

Interaction among neighbouring rectangular finite strike slip faults in a linear viscoelastic half space representing lithosphere-asthenosphere system

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सार – भंश प्रणाली में भूकंपीय रूप से सक्रिय क्षेत्र होते हैं जिनके आस-पास अनेक भूकंप भंश होते हैं। इनमें से किसी एक में भी संचलन उत्पन्न होने से आस-पास के अन्य क्षेत्रों में प्रतिबल संचित होने की प्रकृति प्रभावित होती है। इन परस्पर क्रियाओं और दो लगातार भूकंपीय घटनाओं के बीच में गैर भूकंपीय अवधि के दौरान अंतर-भूकंपीय प्रतिबल के प्रतिरूप का अध्ययन करने के लिए गणितीय मॉडल विकसित किए जा सकते हैं। प्रस्तुत शोध पत्र में स्थल मंडल-स्थेनो स्फीयर प्रणाली को एक रेखीय श्यान प्रत्यास्थ के आधे स्थान (हाफ स्पेस) द्वारा दर्शाया गया है। आधे स्थान की सामग्री में मैक्सवेल और केल्विन प्रकृति दोनों की सामग्री के गुण होने संभावना है। अनुमान है कि यह प्रणाली कुछ विवर्तनी घटनाओं द्वारा उत्पन्न एक स्थिर अपरूपण प्रतिबल के अधीन होती है। परिणामी सीमा मूल्य की समस्या से उत्पन्न विस्थापन झुकाव और प्रतिबल का समाधान प्राप्त करने के लिए इंटीग्रल ट्रांसफॉर्म, ग्रीन फंक्शन तकनीक और अनुकूलता सिद्धांत का उपयोग किया गया है। भंशों में परस्पर क्रियाओं की प्रकृति का पता लगाने के लिए अंकीय गणनाओं हेतु मॉडल प्राचलों के उपयुक्त आकलनों का उपयोग किया गया।

ABSTRACT. There are seismically active regions consisting of fault system with a number of neighbouring earthquake faults. A movement across any one of them may affect the nature of stress accumulation near the others. Mathematical models may be developed to study these interactions and the pattern of interseismic stresses during the aseismic period in between two consecutive seismic events. In this paper, the lithosphere-asthenosphere system is being represented by a linear viscoelastic half space. The material of the half space is expected to possess the properties of both Maxwell and Kelvin type materials. It is assumed that the system is under a steady shear stress generated by some tectonic phenomena. For obtaining the solution for displacement, strain and stresses from the resulting boundary value problem, Integral transform, Green's function technique and correspondence principle have been used. Appropriate estimates of the model parameters were used in carrying out the numerical computations for investigating the nature of interactions among the faults.

Key words – Linear viscoelastic lithosphere-asthenosphere system, Strike slip fault, Aseismic period, Stress accumulation, Interseismic stress, Earthquake prediction.

1. Introduction

Occurrence of an earthquake is a cyclic phenomena, two major seismic events are usually separated by a comparatively long aseismic periods of the order of a few decades or so. To understand the mechanism of earthquake process it is necessary to develop models of both the earthquake phenomena producing seismic disturbance lasting for a short period of time, as well as the slow ground deformation observed during the aseismic period, which may exist over several tens of years.

The small ground deformation during the aseismic period attracts the attention of the seismologists because of the fact that this aseismic period may be looked upon as the preparatory period for the next major seismic event. Some theoretical models have been developed in the lithosphere-asthenosphere system during aseismic period in a seismically active regions by Savage and Prescott (1978); Mukhopadhyay and Mukherjee (1979, 1984, 1986); Cohen (1980a,b); Cohen (1984) and others. In the present paper, we are developing a mathematical model highlighting the essential features of such ground

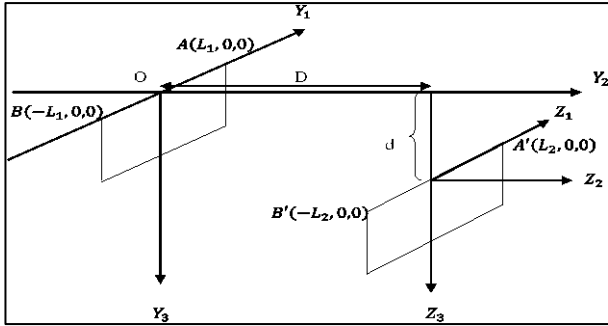


Fig. 1. The faults and the co-ordinate axes

deformation and interseismic and post seismic stress during the aseismic period which typically precedes and follows two major seismic events in a seismically active region in the presence of earthquake faults.

Most of the seismically active regions usually consist of faults systems comprising of a number of faults which are closely located within a specific region. For example, in North America, San Andreas fault system, the Hayward and Calaveras faults are roughly parallel and are closed to the main San Andreas fault. A movement across any one of them is likely to affect the pattern of stress accumulation near the others. Some theoretical models with interacting fault system during aseismic period have been developed by Mukhopadhyay *et al.* (1979c, 1988); Mukhopadhyay and Mukherjee (1984, 1986); Ghosh *et al.* (1992); Debnath and Sen (2014a, 2014b, 2015); Manna and Sen (2017). Most of the earlier work dealt with elastic/viscoelastic layer in welded contact with a viscoelastic half space or in a viscoelastic half space of Maxwell type following Mukhopadhyay *et al.* (1978, 1978a, 1978b, 1979a, 1979b, 1988); Fred (1992). Post seismic relaxation effects in a linear viscoelastic Maxwell type material was discussed by Pollitz. However, the properties of the material in the lithosphere-asthenosphere system indicates that different other types of viscoelastic material may also be relevant. We therefore introduce linear viscoelastic half space in order to represent the lithosphere-asthenosphere system featuring the properties of both Maxwell and Kelvin-Voigt type.

Stresses are accumulated in the model under the action of tectonic phenomena including mantle convection. Now by the effects of interseismic stress it accumulated exceeds the local cohesive and frictional forces which keep the fault locked, the fault starts moving. Depending upon the local rheological nature of the region, the movement across the fault may be sudden in nature leading to an earthquake or alternatively, a creeping movement across the fault which releases the accumulated stress near it.

In most of the earlier studies the faults were assumed to be quite long compared to its depth, so that the problem reduced to a 2D model. In view of the fact that there are numerous other faults which are not so long compared to their depth, a 3D model is imminent.

In view of this we consider two neighbouring strike slip faults of finite length situated in a linear viscoelastic solid, one of which is taken to be surface breaking while the other is buried.

2. Formulation

We are considering two rectangular vertical strike-slip faults F_1 and F_2 of length $2L_1$ and $2L_2$ (L_1, L_2 finite) supposed to be situated in a linearly viscoelastic half space. Let D_1 and D_2 be their respective widths.

We are introducing a rectangular Cartesian coordinate system (Y_1, Y_2, Y_3) is used by the midpoint O of the upper edge of the fault F_1 , which is taken to be surface-breaking, as the origin, the strike of the fault as the Y_1 axis, Y_2 axis perpendicular to the fault and Y_3 axis pointing towards the downwards direction so that the fault F_1 is given by $F_1 : (-L_1 \leq y_1 \leq L_1, y_2 = 0, 0 \leq y_3 \leq D_1)$. For convenience, we introduce another rectangular system (Z_1, Z_2, Z_3) for the second fault F_2 which is taken to be buried as shown in [Fig. 1]. Fault F_2 is given by $F_2 : (-L_2 \leq z_1 \leq L_2, z_2 = 0, d \leq z_3 \leq D_2)$, d being the depth of the upper edge of F_2 below the free surface. The relationships between (Y_1, Y_2, Y_3) & (Z_1, Z_2, Z_3) are given by:

$$z_1 = y_1, \quad z_2 = y_2 - D, \quad z_3 = y_3 - d$$

2.1. Constitutive equations (stress-strain relations)

For the linear viscoelastic type materials the constitutive equations may be taken as:

$$\tau_{11} + \frac{\eta}{\mu} \frac{\partial}{\partial t} (\tau_{11}) = \mu \frac{\partial u}{\partial y_1} + 2\eta \frac{\partial}{\partial t} \left(\frac{\partial u}{\partial y_1} \right) \quad (1)$$

$$\begin{aligned} \tau_{12} + \frac{\eta}{\mu} \frac{\partial}{\partial t} (\tau_{12}) = & \mu \frac{1}{2} \left(\frac{\partial u}{\partial y_2} + \frac{\partial v}{\partial y_1} \right) \\ & + 2\eta \frac{\partial}{\partial t} \frac{1}{2} \left(\frac{\partial u}{\partial y_2} + \frac{\partial v}{\partial y_1} \right) \end{aligned} \quad (2)$$

$$\begin{aligned} \tau_{13} + \frac{\eta}{\mu} \frac{\partial}{\partial t} (\tau_{13}) = & \mu \frac{1}{2} \left(\frac{\partial u}{\partial y_3} + \frac{\partial w}{\partial y_1} \right) \\ & + 2\eta \frac{\partial}{\partial t} \frac{1}{2} \left(\frac{\partial u}{\partial y_3} + \frac{\partial w}{\partial y_1} \right) \end{aligned} \quad (3)$$

$$\tau_{22} + \frac{\eta}{\mu} \frac{\partial}{\partial t} (\tau_{22}) = \mu \frac{\partial v}{\partial y_2} + 2\eta \frac{\partial}{\partial t} \left(\frac{\partial v}{\partial y_2} \right) \quad (4)$$

$$\begin{aligned} \tau_{23} + \frac{\eta}{\mu} \frac{\partial}{\partial t} (\tau_{23}) &= \mu \frac{1}{2} \left(\frac{\partial v}{\partial y_3} + \frac{\partial w}{\partial y_2} \right) \\ &+ 2\eta \frac{\partial}{\partial t} \frac{1}{2} \left(\frac{\partial v}{\partial y_3} + \frac{\partial w}{\partial y_2} \right) \end{aligned} \quad (5)$$

$$\tau_{33} + \frac{\eta}{\mu} \frac{\partial}{\partial t} (\tau_{33}) = \mu \frac{\partial w}{\partial y_3} + 2\eta \frac{\partial}{\partial t} \left(\frac{\partial w}{\partial y_3} \right) \quad (6)$$

where, η is the effective viscosity and μ is the effective rigidity of the material.

2.2. Stress equation of motion

The equation of motion (explained underneath) for quasistatic deformation is satisfied by these stresses; but, in that case, the inertia terms are neglected.

$$\frac{\partial}{\partial y_1} (\tau_{11}) + \frac{\partial}{\partial y_2} (\tau_{12}) + \frac{\partial}{\partial y_3} (\tau_{13}) = 0 \quad (7)$$

$$\frac{\partial}{\partial y_1} (\tau_{21}) + \frac{\partial}{\partial y_2} (\tau_{22}) + \frac{\partial}{\partial y_3} (\tau_{23}) = 0 \quad (8)$$

$$\frac{\partial}{\partial y_1} (\tau_{31}) + \frac{\partial}{\partial y_2} (\tau_{32}) + \frac{\partial}{\partial y_3} (\tau_{33}) = 0 \quad (9)$$

where, $(-\infty < y_1 < \infty, -\infty < y_2 < \infty, y_3 \geq 0, t \geq 0)$

2.3. Boundary conditions

The boundary conditions are assumed as, with $t = 0$ representing a suitable instant when the medium remains in a seismic state:

$$\begin{aligned} \lim_{y_1 \rightarrow L_1^-} \tau_{11}(y_1, y_2, y_3, t) &= \lim_{y_1 \rightarrow L_1^+} \tau_{11}(y_1, y_2, y_3, t) \\ &= \tau_{L_1} \\ y_2 = 0, 0 \leq y_3 \leq D_1, t \geq 0 \end{aligned} \quad (10)$$

$$\begin{aligned} \lim_{y_1 \rightarrow -L_1^-} \tau_{11}(y_1, y_2, y_3, t) &= \lim_{y_1 \rightarrow -L_1^+} \tau_{11}(y_1, y_2, y_3, t) \\ &= \tau_{L_1} \\ y_2 = 0, 0 \leq y_3 \leq D_1, t \geq 0 \end{aligned} \quad (11)$$

where, τ_{L_1} is the constant stress maintained at the tips of the fault along the Y_1 axis. It is likely that its magnitude will be small enough to exclude the possibility of any further extension along the Y_1 direction.

Similarly for the fault F_2 :

$$\begin{aligned} \lim_{y_1 \rightarrow L_2^-} \tau_{11}(y_1, y_2, y_3, t) &= \lim_{y_1 \rightarrow L_2^+} \tau_{11}(y_1, y_2, y_3, t) \\ &= \tau_{L_2} \end{aligned}$$

$$y_2 = 0, d \leq y_3 \leq D_2, t \geq 0$$

$$\begin{aligned} \lim_{y_1 \rightarrow -L_2^-} \tau_{11}(y_1, y_2, y_3, t) &= \lim_{y_1 \rightarrow -L_2^+} \tau_{11}(y_1, y_2, y_3, t) \\ &= \tau_{L_2} \end{aligned}$$

$$y_2 = 0, d \leq y_3 \leq D_2, t \geq 0$$

For the stresses we assume:

$$\tau_{12}(y_1, y_2, y_3, t) \rightarrow \tau_{\infty} \text{ as } |y_2| \rightarrow \infty, -\infty \leq y_1 \leq \infty, y_3 \geq 0, t \geq 0 \quad (12)$$

On the free surface $y_3 = 0, (-\infty \leq y_1, y_2 \leq \infty, t \geq 0)$

$$\tau_{13}(y_1, y_2, y_3, t) = 0 \quad (13)$$

$$\tau_{23}(y_1, y_2, y_3, t) = 0 \quad (14)$$

$$\tau_{33}(y_1, y_2, y_3, t) = 0 \quad (15)$$

Also, as $y_3 \rightarrow \infty, (-\infty \leq y_1, y_2 \leq \infty, t \geq 0)$

$$\tau_{13}(y_1, y_2, y_3, t) = 0 \quad (16)$$

$$\tau_{23}(y_1, y_2, y_3, t) = 0 \quad (17)$$

$$\tau_{33}(y_1, y_2, y_3, t) = 0 \quad (18)$$

$$\tau_{22}(y_1, y_2, y_3, t) = 0, \text{ as } |y_2| \rightarrow \infty, -\infty \leq y_1 \leq \infty, t \geq 0 \quad (19)$$

[where, $\tau_{\infty}(t)$ is the shear stress maintained by mantle convection and other tectonic phenomena far away from the fault].

2.4. Initial conditions

Let $(u)_0, (v)_0, (w)_0, (\tau_{ij})_0, (e_{ij})_0$ $i, j = 1, 2, 3$ are the values of $u, v, w, \tau_{ij}, e_{ij}$ respectively at time $t = 0$. They are functions of y_1, y_2, y_3 and satisfy the relations (1) to (19).

3. Displacements, stresses and strains in the absence of any fault movement

When there is no fault movement, the displacements, stresses and strains are assumed to be continuous throughout the system. For obtaining displacement, strain and stresses, we are introducing Integral transform like Laplace transform of (1) to (19) are taken with respect to t . On taking inverse Laplace transform we get the solutions.

$$(u)_{y_1, y_2, y_3, t} = (u)_0 e^{-\frac{\mu t}{2\eta}} + y_1 \left\{ \tau_{L_1} \left(\frac{1}{\mu} - \frac{1}{2\mu} e^{-\frac{\mu t}{2\eta}} \right) - \frac{(\tau_{11})_0}{2\mu} e^{-\frac{\mu t}{2\eta}} + \frac{y_2 \tau_\infty}{\mu} \left(1 - e^{-\frac{\mu t}{2\eta}} \right) \right\}$$

$$v(y_1, y_2, y_3, t) = (v)_0 e^{-\frac{\mu t}{2\eta}} + \frac{y_1 \tau_\infty}{\mu} \left(1 - e^{-\frac{\mu t}{2\eta}} \right)$$

$$w(y_1, y_2, y_3, t) = (w)_0 e^{-\frac{\mu t}{2\eta}}$$

$$\tau_{11}(y_1, y_2, y_3, t) = (\tau_{11})_0 e^{-\frac{\mu t}{\eta}} + \tau_{L_1} \left(1 - e^{-\frac{\mu t}{\eta}} \right)$$

$$\tau_{12}(y_1, y_2, y_3, t) = (\tau_{12})_0 e^{-\frac{\mu t}{\eta}} + \tau_\infty \left(1 - e^{-\frac{\mu t}{\eta}} \right)$$

$$\tau_{13}(y_1, y_2, y_3, t) = (\tau_{13})_0 e^{-\frac{\mu t}{\eta}}$$

$$\tau_{22}(y_1, y_2, y_3, t) = (\tau_{22})_0 e^{-\frac{\mu t}{\eta}}$$

$$\tau_{23}(y_1, y_2, y_3, t) = (\tau_{23})_0 e^{-\frac{\mu t}{\eta}}$$

$$\tau_{33}(y_1, y_2, y_3, t) = (\tau_{33})_0 e^{-\frac{\mu t}{\eta}}$$

From the above expressions we find that as $t \rightarrow \infty, \tau_{11} \rightarrow \tau_{L_1}, \tau_{12} \rightarrow \tau_\infty$ and all the others stress components $\tau_{13}, \tau_{22}, \tau_{23}, \tau_{33} \rightarrow 0$. However the rheological behaviour of the material near the fault F_1 is assumed to be capable of withstanding stress of magnitude $(\tau_c)_1$ called critical value of the stress where $(\tau_c)_1$ is less than τ_∞ . When the accumulated stress τ_{12} becomes large enough near the fault and exceeds this threshold value $(\tau_c)_1$, a sudden slip across F_1 occurs after a time $t = T_1$ and thereby releasing the accumulated stress to a lower value. Following Debnath and Sen (2013) the magnitude of slip is likely to satisfy the following conditions:

(C1) Its magnitude will be maximum near the middle of the fault on the free surface.

(C2) Its magnitude will gradually decrease to zero at the tips of the fault ($y_1 = \pm L_1, y_2 = 0, 0 \leq y_3 \leq D_1$) along its length.

(C3) The magnitude of the slip will decrease downwards with y_3 and ultimately tends to zero near the lower edge of the fault ($y_1 = \pm L_1, y_2 = 0, y_3 = D_1$).

If $f(y_1, y_3)$ be the slip function, it should satisfy the above conditions.

It is assumed that the critical value of the stress, say $(\tau_c)_2$, near F_2 is greater than $(\tau_c)_1$, so that when F_1 slips, F_2 remains locked.

4. Solutions after the commencement of the fault movement across F_1 and after the fault slip across F_2

We consider first the movement across F_1 , while F_2 remains locked.

Let us suppose that after a time T_1 , the stress component τ_{12} , the main driving force behind the strike-slip motion of the fault F_1 , exceeds the critical value $(\tau_c)_1$, F_1 slips, which is characterized by the following dislocation condition:

$$(u)_{F_1} = U_1 f_1(y_1, y_3) H(t - T_1)$$

where,

$(u)_{F_1}$ = The discontinuity in u across F_1 and $H(t - T_1)$ is Heaviside unit step function.

Similarly, we are considering the fault slips commence across F_2 after time T_2 and calculating in a similar way.

The resulting boundary value problem can be solved by using a modified Green's function technique and correspondence principle, following Maruyama (1964, 1966); Rybicki (1971) and this technique has been explained by Mukhopadhyay *et al.* (1980a) and we get the final solution for displacements, strains and stresses as:

$$\begin{aligned}
 u(y_1, y_2, y_3, t) &= (u)_p e^{\frac{\mu t}{2\eta}} \\
 &+ y_1 \left[\tau_{L_1} \left(\frac{1}{\mu} - \frac{1}{2\mu} e^{-\frac{\mu t}{2\eta}} \right) - \frac{(\tau_{11})_0}{2\mu} e^{-\frac{\mu t}{2\eta}} \right. \\
 &+ \left. \frac{y_2 \tau_\infty}{\mu} \left(1 - e^{-\frac{\mu t}{2\eta}} \right) \right] \\
 &+ \frac{U_1}{2\pi} H(t - T_1) \psi_1(y_1, y_2, y_3) \\
 &+ \frac{U_2}{2\pi} H(t - T_2) \varphi_1(y_1, y_2, y_3)
 \end{aligned}$$

$$v(y_1, y_2, y_3, t) = (v)_p e^{-\frac{\mu t}{2\eta}} + \frac{y_1 \tau_\infty}{\mu} \left(1 - e^{-\frac{\mu t}{2\eta}} \right)$$

$$w(y_1, y_2, y_3, t) = (w)_p e^{-\frac{\mu t}{2\eta}}$$

$$\begin{aligned}
 \tau_{11}(y_1, y_2, y_3, t) &= (\tau_{11})_p e^{-\frac{\mu t}{\eta}} + \tau_{L_1} \left(1 - e^{-\frac{\mu t}{\eta}} \right) \\
 &+ \frac{\mu U_1}{2\pi} H(t - T_1) \left(1 + e^{-\frac{\mu t}{\eta}} \right) \psi_2 \\
 &(y_1, y_2, y_3) + \frac{\mu U_2}{2\pi} H(t - T_2) \\
 &\left(1 + e^{-\frac{\mu t}{\eta}} \right) \varphi_1(y_1, y_2, y_3)
 \end{aligned}$$

$$\tau_{12}(y_1, y_2, y_3, t) = (\tau_{12})_p e^{-\frac{\mu t}{\eta}} + \tau_\infty \left(1 - e^{-\frac{\mu t}{\eta}} \right)$$

$$\begin{aligned}
 &+ \frac{\mu U_1}{2\pi} H(t - T_1) \left(1 + e^{-\frac{\mu t}{\eta}} \right) \psi_3 \\
 &(y_1, y_2, y_3) + \frac{\mu U_2}{2\pi} H(t - T_2) \\
 &\left(1 + e^{-\frac{\mu t}{\eta}} \right) \varphi_3(y_1, y_2, y_3)
 \end{aligned}$$

$$\begin{aligned}
 \tau_{13}(y_1, y_2, y_3, t) &= (\tau_{13})_p e^{-\frac{\mu t}{\eta}} \\
 &+ \frac{\mu U_1}{2\pi} H(t - T_1) \left(1 + e^{-\frac{\mu t}{\eta}} \right) \psi_4 \\
 &(y_1, y_2, y_3) + \frac{\mu U_2}{2\pi} H(t - T_2) \\
 &\left(1 + e^{-\frac{\mu t}{\eta}} \right) \varphi_4(y_1, y_2, y_3)
 \end{aligned}$$

$$\tau_{22}(y_1, y_2, y_3, t) = (\tau_{22})_p e^{-\frac{\mu t}{\eta}}$$

$$\tau_{23}(y_1, y_2, y_3, t) = (\tau_{23})_p e^{-\frac{\mu t}{\eta}}$$

$$\tau_{33}(y_1, y_2, y_3, t) = (\tau_{33})_p e^{-\frac{\mu t}{\eta}}$$

$$\begin{aligned}
 e_{11}(y_1, y_2, y_3, t) &= (e_{11})_p e^{-\frac{\mu t}{2\eta}} \\
 &+ \left[\tau_{L_1} \left(\frac{1}{\mu} - \frac{1}{2\mu} e^{-\frac{\mu t}{2\eta}} \right) - \frac{(\tau_{11})_p}{2\mu} e^{-\frac{\mu t}{2\eta}} \right] \\
 &+ \frac{U_1}{2\pi} H(t - T_1) \psi_2(y_1, y_2, y_3) \\
 &+ \frac{U_2}{2\pi} H(t - T_2) \varphi_2(y_1, y_2, y_3)
 \end{aligned}$$

$$\begin{aligned}
 e_{12}(y_1, y_2, y_3, t) &= (e_{12})_p e^{-\frac{\mu t}{2\eta}} + \frac{\tau_\infty}{\mu} \left(1 - e^{-\frac{\mu t}{2\eta}} \right) \\
 &+ \frac{U_1}{4\pi} H(t - T_1) \psi_3(y_1, y_2, y_3) \\
 &+ \frac{U_2}{4\pi} H(t - T_2) \varphi_3(y_1, y_2, y_3)
 \end{aligned}$$

where,

$$\begin{aligned} \psi_1(y_1, y_2, y_3) &= \int_{-L_1}^{L_1} \int_0^{D_1} f_1(x_1, x_3) \left[\frac{y_2}{[(y_1 - x_1)^2 + y_2^2 + (y_3 - x_3)^2]^{3/2}} \right. \\ &\quad \left. - \frac{y_2}{[(y_1 + x_1)^2 + y_2^2 + (y_3 - x_3)^2]^{3/2}} \right] dx_3 dx_1 \end{aligned}$$

Similarly ψ_2, ψ_3, ψ_4 and $\phi_1, \phi_2, \phi_3, \phi_4$ are obtained from $\psi_1, \psi_2, \psi_3, \psi_4$ by substituting $y_1 = z_1, y_2 = z_2 + D, y_3 = z_3 + d$ and D_1 by D_2 .

5. Results and discussion

Numerical computations have been carried out to compute the following quantities:

$$(i) \quad E_{12}(y_1, y_2, y_3, t) = e_{12}(y_1, y_2, y_3, t) - (e_{12})_p e^{-\frac{\mu t}{2\eta}}$$

$$= \frac{\tau_\infty}{\mu} \left(1 - e^{-\frac{\mu t}{2\eta}} \right)$$

= The strain before the fault movement due to the tectonic forces.

$$(ii) \quad (T_{12})_1(y_1, y_2, y_3, t) = \tau_{12}(y_1, y_2, y_3, t) - (\tau_{12})_p e^{-\frac{\mu t}{\eta}}$$

$$- \tau_\infty \left(1 - e^{-\frac{\mu t}{\eta}} \right)$$

$$= \frac{\mu U_1}{2\pi} H(t - T_1) \left(1 + e^{-\frac{\mu t}{\eta}} \right) + \psi_3(y_1, y_2, y_3)$$

$$+ \frac{\mu U_2}{2\pi} H(t - T_2) \left(1 + e^{-\frac{\mu t}{\eta}} \right) \phi_3(y_1, y_2, y_3)$$

= The stress due to the fault movement across F_1 and F_2 at some particular point.

$$(iii) \quad (T_{12})_2(y_1, y_2, y_3, t) = \frac{\mu U_1}{2\pi} H(t - T_1) \left(1 + e^{-\frac{\mu t}{\eta}} \right)$$

$$+ \psi_3(y_1, y_2, y_3), \text{ at } y_2 = 10 \text{ km and}$$

$$- 25 \text{ km} \leq y_1 \leq 25 \text{ km}, 0 \leq y_3 \leq 25 \text{ km}$$

= Stress contour showing the effect of fault movement across F_1 at different points near F_2 .

$$(iv) \quad (T_{12})_3(y_1, y_2, y_3, t) = \frac{\mu U_2}{2\pi} H(t - T_2) \left(1 + e^{-\frac{\mu t}{\eta}} \right)$$

$$\phi_3(y_1, y_2, y_3)$$

at $y_2 = 0$ km and

$$- 20 \text{ km} \leq y_1 \leq 20 \text{ km}, 0 \leq y_3 \leq 20 \text{ km}$$

= Stress contour showing the effect of fault movement across F_2 at different points near F_1 .

(v) Stress contour map after the sudden movement across F_1 (F_2 remains locked) on different planes given $y_2 = -10$ km, 5 km, 15 km.

(vi) Stress contour map after the sudden movement across both F_1 and F_2 on different planes given by $y_2 = -10$ km, 5 km, 15 km.

Following Cathles (1975); Aki and Richards (1980) and the contemporary studies on rheological behaviour of crust and upper mantle by Chift *et al.* (2002), Karato (2010) the values for the model parameters have been chosen for numerical computation.

We consider $f(x_1, x_3)$ to be

$$f(x_1, x_3) = \left(1 - \frac{x_1^2}{L^2} \right) \left(1 - \frac{3x_3^2}{D_1^2} + \frac{3x_3^3}{D_1^3} \right) \left(\frac{D_1 - x_3}{D_1} \right)$$

which satisfies all conditions (C1) to (C3) stated above.

$$\mu = 3.5 \times 10^{11} \text{ dyne/sq.cm } \eta = 5 \times 10^{20} \text{ poise}$$

For fault F_1 (Surface breaking)

Length $2L_1 = 40$ km, Width $D_1 = 10$ km

For fault F_2 (Buried), length $2L_2 = 50$ km,

Width $D_2 = 15$ km

D = Horizontal distance between F_1 and $F_2 = 10$ km

$$(\tau_{12})_0 = 20 \text{ bar (assumed)}$$

$$(\tau_{12})_p = 40 \text{ bar (assuming 80% stress released}$$

$$[= 80\% \text{ of } (\tau_c)_1])$$

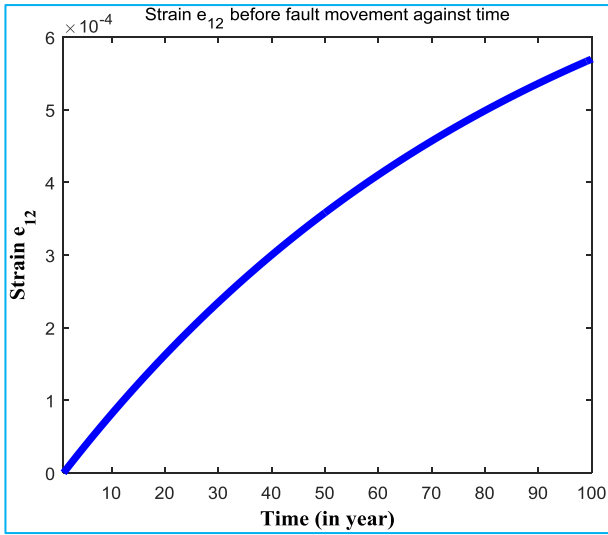


Fig. 2. Surface shear strains under the action of tectonic forces in the absence of fault movement

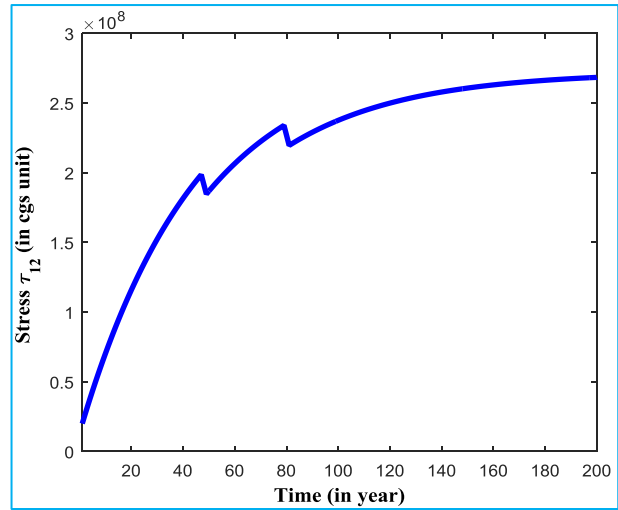


Fig. 3. Stress τ_{12} in the medium against time

$$\tau_{\infty} = 300 \text{ bar}, (\tau_c)_1 = 200 \text{ bar}, (\tau_c)_2 = 250 \text{ bar}$$

T_1 = The time for fault movement across F_1 is found to be 47 years.

T_2 = The time for the second fault movement across F_2 is found to be 79 years.

6. Observations

6.1. Strain at the free surface due to the tectonic forces τ_{∞}

It is observed from [Fig. 2] that the rate of strain accumulation is $\sim 8 \times 10^{-6}$ /year initially with a decreasing rate and reduces approximates to a value 6×10^{-6} /year which is in conformity with the observations in seismically active regions.

6.2. Stress τ_{12} in the medium against time

In [Fig. 3], $(T_{12})_1$ have been shown at a point given by $y_1 = 5 \text{ km}$, $y_2 = 5 \text{ km}$ and $y_3 = 5 \text{ km}$. We observed that for $0 \leq t \leq T_1$, τ_{12} increases with a decreasing rate and attain the value near 200 bar at $t = 47 \text{ years}$ ($= T_1$). Due to the movement across F_1 there is a co-seismic stress drop as shown by [Fig. 3]. The stress again increases, but at a lower rate until it reaches the value near $(\tau_c)_2$ after at a time $t = T_2$ when the second fault F_2 slips with another co-seismic stress drop. The stress at the point is found to increase further, but with a slow increasing rate and reaches the value 265 bar at $t = 200 \text{ years}$.

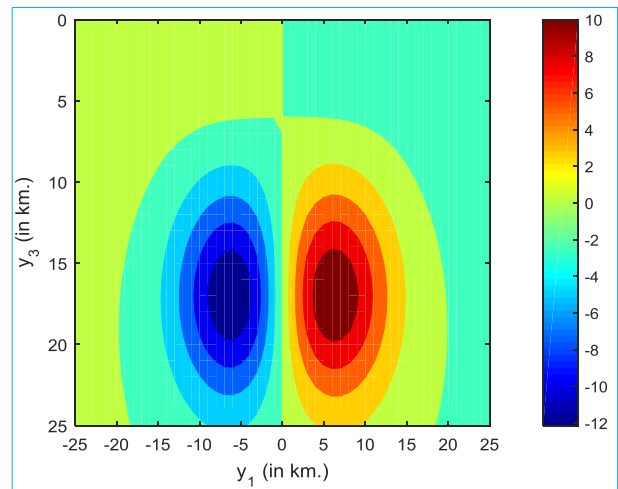


Fig. 4. Stress τ_{12} : Effect of fault movement across F_1 near F_2

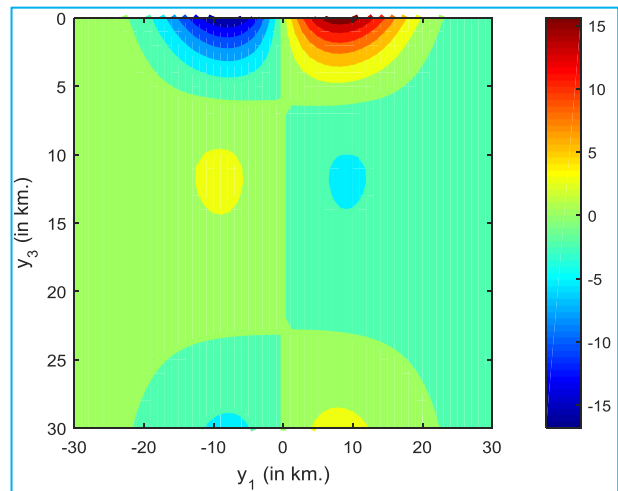
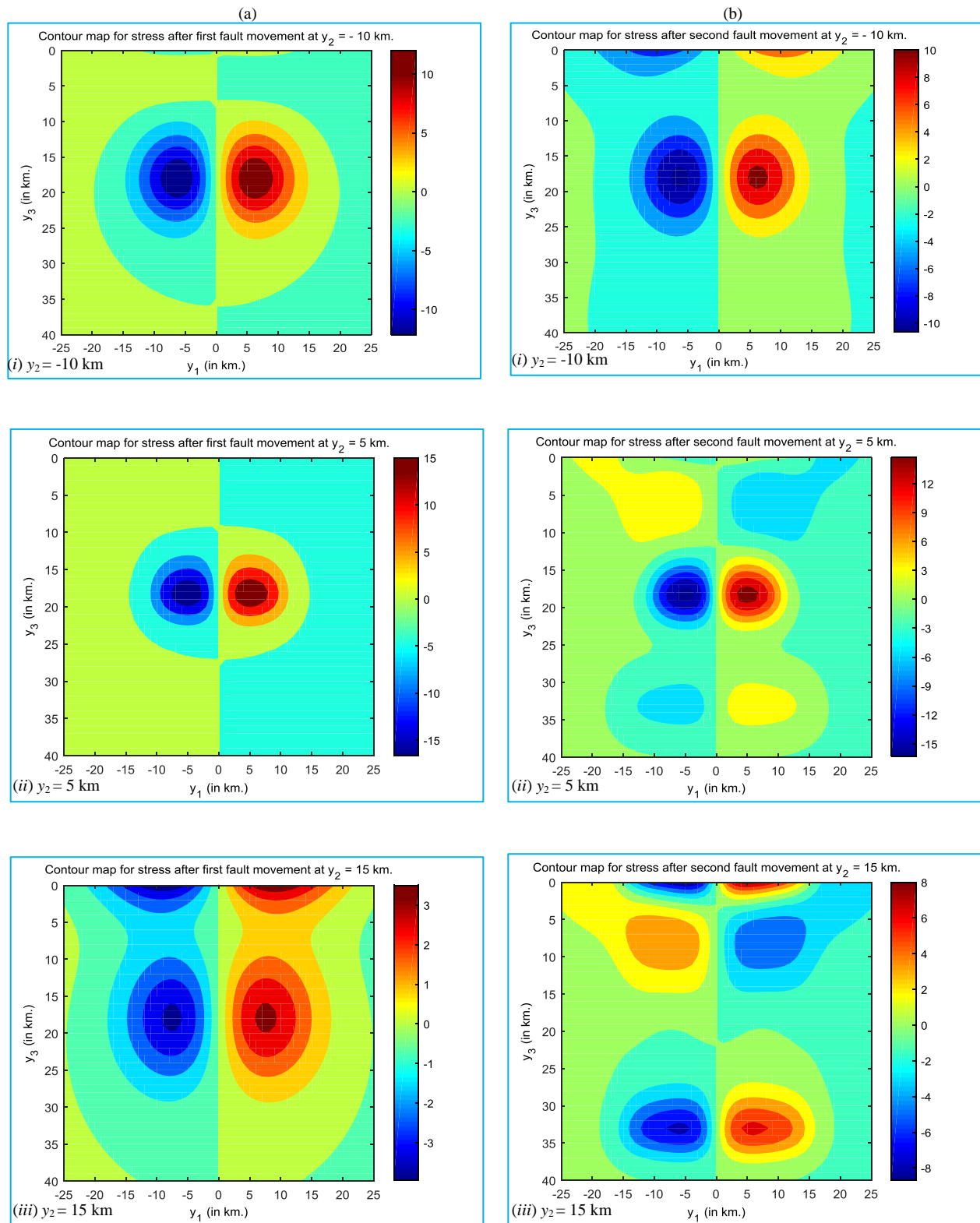


Fig. 5. Stress τ_{12} : Effect of fault movement across F_2 near F_1



Figs. 6(a&b). (a) Stress τ_{12} : Effect of fault movement across F_1 on three different vertical planes given by $y_2 =$ (i) -10 km, (ii) 5 km and (iii) 15 km and (b) Stress τ_{11} : Effect of fault movement across F_2 on three different vertical planes given by $y_2 =$ (i) -10 km, (ii) 5 km and (iii) 15 km

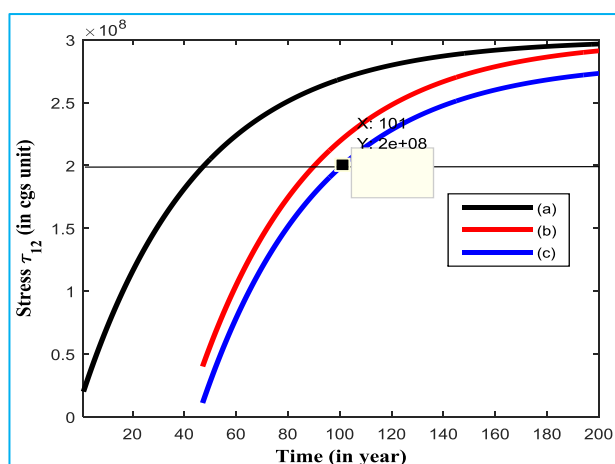


Fig. 7. Estimation of time gap to the next seismic event near F_1 (at $y_1 = 1$ km, $y_2 = 0.5$ km, $y_3 = 5$ km)

6.3. Stress τ_{12} : effect of fault slip across F_1

The stress contour has been shown in [Fig. 4] with $y_2 = 10$ km. The magnitude of the stress varies from -10 bar to 10 bar concentration of the magnitude of the stress have been found to lie on both sides of the middle line $y_1 = 0$ but with opposite direction as is evident from [Fig. 4].

6.4. Stress τ_{12} : effect of fault movement across F_2 near F_1

It is observed from [Fig. 5] that the effect of fault movement across the fault F_2 near the fault F_1 . It is also observed that accumulation pattern of stress is much distinct in this case and the magnitude of the stress are nearly alike to the previous one.

6.5. Stress τ_{12} : effect of fault movement across F_1 [Fig. 6(a)] and F_2 [Fig. 6(b)] respectively on three different vertical planes given by $y_2 = -10$ km, 5 km, 15 km

In the [Figs. 6(a&b)] we find that there are certain areas of stress accumulation and certain other areas of stress release. The magnitude of the stress depends upon the value of y_2 . For small y_2 the magnitude of the stresses is found to be large while for higher values of the y_2 the magnitudes are found to be comparatively smaller.

7. A prediction of the next slipping movement across F_1

We can use the above result suitably for earthquake prediction purposes. In the present work F_1 slips after a

lapse of 48 years when it reaches a value τ_c . The movement across fault F_1 affects the nature of stress accumulation in the regions near it. It is assumed that the slipping movement causes 80% of the accumulated stress near F_1 to be released. This means the initial value $(\tau_{12})_p$ at the second phase of aseismic period becomes 20% of τ_c , that is 40 bars. With the slipping across F_1 , the stress starts accumulating in a different way. [Fig. 7] shows the accumulation of stress at a point near the fault F_1 given by ($y_1 = 1$ km, $y_2 = 0.5$ km, $y_3 = 5$ km) during the second phase of aseismic state. The pink curve shows accumulation of stress under the combined action of τ_∞ and the slip across F_1 , while the blue curve indicates the accumulation of stress under the action of τ_∞ only. The figure shows that after a lapse of another 62 years, the pink curve reaches the threshold level (200 bars). However, in the absence of any movement across F_1 the red curve would have reached the threshold level after 42 years. These observations allow us to conclude that the next possible seismic event will likely be delayed by about 20 years due to the movement across F_1 . Such observations may be expected to prove fruitful in predicting the next seismic event.

- (i) Accumulation of τ_{12} during the first phase of the a seismic state due to the effect of τ_∞ with $(\tau_{12})_0 = 20$ bars.
- (ii) Accumulation of τ_{12} during the second phase of the a seismic state under the action of τ_∞ only with $(\tau_{12})_p = 40$ bars.
- (iii) Accumulation of τ_{12} under the action of τ_∞ along with the effect of the fault movement across F_1 during the second phase of the aseismic state.

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