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STUDY ON STATISTICAL DISTRIBUTION OF MONTHLY RAINFALL IN PUNJAB, INDIA

1. The monsoon in India is erratic. The Punjab state is experiencing sudden dips and peaks in rainfall patterns, resulting in drought and flood conditions, respectively (Kingra *et al.*, 2017). The major disasters of flood and drought result from climate change, which puts massive pressure on available water resources in India. (Guhathakurta and Rajeevan, 2008; Kumar and Bhradwaj, 2015; Krishan *et al.*, 2015a, b; Tarate and Kumar, 2021). However, challenges have not been fulfilled for the management of water resources due to unpredicted climate change which, directly affects various sectors such as socio-economical, industrial and political. Furthermore, the important sector, *i.e.*, agriculture periodically suffers the worst from drought and flood as natural calamities. According to the IPCC report (Solomon *et al.*, 2007), over the past few decades, climate change has drawn attention from a variety of sectors, including the public, political, socioeconomic sectors regarding food production and water availability and security perspectives (Kumar *et al.*, 2017).

The high fluctuation in rainfall amount, frequency, duration, intensity and spatio-temporal extent is due to climate change. Most reviewers analyzed that extreme rainfall events across the globe are due to impacts of climate change (Singh and Sharma, 2003; Alexander *et al.*, 2006; Martinez *et al.*, 2012; Hussain and Lee, 2013; Hussain and Lee, 2014; Bari *et al.*, 2016; Gao and Xie, 2016; Liu *et al.*, 2016; Wang *et al.*, 2017; Alam *et al.*, 2018; Li and Hu, 2019; Shafiq *et al.*, 2019; Zhang *et al.*, 2019). Spatial incoherence in the trends of rainfall was observed by earlier research done for the Indian subcontinent. Researchers also investigated the trends of rainfall frequency; they found that across India's main river basins, the number of rainy days was significantly decreasing (Kumar and Jain, 2011). North and Central India have declining trends, while Peninsular India has rising trends (Guhathakurta *et al.*, 2011). Most of the river basins in the southern western ghats are showing decreasing trends of rainfall magnitude (Sreelash *et al.*, 2018). As a result, inconsistencies in the rainfall data at different spatial and temporal scales have led to contradictory findings regarding the total annual rainfall and the number of rainy days across India.

The geographical area of Punjab is about 50,000 km², with a major contribution of alluvial plain, except in the northeast region, which falls under the Shivaliks sub mountain region. The Punjab state has an identity of land

of five rivers, but only three rivers contribute to two doabs in Punjab, one lying between the Beas and the Sutlej and another lying between the old beds of Beas and the Ravi only [Indian Meteorological Department (IMD), 2021]. Punjab state has a net sown area of 4.1 million ha, which is only 2.94 % of the total net sown area of the country (139.4 million ha). It is necessary to increase production and productivity to meet the needs of the growing population and to achieve food security through sustainable agriculture. The canal irrigated area in Punjab has decreased from 58.4 % to 28 % in the last six decades, while the tube well irrigated area has increased from 41.1 percent to 71.3 percent. Moreover, the net irrigated area increased from 54 % to 98.2 %. Although water resources and groundwater are under considerable stress, the number of over exploited blocks rose from 53 to 109 between 1984 and 2017 (Singla *et al.*, 2023; Madane *et al.*, 2023).

In Punjab state, the rice-wheat cropping system is predominant which consumes extensive water and adversely affects water resources in Punjab. The rainfall distribution pattern in Punjab could be significantly decreasing for a couple of decades. Furthermore, the water demand of various sectors such as agriculture, industrial, drinking and domestic supply has been drastically changed. On the other side, excessive groundwater exploitation decreases the water table alarmingly (CGWB, 2021).

In this study, we have applied different best-fit probability distributions for the monthly rainfall data of Punjab. This proposed study could be useful for the water resource engineers, decision-makers and planners for the management of agricultural activities and conservation of natural resources of Punjab. Limited studies have been carried out for selecting the best fit distributions for rainfall in Punjab. Hence, this study has been undertaken with the following objectives: (i) To apply different best fit distributions for rainfall data of Punjab and (ii) To select the well-performing best fit distribution for different districts of Punjab.

2. *Data sets and methodology* : 2.1. *Study area* - Punjab state has a total area of 50,362 km², with irrigation being provided for the cultivable land. Its mean elevation is 300 meters above mean sea level. Punjab is located between 29.30° N to 32.32° N latitudes and 73.55° E to 76.50° E longitude. Pakistan borders Punjab on the west, Jammu and Kashmir on the north, Himachal Pradesh on the northeast, and Haryana and Rajasthan on the south. Punjab has extremely hot summers and cold winters. The territory along the foothills of the Himalayas experiences considerable rainfall, but the region farther from it experiences limited precipitation and high temperatures.

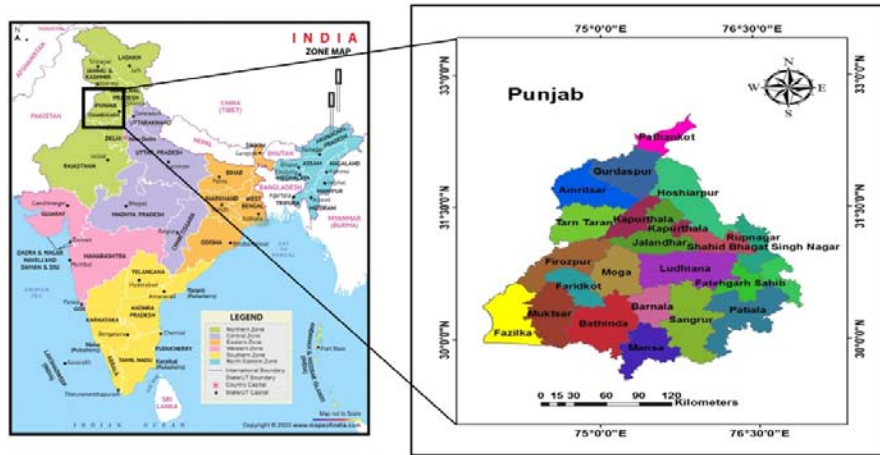


Fig. 1. Map of the study area

The summer months in this state start from mid of April to the end in the last week of June. The Winter season starts in October month (GOP, 2022).

2.2. *Data collection* - The monthly gridded rainfall data of IMD for twenty districts of Punjab for the period of 120 years (1902-2021) were downloaded from the website(<https://indiawris.gov.in/wris/#/>). The details of the metrological location of twenty districts in the state of Punjab are depicted in Fig. 1.

2.3. *Methodology* - Probability distributions are basic concepts in statistics. The results of statistical experiments and their probabilities of occurrence are linked by probability distributions. Different best-fit distributions like Gamma, Gamma (3P), Weibull, Weibull (3P), Chi-squared, Chi-squared (2P), Log-Pearson 3, Generalized extreme value (GEV), Exponential, Exponential (2P) were applied to the collected rainfall data. These best-fit distributions were compared using goodness-of-fit tests like Chi-squared, Anderson–Darling and Kolmogorov-Smirnov test. Finally, it was revealed that the Gamma distribution is the best fit distribution for the monthly rainfall data of twenty districts of Punjab.

2.3.1. *Chi-squared distribution* - The Chi-squared distribution is given by Eqn. (1) as:

$$f(x; n) = \frac{\left[\left(\frac{x}{2} \right)^{\frac{n}{2}-1} \right] e^{-\frac{x}{2}}}{2\Gamma\left(\frac{n}{2}\right)} \quad (1)$$

where the variable $x \geq 0$ and the parameter n is the number of degrees of freedom.

2.3.2. *Exponential distribution* -The exponential distribution is given by Eqn. (2) as:

$$f(x; n) = \frac{1}{\alpha} e^{-\frac{x}{\alpha}} \quad (2)$$

where the variable x and α are positive real quantity.

2.3.3. *Gamma distribution* - The Gamma distribution is given by Eqn. (3) as:

$$f(x; a, b) = a(ax)^{b-1} e^{-ax} / \Gamma(b) \quad (3)$$

where, a and b are positive real quantity.

2.3.4. *Generalized extreme value (GEV) distribution* - The class of GEV distributions is very flexible with the tail shape parameter ξ (and hence the tail index defined as $\alpha = \xi^{-1}$ controlling the shape and size of the tails given by Eqn. (4) as:

$$f_{\xi}^z(x) = \exp\left[-(1 + \xi x)\right]^{\frac{1}{\xi}} \quad (4)$$

with $1 + \xi x > 0, \xi \neq 0$

The standardized GEV distribution incorporates a location parameter μ and a scale parameter σ , in addition to the tail shape parameter, ξ , and is given by Eqn. (5) as:

$$f_{\xi, \mu, \sigma}(x) = \exp\left[1 + \xi \frac{(x - \mu)^{-1/\xi}}{\sigma}\right] \quad (5)$$

2.3.5. *Log-Pearson Type 3* - It is a complicated distribution. It consists of two interacting shape

TABLE 1
Distribution type and their parameters for Ajit Singh Nagar and Amritsar

S. No.	Distribution	Parameters	
		Ajit Singh Nagar	Amritsar
1.	Chi-Squared	$\nu = 96$	$\nu = 59$
2.	Chi-Squared (2P)	$\nu = 35$	$\nu = 20$
3.	Exponential	$\lambda = 0.01035$	$\lambda = 0.01685$
4.	Exponential (2P)	$\lambda = 0.01035 \quad \gamma = -1.0000E-14$	$\lambda = 0.01685 \quad \gamma = -1.0000E-14$
5.	Gamma	$\alpha = 0.53639 \quad \beta = 180.09$	$\alpha = 0.45877 \quad \beta = 129.36$
6.	Gamma (3P)	$\alpha = 0.56052 \quad \beta = 185.06$	$\alpha = 0.53149 \quad \beta = 119.9$
7.	Gen. Extreme Value	$k = 0.40992 \quad \sigma = 52.095 \quad \mu = 31.482$	$k = 0.45142 \quad \sigma = 29.985 \quad \mu = 18.157$
8.	Weibull	$\alpha = 0.38433 \quad \beta = 62.224$	$\alpha = 0.39311 \quad \beta = 36.322$
9.	Weibull (3P)	$\alpha = 0.67945 \quad \beta = 80.56$	$\alpha = 0.65629 \quad \beta = 47.799$
10.	Log-Pearson 3	No fit	No fit

TABLE 2
Distribution type and their parameters for Barnala and Bathinda

S. No.	Distribution	Parameters	
		Barnala	Bathinda
1.	Chi-Squared	$\nu = 40$	$\nu = 32$
2.	Chi-Squared (2P)	$\nu = 13$	$\nu = 9$
3.	Exponential	$\lambda = 0.02476$	$\lambda = 0.03117$
4.	Exponential (2P)	$\lambda = 0.02476 \quad \gamma = -1.0000E-14$	$\lambda = 0.03117 \quad \gamma = -1.0000E-14$
5.	Gamma	$\alpha = 0.38525 \quad \beta = 104.86$	$\alpha = 0.34652 \quad \beta = 92.593$
6.	Gamma (3P)	$\alpha = 0.49241 \quad \beta = 93.385$	$\alpha = 0.45536 \quad \beta = 80.464$
7.	Gen. Extreme Value	$k = 0.48596 \quad \sigma = 19.762 \quad \mu = 10.909$	$k = 0.51129 \quad \sigma = 15.174 \quad \mu = 7.9638$
8.	Weibull	$\alpha = 0.31856 \quad \beta = 17.521$	$\alpha = 0.32705 \quad \beta = 12.381$
9.	Weibull (3P)	$\alpha = 0.62206 \quad \beta = 32.738$	$\alpha = 0.58635 \quad \beta = 24.302$
10.	Log-Pearson 3	No fit	No fit

TABLE 3
Distribution type and their parameters for Faridkot and Fathegadh Saheb

S. No.	Distribution	Parameters	
		Faridkot	Fathegadh Saheb
1.	Chi-Squared	$\nu = 33$	$\nu = 84$
2.	Chi-Squared (2P)	$\nu = 12$	$\nu = 30$
3.	Exponential	$\lambda = 0.02979$	$\lambda = 0.0118$
4.	Exponential (2P)	$\lambda = 0.02979 \quad \gamma = -1.0000E-14$	$\lambda = 0.0118 \quad \gamma = -1.0000E-14$
5.	Gamma	$\alpha = 0.38879 \quad \beta = 86.347$	$\alpha = 0.53886 \quad \beta = 157.3$
6.	Gamma (3P)	$\alpha = 0.52728 \quad \beta = 74.116$	$\alpha = 0.56427 \quad \beta = 156.75$
7.	Gen. Extreme Value	$k = 0.49028 \quad \sigma = 16.351 \quad \mu = 8.9135$	$k = 0.41401 \quad \sigma = 44.519 \quad \mu = 28.607$
8.	Weibull	$\alpha = 0.32338 \quad \beta = 13.894$	$\alpha = 0.45633 \quad \beta = 61.471$
9.	Weibull (3P)	$\alpha = 0.65046 \quad \beta = 28.862$	$\alpha = 0.68455 \quad \beta = 69.21$
10.	Log-Pearson 3	No fit	No fit

TABLE 4

Distribution type and their parameters for Firozpur and Gurdaspur

S. No.	Distribution	Parameters	
		Firozpur	Gurdaspur
1.	Chi-Squared	$\nu = 30$	$\nu = 84$
2.	Chi-Squared (2P)	$\nu = 9$	$\nu = 30$
3.	Exponential	$\lambda = 0.0326$	$\lambda = 0.0118$
4.	Exponential (2P)	$\lambda = 0.0326 \quad \gamma = -1.0000E-14$	$\lambda = 0.0118 \quad \gamma = -1.0000E-14$
5.	Gamma	$\alpha = 0.38798 \quad \beta = 79.07$	$\alpha = 0.53886 \quad \beta = 157.3$
6.	Gamma (3P)	$\alpha = 0.47768 \quad \beta = 70.702$	$\alpha = 0.56427 \quad \beta = 156.75$
7.	Gen. Extreme Value	$k=0.48878 \quad \sigma = 14.891 \quad \mu = 8.3051$	$k=0.41401 \quad \sigma = 44.519 \quad \mu = 28.607$
8.	Weibull	$\alpha = 0.36929 \quad \beta = 14.962$	$\alpha = 0.45633 \quad \beta = 61.471$
9.	Weibull (3P)	$\alpha = 0.60768 \quad \beta = 23.347$	$\alpha = 0.68455 \quad \beta = 69.21$
10.	Log-Pearson 3	No fit	No fit

TABLE 5

Distribution type and their parameters for Hoshiarpur and Jalandhar

S. No.	Distribution	Parameters	
		Hoshiarpur	Jalandhar
1.	Chi-Squared	$\nu = 92$	$\nu = 59$
2.	Chi-Squared (2P)	$\nu = 34$	$\nu = 20$
3.	Exponential	$\lambda = 0.01076$	$\lambda = 0.01683$
4.	Exponential (2P)	$\lambda = 0.01076 \quad \gamma = -1.0000E-14$	$\lambda = 0.01683 \quad \gamma = -1.0000E-14$
5.	Gamma	$\alpha = 0.53857 \quad \beta = 172.59$	$\alpha = 0.49074 \quad \beta = 121.09$
6.	Gamma (3P)	$\alpha = 0.57772 \quad \beta = 168.74$	$\alpha = 0.53652 \quad \beta = 118.67$
7.	Gen. Extreme Value	$k = 0.41862 \quad \sigma = 48.505 \quad \mu = 31.134$	$k = 0.4337 \quad \sigma = 30.914 \quad \mu = 18.658$
8.	Weibull	$\alpha = 0.4457 \quad \beta = 67.392$	$\alpha = 0.4003 \quad \beta = 36.914$
9.	Weibull (3P)	$\alpha = 0.69455 \quad \beta = 77.036$	$\alpha = 0.66016 \quad \beta = 48.134$
10.	Log-Pearson 3	No fit	No fit

TABLE 6

Distribution type and their parameters for Kapurthala and Ludhiana

S. No.	Distribution	Parameters	
		Kapurthala	Ludhiana
1.	Chi-Squared	$\nu = 58$	$\nu = 56$
2.	Chi-Squared (2P)	$\nu = 21$	$\nu = 19$
3.	Exponential	$\lambda = 0.01703$	$\lambda = 0.01765$
4.	Exponential (2P)	$\lambda = 0.01703 \quad \gamma = -1.0000E-14$	$\lambda = 0.01765 \quad \gamma = -1.0000E-14$
5.	Gamma	$\alpha = 0.48192 \quad \beta = 121.85$	$\alpha = 0.47046 \quad \beta = 120.42$
6.	Gamma (3P)	$\alpha = 0.55626 \quad \beta = 114.81$	$\alpha = 0.53297 \quad \beta = 114.75$
7.	Gen. Extreme Value	$k = 0.43983 \quad \sigma = 30.318 \quad \mu = 18.176$	$k = 0.44528 \quad \sigma = 29.121 \quad \mu = 17.217$
8.	Weibull	$\alpha = 0.38517 \quad \beta = 34.947$	$\alpha = 0.39504 \quad \beta = 33.353$
9.	Weibull (3P)	$\alpha = 0.67563 \quad \beta = 49.145$	$\alpha = 0.6558 \quad \beta = 45.763$
10.	Log-Pearson 3	No fit	No fit

TABLE 7

Distribution type and their parameters for Manasa and Moga

S. No.	Distribution	Parameters	
		Manasa	Moga
1.	Chi-Squared	$\nu = 32$	$\nu = 40$
2.	Chi-Squared (2P)	$\nu = 10$	$\nu = 14$
3.	Exponential	$\lambda = 0.03055$	$\lambda = 0.02467$
4.	Exponential (2P)	$\lambda = 0.03055 \quad \gamma = -1.0000E-14$	$\lambda = 0.02467 \quad \gamma = -1.0000E-14$
5.	Gamma	$\alpha = 0.34454 \quad \beta = 95.019$	$\alpha = 0.41954 \quad \beta = 96.608$
6.	Gamma (3P)	$\alpha = 0.4702 \quad \beta = 81.052$	$\alpha = 0.51364 \quad \beta = 87.676$
7.	Gen. Extreme Value	$k = 0.51583 \quad \sigma = 15.356 \quad \mu = 8.0418$	$k = 0.4708 \quad \sigma = 20.162 \quad \mu = 11.533$
8.	Weibull	$\alpha = 0.31054 \quad \beta = 12.007$	$\alpha = 0.35366 \quad \beta = 20.524$
9.	Weibull (3P)	$\alpha = 0.60062 \quad \beta = 25.942$	$\alpha = 0.64031 \quad \beta = 32.942$
10.	Log-Pearson 3	No fit	No fit

TABLE 8

Distribution type and their parameters for Muktsar and Patiala

S. No.	Distribution	Parameters	
		Muktsar	Patiala
1.	Chi-Squared	$\nu = 31$	$\nu = 32$
2.	Chi-Squared (2P)	$\nu = 10$	$\nu = 10$
3.	Exponential	$\lambda = 0.03223$	$\lambda = 0.03055$
4.	Exponential (2P)	$\lambda = 0.03223 \quad \gamma = -1.0000E-14$	$\lambda = 0.03055 \quad \gamma = -1.0000E-14$
5.	Gamma	$\alpha = 0.36763 \quad \beta = 84.4$	$\alpha = 0.34454 \quad \beta = 95.019$
6.	Gamma (3P)	$\alpha = 0.48867 \quad \beta = 73.735$	$\alpha = 0.4702 \quad \beta = 81.052$
7.	Gen. Extreme Value	$k = 0.50562 \quad \sigma = 14.768 \quad \mu = 7.888$	$k = 0.51583 \quad \sigma = 15.356 \quad \mu = 8.0418$
8.	Weibull	$\alpha = 0.3165 \quad \beta = 12.13$	$\alpha = 0.31054 \quad \beta = 12.007$
9.	Weibull (3P)	$\alpha = 0.61766 \quad \beta = 25.317$	$\alpha = 0.60062 \quad \beta = 25.942$
10.	Log-Pearson 3	No fit	No fit

TABLE 9

Distribution type and their parameters for Rupnagar and Sangrur

S. No.	Distribution	Parameters	
		Rupnagar	Sangrur
1.	Chi-Squared	$\nu = 86$	$\nu = 44$
2.	Chi-Squared (2P)	$\nu = 32$	$\nu = 13$
3.	Exponential	$\lambda = 0.01151$	$\lambda = 0.02245$
4.	Exponential (2P)	$\lambda = 0.01151 \quad \gamma = -1.0000E-14$	$\lambda = 0.02245 \quad \gamma = -1.0000E-14$
5.	Gamma	$\alpha = 0.56138 \quad \beta = 154.77$	$\alpha = 0.41062 \quad \beta = 108.46$
6.	Gamma (3P)	$\alpha = 0.57489 \quad \beta = 160.62$	$\alpha = 0.47453 \quad \beta = 103.01$
7.	Gen. Extreme Value	$k = 0.39855 \quad \sigma = 46.939 \quad \mu = 29.663$	$k = 0.47557 \quad \sigma = 22.105 \quad \mu = 12.377$
8.	Weibull	$\alpha = 0.40564 \quad \beta = 60.717$	$\alpha = 0.35532 \quad \beta = 21.895$
9.	Weibull (3P)	$\alpha = 0.69411 \quad \beta = 73.364$	$\alpha = 0.60481 \quad \beta = 33.829$
10.	Log-Pearson 3	No fit	No fit

TABLE 10

Distribution type and their parameters for Shahid Bhagat Singh Nagar and Tarantarn

S. No.	Distribution	Parameters	
		Shahid Bhagat Singh Nagar	Tarantarn
1.	Chi-Squared	$\nu = 72$	$\nu = 49$
2.	Chi-Squared (2P)	$\nu = 26$	$\nu = 18$
3.	Exponential	$\lambda = 0.01385$	$\lambda = 0.02028$
4.	Exponential (2P)	$\lambda = 0.01385 \quad \gamma = -1.0000E-14$	$\lambda = 0.02028 \quad \gamma = -1.0000E-14$
5.	Gamma	$\alpha = 0.50745 \quad \beta = 142.26$	$\alpha = 0.4597 \quad \beta = 107.24$
6.	Gamma (3P)	$\alpha = 0.55231 \quad \beta = 140.67$	$\alpha = 0.55116 \quad \beta = 97.431$
7.	Gen. Extreme Value	$k = 0.42739 \quad \sigma = 37.838 \quad \mu = 23.005$	$k = 0.44874 \quad \sigma = 25.072 \quad \mu = 15.073$
8.	Weibull	$\alpha = 0.38506 \quad \beta = 45.66$	$\alpha = 0.38345 \quad \beta = 29.19$
9.	Weibull (3P)	$\alpha = 0.67375 \quad \beta = 59.875$	$\alpha = 0.67221 \quad \beta = 41.112$
10.	Log-Pearson 3	No fit	No fit

parameters (Griffis and Stedinger, 2007). It is represented by Eqn. (6) as:

$$f(x) = \frac{(\xi - X)^{\alpha-1} e^{-(\xi-X)/\beta}}{\beta^2 \Gamma \alpha} \tag{6}$$

where,

$$\xi = \frac{(\mu - 2\sigma)}{\gamma}$$

2.3.6. Weibull distribution - The Weibull distribution is given by Eqn. (7) as:

$$f(x; \eta, \sigma) = \frac{\eta}{\sigma} \left(\frac{x}{\sigma}\right)^{\eta-1} e^{-\left(\frac{x}{\sigma}\right)^\eta} \tag{7}$$

where, x, η and σ are positive real numbers.

2.4. Testing the goodness of fit - The goodness-of-fit tests like, Kolmogorov-Smirnov, Anderson-Darling and Chi-square test were applied at a 5% level of significance for selecting the best-fit distribution (Kumar *et al.*, 2017). The best distribution was selected based on the minimum error in this study by considering the following tests.

2.4.1. Kolmogorov-Smirnov test (K-S) - This test will be used for comparing an empirical distribution function (F_x) and a specified distribution function (F_y). It is considered an alternative to the Chi-square test. It is given by Eqn. (8) as:

$$.D = \text{Max} |F_x(x) - F_y| \tag{8}$$

Here, a large value of D represents an inconsistency between statistical model prediction and observed data.

2.4.2. Anderson-Darling test (A-D) - The test statistic (AD) is defined by Eqn. (9) as:

$$AD = -n - \frac{1}{n} \sum_{i=1}^n (2i-1) \{ \ln(x_{xi}) + \ln[1 - (x_{[n+1-i]})] \} \tag{9}$$

If the value of AD is large than the critical value then the null hypothesis will be rejected.

2.4.3. Chi-square (χ^2) test - The Chi-square test statistic is given by Eqn. (10) as:

$$(\chi)^2 = \frac{\sum (O - E)^2}{E} \tag{10}$$

The null hypothesis indicates there is no significant difference between expected (E) and observed (O) frequencies while the alternative hypothesis indicates they are different.

2.5. Identification of best-fit probability distribution - The discussed goodness-of-fit tests were fitted to the monthly rainfall data in this study. The test statistic was determined and tested at a 5 % level of significance ($\alpha = 0.05$). The ranking of several probability distributions was therefore determined based on the minimum test statistic value.

3. Results and discussion - 3.1. Probability distribution : Data from all districts of Punjab (1902 to 2021) were studied to determine the best probability

TABLE 11
Goodness of fit summary for Ajit Singh Nagar

S. No.	Distribution	Ajit Singh Nagar						\sum Rank
		Kolmogorov-Smirnov		Anderson-Darling		Chi-squared		
		Statistic	Rank	Statistic	Rank	Statistic	Rank	
1.	Chi-Squared	0.61778	9	22818	9	6714.7	9	27
2.	Chi-Squared (2P)	0.40032	8	4374.6	8	4615.1	8	24
3.	Exponential	0.19874	7	358.33	6	729.9	7	20
4.	Exponential (2P)	0.19874	6	421.44	7	729.9	6	19
5.	Gamma	0.06875	1	167.79	2	91.364	2	5
6.	Gamma (3P)	0.07884	3	179.99	3	90.097	1	7
7.	Gen. Extreme Value	0.135	5	41.124	1	192.12	4	10
8.	Weibull	0.11971	4	204.07	5	228.31	5	14
9.	Weibull (3P)	0.07043	2	181.03	4	95.99	3	9
10.	Log-Pearson 3	No fit						

TABLE 12
Goodness of fit summary for Amritsar

S. No.	Distribution	Amritsar						\sum Rank
		Kolmogorov-Smirnov		Anderson-Darling		Chi-squared		
		Statistic	Rank	Statistic	Rank	Statistic	Rank	
1.	Chi-Squared	0.60	9	16365.00	9	6251.30	9	27
2.	Chi-Squared (2P)	0.39	8	3876.50	8	4051.20	8	24
3.	Exponential	0.19	7	371.67	6	642.11	7	20
4.	Exponential (2P)	0.19	6	430.89	7	642.11	6	19
5.	Gamma	0.07	1	156.97	2	42.92	1	4
6.	Gamma (3P)	0.07	2	176.27	3	76.11	2	7
7.	Gen. Extreme Value	0.13	5	32.25	1	119.30	4	10
8.	Weibull	0.12	4	197.81	5	233.58	5	14
9.	Weibull (3P)	0.08	3	179.07	4	88.38	3	10
10.	Log-Pearson 3	No fit						

TABLE 13
Goodness of fit summary for Barnala

S. No.	Distribution	Barnala						\sum Rank
		Kolmogorov-Smirnov		Anderson-Darling		Chi-squared		
		Statistic	Rank	Statistic	Rank	Statistic	Rank	
1.	Chi-Squared	0.59	9	14474	9	6055.9	9	27
2.	Chi-Squared (2P)	0.36	8	4350.9	8	3900.1	8	24
3.	Exponential	0.24	7	584.96	6	1100.6	7	20
4.	Exponential (2P)	0.24	6	1027.7	7	1100.6	6	19
5.	Gamma	0.12	1	234.59	2	114.74	1	4
6.	Gamma (3P)	0.12	2	288.8	3	229.55	3	8
7.	Gen. Extreme Value	0.15	5	42.237	1	150.46	2	8
8.	Weibull	0.14	4	293.07	4	366.28	5	13
9.	Weibull (3P)	0.13	3	297.16	5	272.29	4	12
10.	Log-Pearson 3	No fit						

TABLE 14

Goodness of fit summary for Bathinda

S. No.	Distribution	Bathinda						
		Kolmogorov-Smirnov		Anderson-Darling		Chi-squared		\sum Rank
		Statistic	Rank	Statistic	Rank	Statistic	Rank	
1.	Chi-Squared	0.59	9	13359	9	5940.2	9	27
2.	Chi-Squared (2P)	0.36	8	3529.7	8	3840.8	8	24
3.	Exponential	0.27	7	679.37	6	1311.1	7	20
4.	Exponential (2P)	0.27	6	1132.4	7	1311.1	6	19
5.	Gamma	0.12	1	234.36	2	100.17	1	4
6.	Gamma (3P)	0.12	2	295.13	4	255.71	3	9
7.	Gen. Extreme Value	0.16	5	50.204	1	180.43	2	8
8.	Weibull	0.12	3	278.48	3	285.32	4	10
9.	Weibull (3P)	0.14	4	302.26	5	333.62	5	14
10.	Log-Pearson 3	No fit						

TABLE 15

Goodness of fit summary for Faridkot

S. No.	Distribution	Faridkot						
		Kolmogorov-Smirnov		Anderson-Darling		Chi-squared		\sum Rank
		Statistic	Rank	Statistic	Rank	Statistic	Rank	
1.	Chi-Squared	0.58	9	11511	9	5893.2	9	27
2.	Chi-Squared (2P)	0.34	8	3354.3	8	3754.7	8	24
3.	Exponential	0.25	7	584.44	6	1071.1	7	20
4.	Exponential (2P)	0.25	6	1241.3	7	1071.1	6	19
5.	Gamma	0.14	1	249.56	2	124.37	1	4
6.	Gamma (3P)	0.14	2	326.9	4	280.59	3	9
7.	Gen. Extreme Value	0.15	5	44.272	1	143.91	2	8
8.	Weibull	0.14	3	311.4	3	381.37	5	11
9.	Weibull (3P)	0.14	4	327.94	5	317.91	4	13
10.	Log-Pearson 3	No fit						

TABLE 16

Goodness of fit summary for Fathegadhisaheb

S. No.	Distribution	Fathegadhisaheb						
		Kolmogorov-Smirnov		Anderson-Darling		Chi-squared		\sum Rank
		Statistic	Rank	Statistic	Rank	Statistic	Rank	
1.	Chi-Squared	0.60708	9	17770	9	6414	9	27
2.	Chi-Squared (2P)	0.42093	8	4186.1	8	4432.4	8	24
3.	Exponential	0.17013	7	254.3	7	510.73	7	21
4.	Exponential (2P)	0.17013	6	230.05	6	510.73	6	18
5.	Gamma	0.04167	1	107.83	2	56.038	2	5
6.	Gamma (3P)	0.05678	3	113.64	3	62.001	3	9
7.	Gen. Extreme Value	0.12114	5	29.559	1	123.97	4	10
8.	Weibull	0.10116	4	132.33	5	144.25	5	14
9.	Weibull (3P)	0.05252	2	114.54	4	55.605	1	7
10.	Log-Pearson 3	No fit						

TABLE 17
Goodness of fit summary for Firozpur

S. No.	Distribution	Firozpur						\sum Rank
		Kolmogorov-Smirnov		Anderson-Darling		Chi-squared		
		Statistic	Rank	Statistic	Rank	Statistic	Rank	
1.	Chi-Squared	0.57741	9	10563	9	5776.2	9	27
2.	Chi-Squared (2P)	0.3533	8	3272.2	8	3428.8	8	24
3.	Exponential	0.23777	7	524.63	6	970.92	7	20
4.	Exponential (2P)	0.23777	6	707.77	7	970.92	6	19
5.	Gamma	0.09167	1	191.59	2	68.01	1	4
6.	Gamma (3P)	0.09167	2	225.82	3	148.91	3	8
7.	Gen. Extreme Value	0.14692	5	40.883	1	138.84	2	8
8.	Weibull	0.11253	4	228.38	4	195.39	5	13
9.	Weibull (3P)	0.10592	3	231.16	5	171.31	4	12
10.	Log-Pearson 3	No fit						

TABLE 18
Goodness of fit summary for Gurdaspur

S. No.	Distribution	Gurdaspur						\sum Rank
		Kolmogorov-Smirnov		Anderson-Darling		Chi-squared		
		Statistic	Rank	Statistic	Rank	Statistic	Rank	
1.	Chi-Squared	0.60708	9	17770	9	6414	9	27
2.	Chi-Squared (2P)	0.42093	8	4186.1	8	4432.4	8	24
3.	Exponential	0.17013	7	254.3	7	510.73	7	21
4.	Exponential (2P)	0.17013	6	230.05	6	510.73	6	18
5.	Gamma	0.04167	1	107.83	2	56.038	2	5
6.	Gamma (3P)	0.05678	3	113.64	3	62.001	3	9
7.	Gen. Extreme Value	0.12114	5	29.559	1	123.97	4	10
8.	Weibull	0.10116	4	132.33	5	144.25	5	14
9.	Weibull (3P)	0.05252	2	114.54	4	55.605	1	7
10.	Log-Pearson 3	No fit						

TABLE 19
Goodness of fit summary for Hoshiarpur

S. No.	Distribution	Hoshiarpur						\sum Rank
		Kolmogorov-Smirnov		Anderson-Darling		Chi-squared		
		Statistic	Rank	Statistic	Rank	Statistic	Rank	
1.	Chi-Squared	0.61603	9	18853	9	6650.5	9	27
2.	Chi-Squared (2P)	0.40418	8	3999.1	8	4210.6	8	24
3.	Exponential	0.17121	6	264.16	7	504.57	7	20
4.	Exponential (2P)	0.17121	7	250.14	6	504.57	6	19
5.	Gamma	0.04728	1	118.6	2	74.042	1	4
6.	Gamma (3P)	0.06761	3	127.33	4	78.485	2	9
7.	Gen. Extreme Value	0.12102	5	30.6	1	137.05	4	10
8.	Weibull	0.10886	4	147.77	5	164.14	5	14
9.	Weibull (3P)	0.05273	2	126.66	3	82.979	3	8
10.	Log-Pearson 3	No fit						

TABLE 20

Goodness of fit summary for Jalandhar

S. No.	Distribution	Jalandhar						
		Kolmogorov-Smirnov		Anderson-Darling		Chi-squared		\sum Rank
		Statistic	Rank	Statistic	Rank	Statistic	Rank	
1.	Chi-Squared	0.59147	9	16087	9	6103.2	9	27
2.	Chi-Squared (2P)	0.39207	8	3975.4	8	4053.5	8	24
3.	Exponential	0.19928	6	366.44	6	709.55	7	19
4.	Exponential (2P)	0.19928	7	415.11	7	709.55	6	20
5.	Gamma	0.06667	1	157.4	2	51.584	1	4
6.	Gamma (3P)	0.06765	2	172.71	3	85.106	2	7
7.	Gen. Extreme Value	0.13356	5	35.761	1	152.79	4	10
8.	Weibull	0.1111	4	190.98	5	213.13	5	14
9.	Weibull (3P)	0.07506	3	175.32	4	93.839	3	10
10.	Log-Pearson 3	No fit						

TABLE 21

Goodness of fit summary for Kapurthala

S. No.	Distribution	Kapurthala						
		Kolmogorov-Smirnov		Anderson-Darling		Chi-squared		\sum Rank
		Statistic	Rank	Statistic	Rank	Statistic	Rank	
1.	Chi-Squared	0.59249	9	16179	9	6147.3	9	27
2.	Chi-Squared (2P)	0.38308	8	4038.4	8	4082.4	8	24
3.	Exponential	0.20272	7	388.62	6	710.14	7	20
4.	Exponential (2P)	0.20272	6	510.94	7	710.14	6	19
5.	Gamma	0.08056	1	177.84	2	57.97	1	4
6.	Gamma (3P)	0.08416	3	203.47	3	112.48	2	8
7.	Gen. Extreme Value	0.13457	5	35.581	1	147.61	4	10
8.	Weibull	0.12201	4	221.27	5	217.54	5	14
9.	Weibull (3P)	0.08281	2	204.51	4	116.81	3	9
10.	Log-Pearson 3	No fit						

TABLE 22

Goodness of fit summary for Ludhiana

S. No.	Distribution	Ludhiana						
		Kolmogorov-Smirnov		Anderson-Darling		Chi-squared		\sum Rank
		Statistic	Rank	Statistic	Rank	Statistic	Rank	
1.	Chi-Squared	0.60257	9	16103	9	6376.5	9	27
2.	Chi-Squared (2P)	0.37405	8	4000.3	8	3971.3	8	24
3.	Exponential	0.20466	7	396.81	6	779.37	7	20
4.	Exponential (2P)	0.20466	6	478.85	7	779.37	6	19
5.	Gamma	0.07361	1	167.04	2	60.718	1	4
6.	Gamma (3P)	0.07871	2	188.42	3	98.291	2	7
7.	Gen. Extreme Value	0.13724	5	38.093	1	140.31	4	10
8.	Weibull	0.10861	4	201.65	5	185.06	5	14
9.	Weibull (3P)	0.07889	3	189.81	4	111.36	3	10
10.	Log-Pearson 3	No fit						

TABLE 23
Goodness of fit summary for Manasa

S. No.	Distribution	Manasa						
		Kolmogorov-Smirnov		Anderson-Darling		Chi-squared		\sum Rank
		Statistic	Rank	Statistic	Rank	Statistic	Rank	
1.	Chi-Squared	0.58577	9	12984	9	5903	9	27
2.	Chi-Squared (2P)	0.34668	8	3453.5	8	3920.9	8	24
3.	Exponential	0.27421	7	690.39	6	1319.2	7	20
4.	Exponential (2P)	0.27421	6	1331.6	7	1319.2	6	19
5.	Gamma	0.14097	1	251.35	2	133.39	1	4
6.	Gamma (3P)	0.14097	2	327.82	4	309.71	3	9
7.	Gen. Extreme Value	0.15862	5	49.178	1	171.08	2	8
8.	Weibull	0.14097	3	305.98	3	378.82	5	11
9.	Weibull (3P)	0.14819	4	333.93	5	351.83	4	13
10.	Log-Pearson 3	No fit						

TABLE 24
Goodness of fit summary for Moga

S. No.	Distribution	Moga						
		Kolmogorov-Smirnov		Anderson-Darling		Chi-squared		\sum Rank
		Statistic	Rank	Statistic	Rank	Statistic	Rank	
1.	Chi-Squared	0.59043	9	13019	9	6105.6	9	27
2.	Chi-Squared (2P)	0.35311	8	3916.9	8	3703.2	8	24
3.	Exponential	0.21892	7	490.34	6	854.29	7	20
4.	Exponential (2P)	0.21892	6	744.73	7	854.29	6	19
5.	Gamma	0.1	1	205.48	2	77.711	1	4
6.	Gamma (3P)	0.1	2	243.71	3	141.66	3	8
7.	Gen. Extreme Value	0.14265	5	38.233	1	128.65	2	8
8.	Weibull	0.1308	4	255.04	5	291.33	5	14
9.	Weibull (3P)	0.10922	3	248.42	4	177.46	4	11
10.	Log-Pearson 3	No fit						

TABLE 25
Goodness of fit summary for Muksar

S. No.	Distribution	Muksar						
		Kolmogorov-Smirnov		Anderson-Darling		Chi-squared		\sum Rank
		Statistic	Rank	Statistic	Rank	Statistic	Rank	
1.	Chi-Squared	0.58945	9	11973	9	5922.4	9	27
2.	Chi-Squared (2P)	0.33764	8	3188.1	8	3712.9	8	24
3.	Exponential	0.25596	7	642.92	6	1162.3	7	20
4.	Exponential (2P)	0.25596	6	1266.6	7	1162.3	6	19
5.	Gamma	0.13889	1	250.4	2	144.31	1	4
6.	Gamma (3P)	0.13889	2	322.82	4	305.85	3	9
7.	Gen. Extreme Value	0.15508	5	45.965	1	156.96	2	8
8.	Weibull	0.13889	3	306.41	3	379.35	5	11
9.	Weibull (3P)	0.14623	4	328.33	5	337.22	4	13
10.	Log-Pearson 3	No fit						

TABLE 26
Goodness of fit summary for Patiala

S. No.	Distribution	Patiala						
		Kolmogorov-Smirnov		Anderson-Darling		Chi-squared		\sum Rank
		Statistic	Rank	Statistic	Rank	Statistic	Rank	
1.	Chi-Squared	0.58577	9	12984	9	5903	9	27
2.	Chi-Squared (2P)	0.34668	8	3453.5	8	3920.9	8	24
3.	Exponential	0.27421	7	690.39	6	1319.2	7	20
4.	Exponential (2P)	0.27421	6	1331.6	7	1319.2	6	19
5.	Gamma	0.14097	1	251.35	2	133.39	1	4
6.	Gamma (3P)	0.14097	2	327.82	4	309.71	3	9
7.	Gen. Extreme Value	0.15862	5	49.178	1	171.08	2	8
8.	Weibull	0.14097	3	305.98	3	378.82	5	11
9.	Weibull (3P)	0.14819	4	333.93	5	351.83	4	13
10.	Log-Pearson 3	No fit						

TABLE 27
Goodness of fit summary for Rupnagar

S. No.	Distribution	Rupnagar						
		Kolmogorov-Smirnov		Anderson-Darling		Chi-squared		\sum Rank
		Statistic	Rank	Statistic	Rank	Statistic	Rank	
1.	Chi-Squared	0.60524	9	18543	9	6474.5	9	27
2.	Chi-Squared (2P)	0.40564	8	4370.5	8	4433.9	8	24
3.	Exponential	0.17085	6	303.1	6	547.73	7	19
4.	Exponential (2P)	0.17085	7	326.25	7	547.73	6	20
5.	Gamma	0.05926	1	148.85	2	73.958	1	4
6.	Gamma (3P)	0.06226	2	155.94	3	78.623	2	7
7.	Gen. Extreme Value	0.12605	5	33.423	1	137.64	4	10
8.	Weibull	0.12139	4	184.9	5	220.27	5	14
9.	Weibull (3P)	0.06965	3	158.52	4	88.984	3	10
10.	Log-Pearson 3	No fit						

TABLE 28
Goodness of fit summary for Sangrur

S. No.	Distribution	Sangrur						
		Kolmogorov-Smirnov		Anderson-Darling		Chi-squared		\sum Rank
		Statistic	Rank	Statistic	Rank	Statistic	Rank	
1.	Chi-Squared	0.58905	9	15567	9	5861.6	9	27
2.	Chi-Squared (2P)	0.3841	8	4102.5	8	4064.5	8	24
3.	Exponential	0.2374	7	522	6	1039.1	7	20
4.	Exponential (2P)	0.2374	6	689.29	7	1039.1	6	19
5.	Gamma	0.08889	1	191.96	2	59.908	1	4
6.	Gamma (3P)	0.09292	2	220.12	3	118.41	2	7
7.	Gen. Extreme Value	0.14696	5	41.445	1	153.24	3	9
8.	Weibull	0.11265	4	229.63	5	224.04	5	14
9.	Weibull (3P)	0.10281	3	227.58	4	170.66	4	11
10.	Log-Pearson 3	No fit						

TABLE 29

Goodness of fit summary for Shahid Bhagat Singh Nagar

S. No.	Distribution	Shahid Bhagat Singh Nagar						\sum Rank
		Kolmogorov-Smirnov		Anderson-Darling		Chi-squared		
		Statistic	Rank	Statistic	Rank	Statistic	Rank	
1.	Chi-Squared	0.61424	9	18516	9	6420.8	9	27
2.	Chi-Squared (2P)	0.38376	8	4176.4	8	4199.5	8	24
3.	Exponential	0.19254	7	362.51	6	632.57	7	20
4.	Exponential (2P)	0.19254	6	436.74	7	632.57	6	19
5.	Gamma	0.07083	1	166.3	2	68.833	1	4
6.	Gamma (3P)	0.07719	3	182.64	3	113.7	2	8
7.	Gen. Extreme Value	0.13241	5	36.045	1	148.01	4	10
8.	Weibull	0.13079	4	207.84	5	245.58	5	14
9.	Weibull (3P)	0.07653	2	184.85	4	114.77	3	9
10.	Log-Pearson 3	No fit						

TABLE 30

Goodness of fit summary for Tarantarn

S. No.	Distribution	Tarantarn						\sum Rank
		Kolmogorov-Smirnov		Anderson-Darling		Chi-squared		
		Statistic	Rank	Statistic	Rank	Statistic	Rank	
1.	Chi-Squared	0.59031	9	13974	9	5969.9	9	27
2.	Chi-Squared (2P)	0.36738	8	3871.5	8	4102.4	8	24
3.	Exponential	0.21245	7	393.08	6	693.76	7	20
4.	Exponential (2P)	0.21245	6	523.42	7	693.76	6	19
5.	Gamma	0.08194	1	178.6	2	58.003	1	4
6.	Gamma (3P)	0.08651	2	206.28	3	98.097	2	7
7.	Gen. Extreme Value	0.13332	5	33.895	1	145.37	4	10
8.	Weibull	0.12485	4	225.36	5	271.47	5	14
9.	Weibull (3P)	0.08853	3	207.91	4	110.28	3	10
10.	Log-Pearson 3	No fit						

distribution for monthly rainfall estimation. Monthly rainfall series data were analyzed for this probability analysis. Different best fit probability distributions like Gamma, Gamma (3P), Weibull, Weibull (3P), Chi-squared, Chi-squared (2P), Log-Pearson 3, Generalized extreme value (GEV), Exponential, Exponential (2P) were considered in this study. Different distribution parameters of the considered probability distributions corresponding to different districts of Punjab are presented in Tables 1-10. In order to select the best-fit probability distribution for these rainfall series data, goodness-of-fit tests like Chi-squared, Anderson-Darling and Kolmogorov-Smirnov were employed. The selection of the best probability distribution was done based on ranks obtained from different goodness of fit tests. Ranks corresponding to different probability distributions were

assigned from one to nine (1-9). Finally, the best fit probability distribution for the given district was selected from the minimum value of the sum of ranks of all three goodness of fit tests. As per the goodness-of-fit tests, it was observed that the Gamma distribution fitted the rainfall series data better than others for all districts of Punjab. In this study, it was found that Log-Pearson 3 distribution is not fitted at all for the rainfall series of entire districts of Punjab. The identification of the best distribution is necessary for risk and uncertainty analysis for water resources management, modeling and planning. These selected models will save time and money during risk and natural hazard assessment.

The goodness of fit test statistics for different districts of Punjab is presented in Tables 11-30. The ranks

were assigned based on the minimum value of test statistics. Here, the best fit distribution was selected based on the minimum value sum of ranks.

4. *Conclusions* - In this study, monthly series of rainfall data for 120 years (1902 to 2021) for different districts of Punjab were analyzed to select the best fit probability distribution. Monthly rainfall series data were fitted in different probability distributions like Gamma, Gamma (3P), Weibull, Weibull (3P), Chi-squared, Chi-squared (2P), Log-Pearson 3, Generalized extreme value (GEV), Exponential, Exponential (2P) to select the best fit probability distribution. Different probability distributions are compared based on goodness of fit tests like Chi-squared, Anderson-Darling and Kolmogorov-Smirnov. In this study, it was revealed that Gamma distribution fitted best for the monthly rainfall series data in all districts of Punjab. Hence, the best fitting Gamma distribution can be used in extrapolation studies of rainfall for the management of water resources and natural hazard assessment. The selection of the best probability distribution can influence decision-making related to local economics and hydrologic safety systems.

This analysis can provide a basement for the selection of district-wise best probability distribution and their distribution parameters. The preliminary result obtained in this study will assist the water resource planner in modeling hydrological processes and it also helps policymakers to frame general guidelines for the best use of rainfall for different activities in Punjab.

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