Probability distributions for long term rainfall data in Ludhiana - A case study

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सार – जलवायु परिवर्तन और कृषि मौसम विज्ञान विद्यालय से 1981 से 2012 तक के 32 वर्षों की अवधि के लुधियाना के वर्षा के आँकड़ों का संग्रहण किया गया। इस अध्ययन का उद्देश्य 2012–1981 तक के विभिन्न संभाव्यता स्तरों पर वर्षा की परिवर्तिता और वर्षा की मात्रा का पता लगाना है। मार्कोव चेन मॉडल का उपयोग करके वर्षा के विभिन्न स्तरों अर्थात 5<मि.मी., 10<मि.मी., 20<मि.मी., 30<मि.मी., 40<मि.मी. और 50<मि.मी. पर प्रारंभिक और पारिस्थितिक संभाव्यता का पता लगाने के लिए साप्ताहिक आधार पर वर्षा के आँकड़ों का विश्लेषण किया गया। इसके अलावा, विभिन्न संभाव्यता का पता लगाने के लिए साप्ताहिक आधार पर वर्षा के आँकड़ों का विश्लेषण किया गया। इसके अलावा, विभिन्न संभाव्यता स्तरों, अर्थात् 20, 30, 40, 50, 65 और 75 प्रतिशत पर वर्षा की घटनाओं का पता लगाने के लिए अपूर्ण गामा वितरण का भी उपयोग किया गया। इस अध्ययन से प्राप्त परिणाम के अनुसार मानक मौसम विज्ञानिक सप्ताह के लिए अधिकतम और न्यूनतम प्रारंभिक संभाव्यता और पारिस्थितिक संभाव्यता (आर्द्र और शुष्क) का आकलन किया गया। यह परिणाम विभिन्न फसलों के लिए फसल उगाने के समय, सिंचाई/खाद डालने और फसल काटने के समय का निर्धारण करने के लिए उपयोगी रहेंगे। इसके अलावा यह अध्ययन अपवाह मात्रा, चरम अपवाह दर का निर्धारण करने में उपयोगी रहेगा और इस प्रकार वर्षा के पानी से फसल की संरचना का डिजाइन तैयार करने में उपयोगी रहेगा।

ABSTRACT. The rainfall data of Ludhiana for a period of 32 years covering 1981 to 2012 have been collected from School of Climate Change and Agricultural Meteorology. The study was planned to find the rainfall variability and amount of rainfall at different probability levels for the year 1981-2012. The rainfall data was analyzed on weekly basis to work out the initial & conditional probability for rainfall at different levels, *i.e.*, > 5 mm, > 10 mm, > 20 mm, > 30 mm, > 40 mm and > 50 mm using Markov chain model. In addition to this, incomplete gamma distribution was also used to find out the occurrence of rainfall events at different probability levels, *i.e.*, 20, 30, 40, 50, 65 and 75 per cent. The study results in estimation of maximum and minimum initial probability and conditional probability (wet and dry) for standard hervesting time for different crops. In addition to this study will be useful for determining the runoff volume, peak runoff rate and hence can be used for designing of rainwater harvesting structures.

Key words - Rainfall, Probability, Variability, Markov chain, Gamma distribution.

1. Introduction

Rain, has a significant effect on agriculture and agricultural production is affected to a large extent by the amount of rainfall. In India only 60 per cent area is irrigated whereas 40 per cent area is rainfed. Whereas, in Punjab more than 97 per cent of the total area is under irrigation and rest of area is rainfed. Although in Punjab only 2-3 per cent area is dependent on rainfall but due to depleting groundwater resources and deficit canal water supply the Punjab water resources greatly influenced by rainfall. The cost of irrigation/cultivation increases drastically in case of deficit rainfall. The states face energy crises too in this kind of scenario, so it is essential to have a thorough record of rainfall data. Rainfall also determines the potential of any region in terms of crops to be grown, farming system to be adopted, the nature and

sequence of farming operations to be followed and to achieve higher agricultural productivity as well.

The main characteristics of rainfall are its amount, frequency and intensity, the values of which vary from place to place, day to day, month to month and also year to year. Precise knowledge of these is essential for planning its full utilization. Simple criteria related to sequential phenomena like dry and wet spells could be used for analyzing rainfall data to obtain specific information needed for crop planning and for carrying out agricultural operations. Barger *et al.* (1959) computed rainfall probabilities for different areas of the United States. Similarly, Biswas and Khambete (1989) computed the lowest amount of rainfall at different probability level by fitting gamma distribution probability model.

So, it is very essential to study the frequency of happening various category of rainfall event. In present study the Markov Chain Model is used to find the probability of dry and wet spells. Incomplete Gamma Distribution is also used to find the occurrence of rainy events. Raudkivi (1979) has also calculated the rainfall probabilities using the Markov chain model to describe the hydrological phenomena. Regular rain pattern is usually vital to healthy plants, too much or too little rainfall can be harmful, even devastating to crops. Plants need varying amounts of rainfall to survive. The annual and seasonal analysis of rainfall will give general idea about the rainfall pattern of the region, whereas the weekly analysis of rainfall will be of much use as far agricultural planning is concerned. Therefore, a study was planned to work out the probabilities of rainfall occurrence at different events with the help of previous records of the rainfall data so that the probabilities of rainfall can be used for a number of agricultural planning purposes like field drying of hay, germination of seeds, applying Fertilizer, applying insecticides, applying herbicides, disease susceptibility in periods of plant growth.

2. Data and methodology

The rainfall data of Ludhiana for a period of 32 years covering 1981 to 2012 has been collected from School of Climate Change and Agricultural Meteorology, PAU, Ludhiana. The annual rainfall for the region is 700-800 mm from which 75% is received during rainy season (June-September) and rest is during winter season through western disturbances. The rainfall probabilities were worked out through different methods as below:

2.1. Markov chain analysis

Markov chain probability model has been found suitable to describe the long term frequency behaviour of wet or dry weather spells. Markov chain probability model assumes that the probability of rainfall occurring on any week depends on whether the previous week was wet or dry. The model calculates the initial probabilities of getting a dry spell/wet spell in a given standard meteorological week. The calculation of conditional probabilities provides the information on the dry spell followed by dry spell or wet spell vice versa. The calculation of initial and conditional probabilities are given below:

2.1.1. Initial rainfall probability (%)

It provides tentative idea of particular week having highest or lowest probability of getting more than 20 mm of rains. Number of year during which > 20 mm rainfall Initial Probability (%)(w) = $\frac{\text{during } x \text{ week}}{\text{Total number}} \times 100$ of year

2.1.2. Conditional rainfall probability (%)

Conditional probability indicates the probability level at which a particular amount of rainfall is anticipated for a particular place over a specified time series data. Conditional probability is useful in predicting the receipt of particular quantity of rainfall for specific purposes based on historical data.

2.1.3. Conditional rainfall probability for wet week (%) (W/Wx)

		Number of years during
Conditional		which next week received,
Rainfall		when this week also
Probability		received > 20 mm × 100
for wet	=	Number of years during ^ 100
week(%)		which this week
(W/Wx)		received > 20 mm
		rainfall

where, W = Wet week and

W/Wx = Wet week followed by wet week

2.1.4. Conditional rainfall probability for dry week (%) (W/Dx)

. . .

Conditional		Number of years during which
		next week received>20mm,
Rainfall		when this week < 20mm
Probability	_	$\frac{\text{when this week < 20 mm}}{100}$
Tiobability	_	Number of years during which
for dry week		
$(0/)(W/D_{W})$		this week received < 20 mm
(70)(W / Dx)		rainfall

where, R = Rainfall,

W = Wet week and

W/Dx = Dry week followed by wet week.

TABL	E 1	l
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Initial probability (%) at different levels

Week	> 5	mm	> 10) mm	> 20) mm	> 30) mm	>40) mm	> 50	mm
No.	Low	High	Low	High	Low	High	Low	High	Low	High	Low	High
1 to 13	13	47	9	38	0	22	0	16	0	6	0	6
14 to 26	13	78	9	69	3	59	3	41	0	38	0	28
27 to 39	22	88	9	81	3	72	3	56	3	50	0	41
40 to 52	3	22	0	16	0	9	0	6	0	3	0	3

2.2. Incomplete gamma distribution

Incomplete gamma distribution can be used to predict the occurrence of rainy events of different probability for crop planning. The rainfall data can be analyzed for probability of weekly rainfall above 5, 10, 20, 30, 40, 50 mm. The gamma distribution has been found to give good fits to precipitation climatologically series. This method is used when no rainfall was recorded in any of the years. In case, these contain zeros, the mixed distribution function of zero and continuous precipitation amounts may be employed.

The rainfall data (x) can be analyzed for probability of weekly rainfall above 5, 10, 20, 30, 40, 50 mm using incomplete gamma distribution. It is defined by

$$g(x) = (1/\beta^{\gamma} \sqrt{\gamma}) x^{\gamma-1} e^{-x/\beta}$$

where, β is a scale parameter, γ is a shape parameter and $\sqrt{\gamma}$ is an ordinary gamma function such that

P + q = 1

The moments in this case give poor estimates of the parameters. Sufficient estimates are available and these are closely approximated :

$$\widehat{\gamma} = \frac{1 + \sqrt{1 + 4A/3}}{4A}$$
$$\widehat{\beta} = \frac{x}{\widehat{\gamma}}$$
$$A = \ln \overline{x} - \sum \frac{\ln X}{N}$$

The distribution function from which probabilities may be obtained is :

$$G(x) = \int_{0}^{x} g(t)dt$$

Pearson "Table of the incomplete gamma function" gives G (u), where,

$$u = \frac{x}{\hat{\beta}\sqrt{\hat{\gamma}}}$$
$$P = \hat{\gamma} - 1$$
$$t_{(f)} = \frac{x}{\hat{\beta}}$$

3. Results and discussion

The initial and conditional probabilities were calculated for different years on weekly basis. The 52 weeks of a year are divided into 4 quarters. 1st - 13th SMWs of the year represents Quarter 1 and consequently, $14^{\text{th}}-26^{\text{th}}$, $27^{\text{th}}-39^{\text{th}}$ and $40^{\text{th}}-52^{\text{nd}}$ SMWs are representing 2^{nd} , 3^{rd} and 4^{th} Quarters, respectively. This analysis will be useful for predicting the irrigation schedules of different crops falling in that period as per crop calendar. The analysis will be helpful for crop diversification depending upon probability of occurrence of rainfall during specific standard meteorological weeks. If the dry spell coincides with critical moisture requiring s tages of the crop, it is damaging whereas during ripening stage, it may be beneficial. One can adjust crop variety/ crop accordingly, e.g., for kharif crops, one can choose between paddy and basmati or even pulses in water scarcity zone whereas in case of rabi season, one can opt early/late sowing varieties of wheat. Also the policy makers can adjust timing of canal water releases/electrical supply to the farmers accordingly.

TABLE 2

Conditional probability wet (%) at different levels

Week	> 5	mm	> 10) mm	> 20) mm	> 30) mm	> 40) mm	> 50	mm
No.	Low	High	Low	High	Low	High	Low	High	Low	High	Low	High
1 to 13	0	67	0	45	0	50	0	50	0	-	0	-
14 to 26	0	73	0	67	0	100	0	62	0	100	0	56
27 to 39	14	83	0	76	0	71	0	67	0	50	0	50
40 to 52	0	100	0	20	0	-	0	-	0	-	0	_

The study will also be useful for agricultural operations, *i.e.*, spraying (insecticide/ pesticides), application of fertilizer can be made more effective by adjusting the timing of their operations according to the probability of occurrence of rainfall. The probability of occurrence of dry followed by a wet spell was comparatively higher during monsoon season and was low during rest of the year. Similarly, the probability of occurrence of wet spell followed by dry spell was less during monsoon and high during rest of the year. In this way, knowledge of rainfall probability analysis, crop sowing dates can be adjusted in such a way that critical stages of the crop coincide with the period of higher rainfall probability. This will helps in reducing the stress on water resources and energy conservation. Apart from saving crops from water deficit, insect-pest disease occurrence can also be predicted based on these sequences of dry and wet spell. The higher level of occurrence of rainfall event will also be helpful in deciding the amount of run-off generated and hence can be used for deciding capacity of rain water harvesting structures.

Initial Rainfall Probability : The data pertaining to initial rainfall probability is presented in Table 1. The initial rainfall can be classified into three categories namely low, moderate and high rainfall level. The low category is for rainfall up to > 10 mm; moderate for > 20 to > 30 mm whereas > 40 to > 50 mm comes under high category. The lowest and highest rainfall probabilities in the form of percentage were worked out for different quarters for Ludhiana region. Similarly, Rama Rao *et al.*, (1975) analyzed the daily rainfall data for Bijapur from 1921 to 1970. Similar studies were also done by Schwab *et al.*, (1966), USDA-SCS (1967) and Keller (1972) for different locations.

Quarter 1 (> 5 mm, > 10 mm and > 20 mm)

The perusal of data reveals that initial probability for > 5 mm was observed lowest (13%) during 13th SMW and highest (47%) during 6th and 7th SMW, for > 10 mm the

probability was lowest (9%) during 9^{th} , 13^{th} SMW and highest (38%) during 6^{th} SMW and for > 20 mm the probability was lowest (0%) during 13^{th} SMW and highest (22%) during 6^{th} SMW.

Quarter 2 (> 5 mm, > 10 mm and > 20 mm)

The perusal of data reveals that initial probability for > 5 mm was observed lowest (13%) during 22^{nd} SMW and highest (78%) during 26^{th} SMW, for > 10 mm the probability was lowest (9%) during 22^{nd} SMW and highest (69%) during 26^{th} SMW and for > 20 mm the probability was lowest (3%) during 17^{th} SMW and highest (59%) during 26^{th} SMW.

Quarter 3 (> 5 mm, > 10 mm and > 20 mm)

The perusal of data reveals that initial probability for > 5 mm was observed lowest (22%) during 39^{th} SMW and highest (88%) during 28^{th} and 31^{th} SMW, for > 10 mm the probability was lowest (9%) during 39^{th} SMW and highest (81%) during 28^{th} SMW and for > 20 mm the probability was lowest (3%) during 39^{th} SMW and highest (72%) during 28^{th} SMW.

Quarter 4 (> 5 mm, > 10 mm and > 20 mm)

The perusal of data reveals that initial probability for > 5 mm was observed lowest (3%) during 43rd, 44th, 46th SMW and highest (22%) during 52nd SMW, for > 10 mm the probability was lowest (0%) during 43rd, 46th, 48th SMW and highest (16%) during 40th, 47th, 52nd SMW and for > 20 mm the probability was lowest (0%) during 43rd, 45-48th SMW and highest (9%) during 40th, 51-52nd SMW.

Quarter 1 (> 30 mm, > 40 mm and > 50 mm)

The perusal of data reveals that initial probability for > 30 mm was observed lowest (0%) during 13^{th} SMW and highest (16%) during 7th SMW, for > 40 mm the

TABLE 3

Conditional probability dry (%) at different levels

Week	> 5 mm		> 10 mm		> 20 mm		> 30 mm		> 40 mm		> 50 mm	
No.	Low	High	Low	High	Low	High	Low	High	Low	High	Low	High
01 to 13	13	47	8	37	0	20	0	13	0	6	0	6
14 to 26	13	82	7	70	4	52	0	35	0	32	0	30
27 to 39	13	100	5	89	4	67	4	53	3	50	0	41
40 to 52	3	25	0	17	0	10	0	3	0	3	0	3

probability was lowest (0%) during 1^{st} , 10^{th} , 13^{th} SMW and highest (6%) during 7^{th} SMW and for > 50 mm the probability was lowest (0%) during 1^{st} to 3^{rd} , 6^{th} , 10^{th} , 13^{th} SMW and highest (6%) during 7^{th} SMW.

Quarter 2 (> 30 mm, > 40 mm and > 50 mm)

The perusal of data reveals that initial probability for > 30 mm was observed lowest (3%) during 14^{th} , $16-17^{th}$ and 19^{th} SMW and highest (41%) during 26^{th} SMW, for > 40 mm the probability was lowest (0%) during $16-17^{th}$ SMW and highest (38%) during 26^{th} SMW and for > 50 mm the probability was lowest (0%) during $16-17^{th}$, 19^{th} SMW and highest (28%) during 26^{th} SMW.

Quarter 3 (> 30 mm, > 40 mm and > 50 mm)

The perusal of data reveals that initial probability for > 30 mm was observed lowest (3%) during 39^{th} SMW and highest (56%) during 29^{th} SMW, for > 40 mm the probability was lowest (3%) during 39^{th} SMW and highest (50%) during 29^{th} and 31^{th} SMW and for > 50 mm the probability was lowest (0%) during 39^{th} SMW and highest (41%) during 31^{th} SMW.

Quarter 4 (> 30 mm, > 40 mm and > 50 mm)

The perusal of data reveals that initial probability for > 30 mm was observed lowest (0%) during 41^{th} , 43^{rd} , $45^{\text{th}}-48^{\text{th}}$, 50^{th} , 52^{nd} SMW and highest (6%) during 40^{th} SMW, for > 40 mm the probability was lowest (0%) during 41^{th} , 43^{rd} , $45^{\text{th}}-48^{\text{th}}$, 50^{th} , 52^{nd} SMW and highest (3%) during 40^{th} , 42^{th} , 44^{th} , 49^{th} , 51^{th} SMW and for > 50 mm the probability was lowest (0%) during $40-41^{\text{th}}$, 43^{th} , $45^{\text{th}}-48^{\text{th}}$, 50^{th} , 52^{nd} SMW and highest (3%) during 42^{th} , 44^{th} , 49^{th} , 51^{th} SMW.

Conditional Rainfall Probability (wet): The data pertaining to conditional probability (wet) is presented in Table 2. The detailed description of different quarters is as follows:

Quarter 1 (> 5 mm, > 10 mm and > 20 mm)

The perusal of data reveals that conditional probability for > 5 mm was observed lowest (0%) during 3^{rd} SMW and highest (67%) during 7^{th} SMW, for > 10 mm the probability was lowest (0%) during $1-3^{rd}$, 10^{th} , 12^{th} and highest (45%) during 7^{th} SMW and for > 20 mm the probability was lowest (0%) during $1-4^{th}$, 8^{th} , $10-13^{th}$ SMW and highest (50%) during 5^{th} , 9^{th} SMW.

Quarter 2 (> 5 mm, > 10 mm and > 20 mm)

The perusal of data reveals that conditional probability for > 5 mm was observed lowest (0%) during 14^{th} and 21^{st} SMW and highest (73%) during 25^{th} SMW, for > 10 mm the probability was lowest (0%) during 14^{th} , 17^{th} , 19^{th} , 21^{st} SMW and highest (67%) during 25^{th} SMW and for > 20 mm the highest probability was observed for 15^{th} SMW (100%).

Quarter 3 (> 5 mm, > 10 mm and > 20 mm)

The perusal of data reveals that conditional probability for > 5 mm was observed lowest (14%) during 39^{th} SMW and highest (83%) during 34^{th} SMW, for > 10 mm the probability was lowest (0%) during 39^{th} SMW and highest (76%) during 31^{st} SMW and for > 20 mm the probability was highest (71%) during 27^{th} SMW.

Quarter 4 (> 5 mm, > 10 mm and > 20 mm)

The perusal of data reveals that conditional probability for > 5 mm was observed lowest (0%) during $41-43^{rd}$, $45-51^{th}$ SMW and highest (100%) during 44^{th} SMW, for > 10 mm the probability was lowest (0%) during $41-51^{th}$ SMW and highest (20%) during 40^{th} , 52^{nd} SMW and for > 20 mm the probability was (0%) during 40^{th} to 52^{nd} week.

TABLE 4

Rainfall probability by using incomplete gamma distribution

Year	Rainfall (X) mm	ln X	Year	Rainfall (X) mm	ln X			
1981	551.1	6.31	1997	971.1	6.88			
1982	681.7	6.52	1998	1004	6.91			
1983	995.8	6.9	1999	625.9	6.44			
1984	733.2	6.6	2000	706	6.56			
1985	856.3	6.75	2001	946.8	6.85			
1986	653.4	6.48	2002	407.2	6.01			
1987	341.9	5.83	2003	815.3	6.7			
1988	1289.1	7.16	2004	514.9	6.24			
1989	446.6	6.1	2005	710.8	6.57			
1990	333.3	5.81	2006	608.7	6.41			
1991	545.7	6.3	2007	596.5	6.39			
1992	846.2	6.74	2008	1049	6.96			
1993	838.6	6.73	2009	914.9	6.82			
1994	715.6	6.57	2010	727.01	6.59			
1995	1214.5	7.1	2011	1284.3	7.16			
1996	843.2	6.74	2012	498.3	6.21			
Paramete	ers of incomplete gamma di	stribution	Rainfall at different probability levels					
\overline{x}	$\frac{\sum x}{N}$	758.34	Rainfall	Probability	mm			
	$\frac{\sum \ln x}{N}$	6.57	$x = 0.1 * \hat{\beta}$	20%	8.93			
	$\ln \overline{x}$	6.63	$x = 0.2 * \hat{\beta}$	30%	17.86			

 $x = 0.4 * \hat{\beta}$

 $x = 0.6 * \hat{\beta}$

 $x = 0.9 * \hat{\beta}$

 $x = 1.3 * \hat{\beta}$

All Quarters (> 30 mm, > 40 mm and > 50 mm)

 $\ln \overline{x} - \sum \frac{\ln x}{N}$

 $\frac{1+\sqrt{1+4A/3}}{4A}$

 $\frac{x}{\overline{\gamma}}$

А

 $\overline{\gamma}$

Â

0.06

8.49

89.32

The perusal of data reveals that conditional probability for > 30 mm was observed highest during 27^{th} SMW (67%), for > 40 mm the probability highest for 22^{nd} SMW (100%) and for >50 mm the probability was highest for 26^{th} SMW (56%).

Conditional Probability (dry) : The data pertaining to conditional probability (dry) is presented in Table 3. The detailed description of different quarters is as follows:

40%

50%

65%

75%

35.73

53.59

80.39

116.12

Quarter 1 (> 5 mm, > 10 mm and > 20 mm)

The perusal of data reveals that conditional probability for > 5 mm was observed lowest (13%) during



Fig. 1. Initial P, Conditional P (wet) & Conditional P (dry) at rainfall > 5 mm



Fig. 2. Initial P, Conditional P (wet) & Conditional P (dry) at rainfall > 10 mm



Fig. 3. Initial P, Conditional P (wet) & Conditional P (dry) at rainfall > 20 mm

 4^{rth} SMW and highest (47%) during 6^{th} SMW, for > 10 mm the probability was lowest (8%) during 4^{rth} SMW and highest (37%) during 5^{th} SMW and for > 20 mm the probability was lowest (0%) during 12^{th} SMW and highest (20%) during 3^{rd} , 5^{th} , 6^{th} SMW.

Quarter 2 (> 5 mm, > 10 mm and > 20 mm)

The perusal of data reveals that conditional probability for > 5 mm was observed lowest (13%) during 21^{st} SMW and highest (82%) during 25^{th} SMW,







Fig. 5. Initial P, Conditional P (wet) & Conditional P (dry) at rainfall > 40 mm



Fig. 6. Nitial P, Conditional P (wet) & Conditional P (dry) at rainfall > 50 mm

for > 10 mm the probability was lowest (7%) during 21^{st} SMW and highest (70%) during 25^{th} SMW and for > 20 mm the probability was lowest (4%) during 21^{st} SMW and highest (52%) during 25^{th} SMW.

Quarter 3 (> 5 mm, > 10 mm and > 20 mm)

The perusal of data reveals that conditional probability for > 5 mm was observed lowest (13%) during 34^{th} SMW and highest (100%) during 30^{th} and 32^{th} SMW, for > 10 mm the probability was lowest (5%) during 38^{th} SMW and highest (89%) during 30^{th} SMW and for > 20

mm the probability was lowest (4%) during 38^{th} SMW and highest (67%) during 30^{th} SMW.

Quarter 4 (> 5 mm, > 10 mm and > 20 mm)

The perusal of data reveals that conditional probability for > 5 mm was observed lowest (3%) during 43^{rd} SMW and highest (25%) during 51^{st} SMW, for > 10 mm the probability was lowest (0%) during 42^{rd} , 45^{th} , 47^{th} SMW and highest (17%) during 51^{st} SMW and for > 20 mm the probability was lowest (0%) during 42^{rd} , 42^{th} , 44^{th} - 47^{th} SMW and highest (10%) during 51^{st} , 52^{rd} SMW.

Quarter 1 (> 30 mm, > 40 mm and > 50 mm)

The perusal of data reveals that conditional probability for > 30 mm was observed lowest (0%) during 12^{th} SMW and highest (13%) during 6^{th} SMW, for > 40 mm the probability was lowest (0%) during 9^{th} , 12^{th} SMW and highest (6%) during 6^{th} SMW and for > 50 mm the probability was lowest (0%) during $1-2^{\text{nd}}$, 5^{th} , 9^{th} , 12^{th} SMW and highest (6%) during 6^{th} SMW.

Quarter 2 (> 30 mm, > 40 mm and > 50 mm)

The perusal of data reveals that conditional probability for > 30 mm was observed lowest (0%) during 18^{th} SMW and highest (35%) during 25^{th} SMW, for > 40 mm the probability was lowest (0%) during 15^{th} , 16^{th} SMW and highest (32%) during 25^{th} SMW and for > 50 mm the probability was lowest (0%) during 15^{th} , 16^{th} , 18^{th} SMW and highest (30%) during 26^{th} SMW.

Quarter 3 (> 30 mm, > 40 mm and > 50 mm)

The perusal of data reveals that conditional probability for > 30 mm was observed lowest (4%) during 38^{th} SMW and highest (53%) during 28^{th} SMW, for > 40 mm the probability was lowest (3%) during 39^{th} SMW and highest (50%) during 28^{th} , 30^{th} SMW and for > 50 mm the probability was lowest (0%) during 38^{th} , 39^{th} SMW and highest (41%) during 30^{th} SMW.

Quarter 4 (> 30 mm, > 40 mm and > 50 mm)

The perusal of data reveals that conditional probability for > 30 mm was observed lowest (0%) during 40th, 42nd, 44th-47th, 49th, 51st SMW and highest (3%) during 41st, 43rd, 48th, 50th, 52nd SMW, for > 40 mm the probability was lowest (0%) during 40th, 42nd, 44-47th, 49th, 51st, 52nd SMW and highest (3%) during 41st, 43rd, 48th, 50th SMW and for > 50 mm the probability was lowest (0%) during 40th, 42nd, 44-47th, 49th, 51st, 52nd SMW and for > 50 mm the probability was lowest (0%) during 40th, 42nd, 44-47th, 49th, 51st, 52nd SMW and highest (3%) during 41st, 43rd, 48th, 50th SMW and highest (3%) during 41st, 43rd, 48th, 50th SMW.

The graphical presentation of Tables 1 to 3 can be visualized in Figs. 1-6 for the initial probability, conditional probability (wet & dry) for rainfall > 5 mm, > 10 mm, > 20 mm, > 30 mm, > 40 mm & > 50 mm respectively.

4. Impact of initial rainfall probability on agricultural activities

The above analysis can be interpreted in the form of low, moderate and high level for different agricultural activities in terms of *rabi* (wheat) and kharif (rice) crops in Punjab (Ashu, 2011). As the lowest rainfall probability in 13th SMW for >5 - 50 mm will be useful for the wheat crop as crop is at maturity and requires minimum water. So it will protect crop from the chances of lodging. According to this analysis, expected initial rainfall probability (> 5 - 50 mm) is maximum in 6th - 7th SMW, *i.e.*, mid-February so farmers can depend upon this event and can save one irrigation required for the crop during that period. Farmers can go for light irrigation depending upon *in-situ* conditions.

The lowest rainfall probability is in 22^{nd} SMW for > 5, 10 and 20 mm that falls in first week of June so farmers should ensure irrigation to the nursery of Paddy crop and transplanting of rice should be done after first week, which is already recommended for the Punjab and Haryana states. According to this analysis, expected initial probability (> 5 - 50 mm) is maximum during end June so farmers can depend upon rain for transplanting basmati and hence can save water and skip/provide light irrigation for paddy crop during this period.

The lowest probability for >30 - > 50 mm falls in 16, 17 and 19th SMW which falls in end April to May so farmers should not go for crops demanding more water like paddy but they can opt for low water demanding crops like pulses to reduce the stress on water resources or they can go for green manuring during this period to improve soil health.

The lowest probability is in 39th SMW for >5 - 50 mm that falls in last week of September so farmers should ensure irrigation to rice crop as crop is at physiological maturity. Irrigation department can ensure canal water supply during this period. The highest initial probability is maximum during $28^{th} - 31^{st}$ SMW, *i.e.*, from mid-July to first week of August for >5 - 50 mm so farmers can skip/ provide light irrigation depending upon *in-situ* condition and can depend upon rainfall during this period.

The lowest probability is from 43 - 48th SMW for >5 - 50 mm that falls in mid October to November months so farmers can adopt zero till drill for sowing of wheat and can save one irrigation. Otherwise farmers can adopt late sowing varieties of wheat as the initial probability of >5, 10 and 20 mm is highest in 52^{nd} SMW, *i.e.*, in end of December particularly in water scarce area to save water and energy. As the analysis done is for Ludhiana district which is in the central Punjab and whole of the central Punjab falls in over exploited zone.

Similar trend was found for conditional rainfall probability for dry and wet conditions. Analysis interpret that that conditional rainfall probability for wet period is 100 per cent for 22 SMW, *i.e.*, in 1st week of June so farmers can transplant paddy after 1st week which is already recommended.

Incomplete gamma distribution : The data pertaining to incomplete gamma distribution is presented in Table 4

The perusal of data reveals that different parameters of incomplete gamma distribution are determined. Rainfall at different probability levels is determined. At different Probabilities ranging 20%, 30%, 40%, 50%, 65%, 75%, the rainfall is 8.93, 17.86, 35.73, 53.59, 80.39, 116.12 mm respectively. At 20% probability, rainfall is minimum and maximum at 75%.

5. Conclusions

The information generated for maximum and minimum initial probability and conditional probability (Wet and dry) for different standard meteorological weeks will be helpful for deciding the sowing time, irrigation/fertilizer scheduling and harvesting time. The information generated will be helpful for probability of estimating the insect/pest attack depending upon the crop and probability of occurrence of diseases. In addition to this study will be useful for determining the runoff volume and peak rate of runoff and hence can be used for designing of rainwater harvesting structures.

It was revealed from this study that rainfall during the pre-sowing and sowing periods is an important determinant of crop acreage. The purpose of this study is to analyze the rainfall distribution patterns during monsoon (July-September), *rabi* crops sowing (October-December) and growing periods (June - July) of kharif crops. Rainfall during the pre-sowing period is prerequisite for land preparation as well as for moisture conservation for *rabi* planting. Low rainfall is likely to give a poor crop, while good rainfall will give a bumper kharif crops.

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