96	2313.67	582.98	30247.62	910.14	12855.06	4069.21
Null Hypothesis Testing		Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject
NSE		0.6049	0.9849	0.9430	0.9741	0.8909	0.9576	0.9077	0.9212
Rating		Excellent	Excellent	Excellent	Excellent	Excellent	Excellent	Excellent	Excellent
Rank		8	1	4	2	7	3	6	5
Parameters		$\mu=1186.75$, $\sigma=1038.69$, $Y_n=0.5424$, $S_n=1.1363$	$A=-1174.91$, $B=-3950.76$	$Z=3.01, \sigma_z=0.24$, $C_S=-118.18$	$Z=3.01, \sigma_z=0.24$, $C_S=0$	$\alpha=1038.69$, $\xi=148.06$	$\alpha=901.89$, $\xi=1287.58$, $k=1.26$	$\alpha=-$, $\xi=3695.51$, $k=4.00$	$\alpha=-$, $\xi=836.25$, $\epsilon=2023.00$

APPENDIX 2

Estimation of two days annual maximum rainfall and goodness of fit for theoretical probability distributions

Return Period (T), Years	Observed Rainfall, mm	Gumbel	V T Chow	LP-III	LN	EXP	GEV	GP	Gamma
3	2199	2966	3516	2609	2374	1195	3340	1230	3126
4	2331	3529	4663	2691	2732	973	3461	1196	2932
5	2534	3946	5513	2775	3143	852	3521	1188	2781
10	3566	5178	8021	2794	3917	630	3613	1183	2312
15	6817	5873	9436	2795	4289	562	3635	1183	2038
20	7005	6359	10427	2797	4697	529	3645	1183	1843
25	7919	6734	11190	2797	4860	509	3650	1183	1692
30	8833	7039	11811	2797	5028	496	3653	1183	1569
35	9747	7296	12334	2797	5203	487	3655	1183	1465
Chi-Square value		9.35276	973.691	3045.6768	663.2323	35645.41	1217.44	16795.58	5471.56
Null Hypothesis Testing		Accept	Reject	Reject	Reject	Reject	Reject	Reject	Reject
NSE		0.9166	0.7507	0.5498	0.8243	0.0566	0.6692	0.2201	0.3439
Rating		Excellent	Excellent	Excellent	Excellent	Poor	Excellent	Fair	Good
Rank		1	3	5	2	8	4	7	6
Parameters		$\mu=2374.50$ A=-2288.61, $\sigma=1865.08$, B=-7696.28 Yn=0.5424, Sn=1.1363	$Z^- = 3.31, \sigma_z = 0.22$, C _s = -201.19	$Z^- = 3.31, \sigma_z = 0.22$, C _s = 0	$\alpha=1879.60, \alpha=1519.94$, $\xi=432.41$ $\xi=2551.59$, 27327.28, k=1.37 $\xi=7330.68, \epsilon=3869.37$ k=4.45				

APPENDIX 3

Estimation of three days annual maximum rainfall and goodness of fit for theoretical probability distributions

Return Period (T), Years	Observed Rainfall, mm	Gumbel	V T Chow	LP-III	LN	EXP	GEV	GP	Gamma
3	3191	4364	5142	3873	3541	1853	4869	1747	4585
4	3544	5121	6819	3968	4069	1551	5104	1646	4297
5	3709	5682	8061	4066	4675	1385	5226	1615	4074
10	7762	7338	11728	4081	5813	1082	5434	1592	3381
15	9257	8272	13798	4082	6360	989	5492	1590	2975
20	12003	8926	15246	4083	6958	943	5518	1590	2687
25	12046	9430	16362	4083	7197	917	5533	1590	2464
30	12089	9840	17270	4083	7444	899	5543	1590	2282
35	12133	10186	18036	4083	7699	886	5550	1590	2128
Chi-Square value		69.47	1332.72	4721.72	998.40	45003.91	1736.58	28593.30	8450.27
Null Hypothesis Testing		Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject
NSE		0.8649	0.6785	0.4074	0.7925	-0.2447	0.6042	-0.0927	0.1272
Rating		Excellent	Excellent	Very Good	Excellent	Poor	Excellent	Poor	Poor
Rank		1	3	5	2	8	4	7	6
Parameters		$\mu=3568.57$ A=-3346.36, $\sigma=2507.61$, B=-11253.61 Yn=0.5424, Sn=1.1363	$Z^- = 3.49, \sigma_z = 0.22$, C _s = -242.38	$Z^- = 3.49, \sigma_z = 0.22$, C _s = 0	$\alpha=2569.11, \alpha=2370.26$, $\xi=811.53$ $\xi=3543.80$, 29660.61, k=1.16 $\xi=9828.84, \epsilon=5683.68$ k=3.6				

APPENDIX 4

Estimation of four days annual maximum rainfall and goodness of fit for theoretical probability distributions

Return Period (T), Years	Observed Rainfall, mm	Gumbel	V T Chow	LP-III	LN	EXP	GEV	GP	Gamma
3	4466	5728	6678	5203	4776	2498	6283	2229	5982
4	4689	6643	8856	5326	5451	2123	6656	2014	5602
5	4936	7320	10468	5452	6223	1917	6860	1938	5307
10	9259	9322	15232	5472	7658	1543	7231	1869	4391
15	13076	10451	17919	5474	8342	1427	7346	1862	3855
20	13273	11241	19800	5475	9088	1370	7402	1860	3474
25	13382	11850	21250	5475	9385	1337	7435	1860	3179
30	13491	12346	22429	5475	9691	1315	7456	1860	2938
35	13600	12763	23423	5475	10008	1299	7472	1859	2734
Chi-Square value		7.98	2375.63	3890.14	601.41	42532.72	1174.07	33955.24	8240.51
Null Hypothesis Testing		Accept	Reject	Reject	Reject	Reject	Reject	Reject	Reject
NSE		0.8971	0.6242	0.6872	0.9157	0.2335	0.8167	0.2943	0.4819
Rating		Excellent	Excellent	Excellent	Excellent	Fair	Excellent	Fair	Very Good
Rank		2	5	4	1	8	3	7	6
Parameters		$\mu=4766.95$ $\sigma=3030.23$ $Y_n=0.5424$ $S_n=1.1363$	A=- 4345.84, B=- 14615.00	$Z^- = 3.62, \sigma_z = 0.20$, $C_s = -471.90$	$Z^- = 3.62, \sigma_z = 0.20$, $C_s = 0$	$\alpha = 3183.52$, $\alpha = 3178.47$, $\xi = 1207.09$	$\alpha = 3178.47$, $\xi = 4395.63$, k=1.00	$\alpha = -$ 30675.68, $\xi = 12027.37$, k=3.02	$\alpha = -$ 3044.45, $\epsilon = 7435.06$

APPENDIX 5

Estimation of five days annual maximum rainfall and goodness of fit for theoretical probability distributions

Return Period (T), Years	Observed Rainfall, mm	Gumbel	V T Chow	LP-III	LN	EXP	GEV	GP	Gamma
3	5448	7117	8176	6573	6065	3120	7642	2706	7370
4	5909	8173	10843	6704	6892	2673	8183	2326	6894
5	6202	8955	12818	6837	7832	2428	8492	2174	6524
10	14012	11264	18650	6855	9570	1982	9093	2010	5376
15	14297	12567	21940	6856	10394	1844	9294	1988	4704
20	14524	13479	24244	6856	11290	1776	9397	1982	4228
25	14637	14182	26019	6856	11646	1737	9460	1979	3858
30	14749	14754	27462	6857	12013	1710	9503	1978	3556
35	14862	15235	28679	6857	12392	1692	9534	1977	3301
Chi-Square value		1.24	3420.01	3415.08	345.35	43015.08	796.90	42501.32	8393.84
Null Hypothesis Testing		Accept	Reject	Reject	Reject	Reject	Reject	Reject	Reject
NSE		0.8879	0.4646	0.7407	0.9474	0.2670	0.8731	0.2842	0.5228
Rating		Excellent	Very Good	Excellent	Excellent	Fair	Excellent	Fair	Excellent
Rank		2	6	4	1	8	3	7	5
Parameters		$\mu=6008.50$ $\sigma=3496.62$ $Y_n=0.5424$ $S_n=1.1363$	A=- 5321.13, B=- 17895.02	$Z^- = 3.73, \sigma_z = 0.20$, $C_s = -762.85$	$Z^- = 3.73, \sigma_z = 0.20$, $C_s = 0$	$\alpha = 3794.29$, $\alpha = 3981.57$, $\xi = 1581.73$	$\alpha = 3981.57$, $\xi = 5147.51$, k=0.87	$\alpha = -$ 30887.11, $\xi = 14063.74$, k=2.56	$\alpha = -$ 3814.36, $\epsilon = 9190.39$

Estimation of one day monthly (June) maximum rainfall and goodness of fit for theoretical probability distributions

Return Period (T), Years	Observed Rainfall, mm	Gumbel	V T Chow	LP-III	LN	EXP	GEV	GP	Gamma
3	75	88	105	80	64	34	104	24	104
4	120	106	140	86	88	27	114	15	96
5	127	119	165	94	122	23	120	12	89
10	156	159	240	96	201	16	132	7	69
15	177	181	283	96	247	14	136	6	57
20	196	196	312	96	304	13	139	6	49
25	203	208	335	96	329	12	140	6	43
30	210	218	354	96	356	12	141	6	37
35	217	226	369	96	385	11	142	6	33
Chi-Square value		0.03	32.49	55.38	19.96	1217.02	9.36	2496.84	157.83
Null Hypothesis Testing		Accept	Reject	Reject	Reject	Reject	Reject	Reject	Reject
NSE		0.9674	-2.5953	-1.3203	-2.1944	-7.2858	0.2106	-8.0486	-4.1251
Rating		Excellent	Poor	Poor	Poor	Poor	Fair	Poor	Poor
Rank		1	5	3	4	7	2	8	6
Parameters		$\mu=69.11, \sigma=59.61, Y_n=0.5424, S_n=1.1363$	$A=-68.77, B=-230.65$	$Z^- = 1.67, \sigma_z=0.50, C_s=-3.13$	$Z^- = 1.67, \sigma_z=0.50, C_s=0$	$\alpha=59.61, \xi=9.50$	$\alpha=64.24, \xi=62.13, k=0.75$	$\alpha=-445.51, \xi=208.53, k=2.20$	$\alpha=-66.46, \epsilon=135.57$

Estimation of two days monthly (June) maximum rainfall and goodness of fit for theoretical probability distributions

Return Period (T), Years	Observed Rainfall, mm	Gumbel	V T Chow	LP-III	LN	EXP	GEV	GP	Gamma
3	113	117	140	106	84	45	139	31	138
4	145	141	185	114	118	35	152	20	127
5	167	159	219	123	166	30	160	14	119
10	236	211	319	125	282	21	176	8	92
15	241	241	375	125	352	18	182	7	76
20	249	262	415	125	438	16	185	6	65
25	257	278	445	125	476	16	187	6	56
30	265	291	470	125	517	15	189	6	49
35	272	302	491	125	561	15	190	6	43
Chi-Square value		0.16	45.01	74.62	40.70	1589.48	10.63	3580.32	201.89
Null Hypothesis Testing		Accept	Reject	Reject	Reject	Reject	Accept	Reject	Reject
NSE		0.9496	0.5771	0.7606	0.4291	0.1356	0.9320	0.0445	0.4798
Rating		Excellent	Excellent	Excellent	Very Good	Poor	Excellent	Poor	Very Good
Rank		1	4	3	6	7	2	8	5
Parameters		$\mu=91.86, \sigma=79.45, Y_n=0.5424, S_n=1.1363$	$A=-91.28, B=-306.38$	$Z^- = 1.78, \sigma_z=0.53, C_s=-3.29$	$Z^- = 1.78, \sigma_z=0.53, C_s=0$	$\alpha=79.45, \xi=12.42$	$\alpha=85.75, \xi=82.14, k=0.74$	$\alpha=-587.87, \xi=277.60, k=2.17$	$\alpha=-89.19, \epsilon=181.05$

APPENDIX 8

Estimation of three days monthly (June) maximum rainfall and goodness of fit for theoretical probability distributions

Return Period (T), Years	Observed Rainfall, mm	Gumbel	V T Chow	LP-III	LN	EXP	GEV	GP	Gamma
3	135	130	157	119	95	53	152	40	154
4	159	156	209	126	134	42	169	24	142
5	184	175	247	133	188	37	179	17	133
10	246	232	359	135	319	27	201	6	103
15	253	264	422	135	397	24	209	3	86
20	259	286	467	135	495	22	214	3	74
25	265	303	501	135	537	21	217	2	65
30	270	317	529	135	583	21	219	2	57
35	276	329	552	135	633	20	221	2	50
Chi-Square value		1.05	62.71	69.61	58.31	1317.25	4.50	4276.05	180.24
Null Hypothesis Testing		Accept	Reject	Reject	Reject	Reject	Accept	Reject	Reject
NSE		0.8243	0.0269	0.6608	-0.2859	-0.2945	0.9518	-0.5347	0.2537
Rating		Excellent	Poor	Excellent	Poor	Poor	Excellent	Poor	Fair
Rank		2	5	3	6	7	1	8	4
Parameters		$\mu=103.36, \sigma=85.45, Y_n=0.5424, S_n=1.1363$	$A=-102.66, B=-344.63$	$Z^-1.83, \sigma_z=0.52, C_s=-3.52$	$Z^-1.83, \sigma_z=0.52, C_s=0$	$\alpha=85.45, \xi=17.90$	$\alpha=93.09, \xi=87.80, k=0.62$	$\alpha=-530.02, \xi=290.30, k=1.84$	$\alpha=-97.49, \epsilon=200.84$

APPENDIX 9

Estimation of four days monthly (June) maximum rainfall and goodness of fit for theoretical probability distributions

Return Period (T), Years	Observed Rainfall, mm	Gumbel	V T Chow	LP-III	LN	EXP	GEV	GP	Gamma
3	142	138	166	125	100	54	163	40	164
4	160	166	220	132	141	43	180	24	151
5	192	187	260	141	198	37	190	17	140
10	252	248	379	142	339	26	211	8	109
15	264	283	446	142	424	23	219	6	91
20	270	307	492	142	529	21	224	5	78
25	295	326	528	142	575	20	227	5	68
30	319	341	558	142	625	20	229	5	59
35	344	353	583	142	679	19	230	5	52
Chi-Square value		0.59	59.46	86.46	58.02	1646.19	7.93	4357.63	214.58
Null Hypothesis Testing		Accept	Reject	Reject	Reject	Reject	Accept	Reject	Reject
NSE		0.9725	0.5367	0.7615	0.3495	0.1824	0.9458	0.0709	0.5054
Rating		Excellent	Excellent	Excellent	Good	Poor	Excellent	Poor	Excellent
Rank		1	4	3	6	7	2	8	5
Parameters		$\mu=109.05, \sigma=92.63, Y_n=0.5424, S_n=1.1363$	$A=-108.29, B=-363.59$	$Z^-1.85, \sigma_z=0.53, C_s=-3.50$	$Z^-1.85, \sigma_z=0.53, C_s=0$	$\alpha=92.63, \xi=16.42$	$\alpha=100.69, \xi=94.76, k=0.68$	$\alpha=-619.28, \xi=316.57, k=1.98$	$\alpha=-104.17, \epsilon=213.22$

Estimation of five days monthly (June) maximum rainfall and goodness of fit for theoretical probability distributions

Return Period (T), Years	Observed Rainfall, mm	Gumbel	V T Chow	LP-III	LN	EXP	GEV	GP	Gamma
3	145	146	175	131	106	57	173	42	173
4	161	176	233	138	149	45	190	26	159
5	207	198	275	146	210	39	200	19	148
10	253	263	400	147	360	27	222	10	115
15	295	299	471	147	450	24	230	8	96
20	345	325	520	147	561	22	235	8	82
25	354	345	558	147	610	21	238	7	72
30	362	361	589	147	663	20	240	7	63
35	370	374	616	147	721	20	241	7	56
Chi-Square value		0.00	52.46	121.90	52.04	1976.86	15.47	4611.77	269.51
Null Hypothesis Testing		Accept	Reject	Reject	Reject	Reject	Accept	Reject	Reject
NSE		0.9933	0.6705	0.7142	0.5164	0.1660	0.9151	0.0741	0.4692
Rating		Excellent	Excellent	Excellent	Excellent	Poor	Excellent	Poor	Very Good
Rank		1	4	3	5	7	2	8	6
Parameters		$\mu=115.24, \sigma=98.17, Y_n=0.5424, S_n=1.1363$	$A=-114.42, B=-384.20$	$Z=1.88, \sigma_z=0.53, C_s=-3.64$	$Z=1.88, \sigma_z=0.53, C_s=0$	$\alpha=98.17, \xi=17.08$	$\alpha=106.51, \xi=101.18, k=0.70$	$\alpha=-675.34, \xi=337.00, k=2.05$	$\alpha=-109.64, \xi=224.88$

Estimation of one day monthly (July) maximum rainfall and goodness of fit for theoretical probability distributions

Return Period (T), Years	Observed Rainfall, mm	Gumbel	V T Chow	LP-III	LN	EXP	GEV	GP	Gamma
3	119	116	143	115	97	53	131	46	253
4	151	137	189	123	125	45	146	30	229
5	162	152	224	131	160	40	156	22	211
10	187	198	326	133	236	32	178	8	154
15	202	223	383	133	277	29	188	5	120
20	206	241	424	134	326	28	194	3	97
25	224	255	455	134	346	27	197	2	78
30	242	267	480	134	367	27	200	2	63
35	260	276	501	134	390	27	202	1	51
Chi-Square value		0.76	66.92	32.17	15.65	752.55	1.81	2479.18	21.70
Null Hypothesis Testing		Accept	Reject	Reject	Reject	Reject	Accept	Reject	Reject
NSE		0.7547	-10.9648	-1.2136	-2.1103	-10.1783	0.7166	-13.2470	-5.2789
Rating		Excellent	Poor	Poor	Poor	Poor	Excellent	Poor	Poor
Rank		1	7	3	4	6	2	8	5
Parameters		$\mu=93.79, \sigma=69.13, Y_n=0.5424, S_n=1.1363$	$A=-93.19, B=-312.79$	$Z=1.88, \sigma_z=0.38, C_s=-10.85$	$Z=1.88, \sigma_z=0.38, C_s=0$	$\alpha=69.13, \xi=24.66$	$\alpha=74.39, \xi=76.20, k=0.48$	$\alpha=-340.36, \xi=231.48, k=1.47$	$\alpha=-189.82, \xi=343.71$

APPENDIX 12

Estimation of two days monthly (July) maximum rainfall and goodness of fit for theoretical probability distributions

Return Period (T), Years	Observed Rainfall, mm	Gumbel	V T Chow	LP-III	LN	EXP	GEV	GP	Gamma
3	153	148	184	149	127	70	167	62	323
4	204	175	244	157	161	59	186	41	293
5	215	194	289	165	204	54	198	30	269
10	257	251	420	167	297	44	228	12	196
15	260	284	494	168	346	40	240	7	154
20	264	306	546	168	404	39	248	5	123
25	276	324	586	168	428	38	253	4	100
30	289	338	619	168	453	37	257	3	81
35	302	350	646	168	480	37	259	3	64
Chi-Square value		1.04	90.30	41.54	17.76	865.51	1.86	2825.00	26.49
Null Hypothesis Testing		Accept	Reject	Reject	Reject	Reject	Accept	Reject	Reject
NSE		0.7627	0.1237	0.8668	0.7987	0.3096	0.9899	0.1016	0.6355
Rating		Excellent	Poor	Excellent	Excellent	Good	Excellent	Poor	Excellent
Rank		4	7	2	3	6	1	8	5
Parameters		$\mu=121.02, \sigma=86.64, A=-120.14, Y_n=0.5424, S_n=1.1363$	$Z=2.00, \sigma_z=0.37, B=-403.44, C_S=-10.85$	$Z=2.00, \sigma_z=0.37, C_S=0$	$\alpha=86.64, \xi=34.38$	$\alpha=92.90, \xi=98.17, k=0.46$	$\alpha=-416.15, \xi=292.52, k=1.43$	$\alpha=-242.35, \epsilon=438.54$	

APPENDIX 13

Estimation of three days monthly (July) maximum rainfall and goodness of fit for theoretical probability distributions

Return Period (T), Years	Observed Rainfall, mm	Gumbel	V T Chow	LP-III	LN	EXP	GEV	GP	Gamma
3	177	174	214	172	147	79	197	70	376
4	223	205	284	180	188	67	219	47	340
5	234	228	336	188	240	60	233	35	313
10	260	297	489	189	353	48	266	16	228
15	323	335	576	189	413	44	280	11	178
20	393	362	636	189	484	43	288	9	142
25	396	383	683	189	514	42	293	8	115
30	399	400	721	189	545	41	297	8	92
35	402	414	753	189	579	40	300	7	73
Chi-Square value		0.00	84.24	84.71	13.90	1309.21	8.75	3542.47	53.78
Null Hypothesis Testing		Accept	Reject	Reject	Accept	Reject	Accept	Reject	Reject
NSE		0.9775	0.1803	0.6714	0.8461	-0.1375	0.9266	-0.3860	0.3015
Rating		Excellent	Poor	Excellent	Excellent	Poor	Excellent	Poor	Good
Rank		1	6	4	3	7	2	8	5
Parameters		$\mu=140.93, \sigma=103.58, A=-139.84, Y_n=0.5424, S_n=1.1363$	$Z=2.06, \sigma_z=0.38, B=-469.69, C_S=-14.64$	$Z=2.06, \sigma_z=0.38, C_S=0$	$\alpha=103.58, \xi=37.34$	$\alpha=111.90, \xi=115.79, k=0.51$	$\alpha=-522.38, \xi=347.49, k=1.53$	$\alpha=-283.49, \epsilon=511.16$	

APPENDIX 14

Estimation of four days monthly (July) maximum rainfall and goodness of fit for theoretical probability distributions

Return Period (T), Years	Observed Rainfall, mm	Gumbel	V T Chow	LP-III	LN	EXP	GEV	GP	Gamma
3	197	190	235	190	162	88	214	79	408
4	235	224	312	198	207	75	239	54	369
5	242	249	368	207	265	67	254	40	340
10	274	323	536	209	391	54	292	17	248
15	346	364	631	209	459	50	308	11	194
20	435	394	697	209	538	48	317	9	155
25	437	416	748	209	571	47	324	7	126
30	439	434	790	209	607	46	328	7	101
35	441	450	825	209	644	46	332	6	81
Chi-Square value		0.00	94.87	86.21	18.39	1356.87	8.17	3825.09	57.74
Null Hypothesis Testing		Accept	Reject	Reject	Reject	Reject	Accept	Reject	Reject
NSE		0.9708	0.4542	0.7885	0.8862	0.2680	0.9524	0.0959	0.5380
Rating		Excellent	Very Good	Excellent	Excellent	Fair	Excellent	Poor	Excellent
Rank		1	6	4	3	7	2	8	5
Parameters		$\mu=154.42, \sigma=111.94, A=-153.19, Y_n=0.5424, S_n=1.1363$	$Z=2.10, \sigma_z=0.38, B=-514.59, C_S=-15.15$	$Z=2.10, \sigma_z=0.38, C_S=0$	$\alpha=111.94, \xi=42.48$	$\alpha=120.19, \xi=125.28, k=0.47$	$\alpha=-530.61, \xi=371.60, k=1.44$	$\alpha=-306.40, \epsilon=553.92$	

APPENDIX 15

Estimation of five days monthly (July) maximum rainfall and goodness of fit for theoretical probability distributions

Return Period (T), Years	Observed Rainfall, mm	Gumbel	V T Chow	LP-III	LN	EXP	GEV	GP	Gamma
3	211	199	246	198	169	91	225	81	427
4	237	235	326	205	217	77	251	55	387
5	251	262	386	213	277	70	268	41	356
10	299	340	561	214	408	56	306	19	259
15	368	384	660	214	478	51	321	14	202
20	470	415	730	214	560	49	331	12	162
25	471	439	783	214	595	48	337	10	130
30	473	458	827	214	632	47	342	10	105
35	474	475	863	214	670	47	345	9	83
Chi-Square value		0.08	93.48	106.63	15.67	1533.65	11.36	3981.62	68.62
Null Hypothesis Testing		Accept	Reject	Reject	Reject	Reject	Accept	Reject	Reject
NSE		0.9702	0.5133	0.7665	0.9109	0.2556	0.9422	0.1008	0.5244
Rating		Excellent	Excellent	Excellent	Excellent	Fair	Excellent	Poor	Excellent
Rank		1	6	4	3	7	2	8	5
Parameters		$\mu=161.65, \sigma=118.58, A=-160.36, Y_n=0.5424, S_n=1.1363$	$Z=2.12, \sigma_z=0.38, B=-538.68, C_S=-15.73$	$Z=2.12, \sigma_z=0.38, C_S=0$	$\alpha=118.58, \xi=43.08$	$\alpha=127.94, \xi=132.42, k=0.50$	$\alpha=-584.66, \xi=394.61, k=1.51$	$\alpha=-322.53, \epsilon=581.35$	

APPENDIX 16

Estimation of one day monthly (August) maximum rainfall and goodness of fit for theoretical probability distributions

Return Period (T), Years	Observed Rainfall, mm	Gumbel	V T Chow	LP-III	LN	EXP	GEV	GP	Gamma
3	63	70	81	60	49	23	84	16	157
4	71	85	108	65	66	17	90	12	142
5	79	97	128	70	89	14	94	10	130
10	121	131	186	71	141	8	100	9	92
15	185	150	219	72	170	6	102	9	70
20	195	163	242	72	206	5	103	9	55
25	196	173	259	72	221	4	103	9	43
30	198	182	274	72	238	4	104	9	33
35	199	189	286	72	255	4	104	9	25
Chi-Square value		0.08	93.48	106.63	15.67	1533.65	11.36	3981.62	68.62
Null Hypothesis Testing		Accept	Accept	Reject	Accept	Reject	Reject	Reject	Reject
NSE		0.8716	0.9698	0.9177	0.9933	0.7869	0.9545	0.7941	0.8623
Rating		Excellent	Excellent	Excellent	Excellent	Excellent	Excellent	Excellent	Excellent
Rank		5	2	4	1	8	3	7	6
Parameters		$\mu=53.43, \sigma=51.32, A=-53.24, Y_n=0.5424, S_n=1.1363$	$B=-178.45$	$Z=1.56, \sigma_z=0.46, C_s=-3.05$	$Z=1.56, \sigma_z=0.46, C_s=0$	$\alpha=51.32, \xi=2.10$	$\alpha=51.70, \xi=53.04, k=0.98$	$\alpha=-518.65, \xi=184.98, k=2.94$	$\alpha=-124.17, \epsilon=216.41$

APPENDIX 17

Estimation of two days monthly (August) maximum rainfall and goodness of fit for theoretical probability distributions

Return Period (T), Years	Observed Rainfall, mm	Gumbel	V T Chow	LP-III	LN	EXP	GEV	GP	Gamma
3	85	99	115	85	69	32	119	22	223
4	105	121	152	91	93	23	127	17	201
5	109	138	180	97	126	18	132	15	184
10	203	187	262	98	201	9	140	14	130
15	231	214	309	98	243	7	142	14	99
20	252	234	341	98	295	5	143	13	77
25	271	248	366	98	317	5	144	13	60
30	290	260	386	98	340	4	144	13	46
35	310	271	404	98	366	4	145	13	34
Chi-Square value		0.45	19.19	127.70	2.02	3215.68	34.41	2448.74	68.18
Null Hypothesis Testing		Accept	Reject	Reject	Accept	Reject	Reject	Reject	Reject
NSE		0.9670	0.7100	0.0964	0.9432	-1.2754	0.5017	-1.1551	-0.4744
Rating		Excellent	Excellent	Poor	Excellent	Poor	Excellent	Poor	Poor
Rank		1	3	5	2	8	4	7	6
Parameters		$\mu=75.49, \sigma=73.96, A=-75.08, Y_n=0.5424, S_n=1.1363$	$B=-251.89$	$Z=1.71, \sigma_z=0.46, C_s=-3.77$	$Z=1.71, \sigma_z=0.46, C_s=0$	$\alpha=73.96, \xi=1.53$	$\alpha=72.65, \xi=76.74, k=1.04$	$\alpha=-809.37, \xi=270.66, k=3.15$	$\alpha=-177.26, \epsilon=307.51$

Estimation of three days monthly (August) maximum rainfall and goodness of fit for theoretical probability distributions

Return Period (T), Years	Observed Rainfall, mm	Gumbel	V T Chow	LP-III	LN	EXP	GEV	GP	Gamma
3	93	115	133	97	80	35	139	25	257
4	112	142	176	103	108	25	148	20	232
5	125	161	209	108	147	19	153	18	212
10	239	219	303	109	235	9	161	17	149
15	262	252	357	109	286	6	163	17	113
20	302	275	395	109	348	4	165	17	87
25	328	292	423	109	374	3	165	17	67
30	355	307	447	109	403	3	166	17	51
35	382	319	467	109	433	2	166	17	37
Chi-Square value		0.72	19.34	175.56	2.14	4594.52	46.60	2774.34	90.70
Null Hypothesis Testing		Accept	Reject	Reject	Accept	Reject	Reject	Reject	Reject
NSE		0.9568	0.8603	0.3850	0.9757	-0.4256	0.6506	-0.3208	0.0452
Rating		Excellent	Excellent	Good	Excellent	Poor	Excellent	Poor	Poor
Rank		2	3	5	1	8	4	7	6
Parameters		$\mu=87.35, \sigma=87.73, A=-86.82, Y_n=0.5424, S_n=1.1363$	$B=-291.37$	$Z^- = 1.77, \sigma_z = 0.47, C_s = -4.05$	$Z^- = 1.77, \sigma_z = 0.47, C_s = 0$	$\alpha=87.73, \xi=-0.38$	$\alpha=84.06, \xi=90.62, k=1.09$	$\alpha=-1016.06, \xi=321.83, \epsilon=355.69, k=3.33$	$\alpha=-206.22, \epsilon=355.69$

Estimation of four days monthly (August) maximum rainfall and goodness of fit for theoretical probability distributions

Return Period (T), Years	Observed Rainfall, mm	Gumbel	V T Chow	LP-III	LN	EXP	GEV	GP	Gamma
3	101	127	146	107	88	37	154	27	284
4	118	157	194	113	119	26	163	22	255
5	132	179	229	119	160	19	169	21	233
10	242	244	334	120	256	8	177	19	165
15	298	281	393	120	311	4	180	19	125
20	410	307	434	120	377	2	181	19	96
25	414	327	466	120	405	1	182	19	74
30	419	343	491	120	436	1	182	19	56
35	424	356	513	120	469	0	183	19	41
Chi-Square value		2.68	14.29	235.80	0.16	6743.57	68.88	3361.13	126.26
Null Hypothesis Testing		Accept	Accept	Reject	Accept	Reject	Reject	Reject	Reject
NSE		0.8996	0.9423	0.5487	0.9946	0.0302	0.7233	0.1104	0.3324
Rating		Excellent	Excellent	Excellent	Excellent	Poor	Excellent	Poor	Good
Rank		3	2	5	1	8	4	7	6
Parameters		$\mu=96.07, \sigma=98.65, A=-95.44, Y_n=0.5424, S_n=1.1363$	$B=-320.37$	$Z^- = 1.82, \sigma_z = 0.46, C_s = -4.40$	$Z^- = 1.82, \sigma_z = 0.46, C_s = 0$	$\alpha=98.65, \xi=-2.59$	$\alpha=93.33, \xi=100.66, k=1.12$	$\alpha=-1163.36, \xi=358.98, k=3.42$	$\alpha=-227.23, \epsilon=391.96$

APPENDIX 20

Estimation of five days monthly (August) maximum rainfall and goodness of fit for theoretical probability distributions

Return Period (T), Years	Observed Rainfall, mm	Gumbel	V T Chow	LP-III	LN	EXP	GEV	GP	Gamma
3	116	135	155	114	94	41	163	30	298
4	125	166	206	119	127	28	174	24	269
5	136	189	243	125	172	22	179	22	246
10	244	257	354	126	274	9	190	21	174
15	301	296	417	126	332	6	193	21	132
20	414	323	460	126	403	4	194	21	102
25	431	344	494	126	433	3	195	21	79
30	448	360	522	126	466	2	195	21	61
35	464	375	545	126	501	2	196	21	45
Chi-Square value		2.51	16.79	244.98	0.59	6288.73	66.38	3405.75	128.16
Null Hypothesis Testing		Accept	Reject	Reject	Accept	Reject	Reject	Reject	Reject
NSE		0.9135	0.9351	0.5476	0.9945	0.0375	0.7312	0.1128	0.3347
Rating		Excellent	Excellent	Excellent	Excellent	Poor	Excellent	Poor	Good
Rank		3	2	5	1	8	4	7	6
Parameters		$\mu=96.07, \sigma=98.65,$ $Y_n=0.5424,$ $S_n=1.1363$	$A=-101.27,$ $B=-339.96$	$Z^- = 1.85, \sigma_z = 0.46,$ $C_s = -4.69$	$Z^- = 1.85, \sigma_z = 0.46,$ $C_s = 0$	$\alpha = 103.36,$ $\xi = -1.41$	$\alpha = 99.76,$ $\xi = 105.21,$ $k = 1.08$	$\alpha = 1142.02,$ $\xi = 368.80,$ $k = 3.28$	$\alpha = 237.91,$ $\epsilon = 411.95$

APPENDIX 21

Estimation of one day monthly (September) maximum rainfall and goodness of fit for theoretical probability distributions

Return Period (T), Years	Observed Rainfall, mm	Gumbel	V T Chow	LP-III	LN	EXP	GEV	GP	Gamma
3	77	80	93	71	57	26	96	18	181
4	87	97	124	77	78	20	105	12	163
5	94	110	146	83	107	16	109	9	150
10	142	149	213	85	174	9	119	6	107
15	183	171	250	85	212	7	122	6	82
20	232	186	277	85	259	6	124	6	65
25	232	198	297	85	280	5	125	6	51
30	232	207	314	85	302	5	125	6	40
35	233	215	327	85	325	4	126	6	30
Chi-Square value		0.74	15.28	89.02	4.93	2265.45	22.39	3169.18	52.55
Null Hypothesis Testing		Accept	Accept	Reject	Accept	Reject	Reject	Reject	Reject
NSE		0.8740	0.8436	0.5647	0.9176	-0.1718	0.7769	-0.1867	0.2591
Rating		Excellent	Excellent	Excellent	Excellent	Poor	Excellent	Poor	Fair
Rank		2	3	5	1	7	4	8	6
Parameters		$\mu=61.23, \sigma=58.43, A=-60.96,$ $Y_n=0.5424,$ $S_n=1.1363$	$B=-204.41$	$Z^- = 1.62, \sigma_z = 0.48,$ $C_s = -3.94$	$Z^- = 1.62, \sigma_z = 0.48,$ $C_s = 0$	$\alpha = 58.43,$ $\xi = 2.80$	$\alpha = 61.52,$ $\xi = 57.38,$ $k = 0.85$	$\alpha = -492.93,$ $\xi = 201.40,$ $k = 2.52$	$\alpha = 141.13,$ $\epsilon = 248.37$

Estimation of two days monthly (September) maximum rainfall and goodness of fit for theoretical probability distributions

Return Period (T), Years	Observed Rainfall, mm	Gumbel	V T Chow	LP-III	LN	EXP	GEV	GP	Gamma
3	95	110	127	94	76	33	134	21	251
4	130	136	168	101	105	23	145	14	226
5	136	155	199	109	145	17	150	12	207
10	176	211	289	111	241	7	161	9	147
15	260	243	340	111	297	4	165	9	112
20	345	265	376	111	366	2	167	9	87
25	348	282	403	111	396	2	168	9	68
30	351	296	426	111	428	1	168	9	53
35	354	308	445	111	463	0	169	9	39
Chi-Square value		1.97	13.32	172.46	4.59	5582.36	46.03	4819.73	94.11
Null Hypothesis Testing		Accept	Accept	Reject	Accept	Reject	Reject	Reject	Reject
NSE		0.9126	0.9195	0.5229	0.9492	-0.1339	0.7330	-0.0974	0.2663
Rating		Excellent	Excellent	Excellent	Excellent	Poor	Excellent	Poor	Fair
Rank		3	2	5	1	8	4	7	6
Parameters		$\mu=83.18, \sigma=85.15, A=-82.69, Y_n=0.5424, S_n=1.1363$	$B=-277.47$	$Z^- = 1.74, \sigma_z = 0.50, C_s = -4.25$	$Z^- = 1.74, \sigma_z = 0.50, C_s = 0$	$\alpha=85.15, \xi=-1.97$	$\alpha=86.48, \xi=81.77, k=0.96$	$\alpha=-826.14, \xi=296.54, k=2.87$	$\alpha=-198.24, \epsilon=345.39$

Estimation of three days monthly (September) maximum rainfall and goodness of fit for theoretical probability distributions

Return Period (T), Years	Observed Rainfall, mm	Gumbel	V T Chow	LP-III	LN	EXP	GEV	GP	Gamma
3	106	128	146	107	87	36	156	23	293
4	133	159	194	114	121	25	168	16	264
5	155	181	230	121	169	18	174	13	241
10	228	248	334	122	284	6	185	11	171
15	323	285	393	122	352	3	189	11	130
20	391	311	435	122	436	1	191	11	101
25	397	331	467	122	473	0	192	11	78
30	404	348	492	122	513	-1	192	11	60
35	411	362	514	122	556	-1	193	11	44
Chi-Square value		1.82	14.97	225.60	7.25	7834.38	56.11	5608.22	109.48
Null Hypothesis Testing		Accept	Accept	Reject	Accept	Reject	Reject	Reject	Reject
NSE		0.9284	0.9065	0.3478	0.9188	-0.4871	0.6448	-0.4197	0.0373
Rating		Excellent	Excellent	Good	Excellent	Poor	Excellent	Poor	Poor
Rank		1	3	5	2	8	4	7	6
Parameters		$\mu=96.26, \sigma=100.62, A=-95.63, Y_n=0.5424, S_n=1.1363$	$B=-321.01$	$Z^- = 1.79, \sigma_z = 0.51, C_s = -4.31$	$Z^- = 1.79, \sigma_z = 0.51, C_s = 0$	$\alpha=100.62, \xi=-4.37$	$\alpha=100.03, \xi=96.84, k=1.01$	$\alpha=-1057.28, \xi=357.17, k=3.05$	$\alpha=-232.86, \epsilon=403.88$

Estimation of four days monthly (September) maximum rainfall and goodness of fit for theoretical probability distributions

Return Period (T), Years	Observed Rainfall, mm	Gumbel	V T Chow	LP-III	LN	EXP	GEV	GP	Gamma
3	119	138	158	116	93	40	168	25	318
4	148	171	210	123	131	28	181	16	286
5	175	195	249	131	184	21	188	13	262
10	252	265	362	132	313	8	201	11	186
15	346	305	426	132	391	4	205	10	142
20	394	333	470	132	487	2	207	10	110
25	406	355	505	132	529	1	208	10	86
30	417	373	533	132	574	0	209	10	66
35	429	387	556	132	624	0	210	10	49
Chi-Square value		1.19	19.59	221.62	13.63	7043.01	51.66	6347.37	103.50
Null Hypothesis Testing		Accept	Reject	Reject	Accept	Reject	Reject	Reject	Reject
NSE		0.9531	0.9201	0.5955	0.8998	0.0348	0.7936	0.0641	0.3928
Rating		Excellent	Excellent	Excellent	Excellent	Poor	Excellent	Poor	Good
Rank		1	2	5	3	8	4	7	6
Parameters		$\mu=104.15, \sigma=107.32, Y_n=0.5424, S_n=1.1363$	$A=-103.45, B=-347.29$	$Z^-=1.82, \sigma_z=0.53, C_S=-4.23$	$Z^-=1.82, \sigma_z=0.53, C_S=0$	$\alpha=107.32, \xi=-3.17$	$\alpha=107.97, \xi=103.49, k=0.99$	$\alpha=-1097.77, \xi=381.83, k=2.95$	$\alpha=-251.94, \epsilon=437.88$

Estimation of five days monthly (September) maximum rainfall and goodness of fit for theoretical probability distributions

Return Period (T), Years	Observed Rainfall, mm	Gumbel	V T Chow	LP-III	LN	EXP	GEV	GP	Gamma
3	120	148	171	124	100	45	180	27	344
4	170	183	227	131	141	31	195	17	310
5	214	208	268	138	200	24	203	13	284
10	270	283	390	139	345	11	218	9	202
15	358	326	459	139	432	7	223	9	154
20	419	355	508	139	541	5	226	9	120
25	424	378	545	139	589	3	227	9	94
30	430	397	575	139	641	3	228	9	72
35	435	412	601	139	698	2	229	9	54
Chi-Square value		0.92	24.24	235.52	21.61	6269.59	47.87	7638.26	99.18
Null Hypothesis Testing		Accept	Reject	Reject	Reject	Reject	Reject	Reject	Reject
NSE		0.9603	0.8977	0.6036	0.8351	0.0482	0.8186	0.0555	0.4190
Rating		Excellent	Excellent	Excellent	Excellent	Poor	Excellent	Poor	Very Good
Rank		1	2	5	3	8	4	7	6
Parameters		$\mu=112.43, \sigma=113.70, Y_n=0.5424, S_n=1.1363$	$A=-111.64, B=-374.85$	$Z^-=1.85, \sigma_z=0.54, C_S=-4.13$	$Z^-=1.85, \sigma_z=0.54, C_S=0$	$\alpha=113.70, \xi=-1.27$	$\alpha=116.35, \xi=109.56, k=0.94$	$\alpha=-1108.78, \xi=403.82, k=2.81$	$\alpha=-271.57, \epsilon=473.38$

