

A mathematical model for the 3-D dynamics of lee wave across a meso-scale mountain corner

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सार – मध्य-मापक्रम पर्वतीय कोने में ली वेव का 3-डी डायनेमिकल स्ट्रक्चर का अध्ययन करने के लिए एक गणितीय मॉडल प्रस्तावित किया गया है जिससे कि पवन गति एवं तापमान की ऊर्ध्वाधर भिन्नता का औसत बहाव वास्तविक रूप से जाना जा सके। मूलभूत बहाव में जोनल पवन गति अवयव (U) और रेखांशिक अवयव (meridional component) (V) दोनों होते हैं जिन्हें ऊँचाई से संबंधित माना जाता है। ब्रंट-वैस्ला आवृत्ति (N) भी ऊँचाई से संबंधित मानी जाती है। इस मॉडल को पूर्वोत्तर भारत में पर्वतीय क्षेत्र के कोने वाले हिस्से के लिए लागू किया गया है जो मुख्य रूप से असम बर्मा पहाड़ियों (ABH) के उत्तरी-दक्षिणी भाग तथा खासी जयंतिया पहाड़ियों (KJH) के पूर्वी-पश्चिमी क्षेत्र में फैला है। इस मॉडल को अर्द्ध संख्यात्मक पहल के जरिए तैयार किया गया है। ऊर्ध्वाधर वेग विक्रोभ (w') को द्विगुणित योग के रूप में दर्शाया गया है। इसमें तीन स्थितियों का अध्ययन किया गया है और सभी स्थितियों में यथासंभव अनुप्रस्थ और अपसारी ली वेव संख्या (k, l) के बीच के संबंध और भिन्न-भिन्न ऊँचाईयों पर ऊर्ध्ववाह/ अधोवाह क्षेत्र को भी लिया गया है जो ली वेव से संबंधित हैं और उस पर चर्चा की गई है।

ABSTRACT. A mathematical model for studying the 3-D dynamical structure of lee wave across a meso-scale mountain corner has been proposed for a mean flow with realistic vertical variation of wind and temperature. The basic flow consists of both zonal wind component (U) and meridional component (V), which are assumed to be dependent of height. The Brunt-Vaisala frequency (N) is also assumed to be dependent of height. This model has been applied to the mountain corner, in the North East India, formed by broadly North-South oriented Assam Burma Hills (ABH) and broadly East-West oriented Khasi Jayantia hills (KJH). The model has been solved following the quasi-numerical approach. The perturbation vertical velocity (w') is expressed as a double integral. Three cases have been studied and in all cases the relation between the possible transverse and divergent lee wave numbers (k, l) and also the updraft/downdraft regions associated with lee waves at different heights has been mapped and discussed.

Key words – Corner mountain hills, Lee wave, Quasi-numerical solution.

1. Introduction

It is well-established fact that, weather and climate of a place is crucially influenced by the presence of orographic barriers at that place. Influence of orographic barriers on airflow depends on the scale of the obstacle as well as scale of flow.

In a stably stratified atmosphere an air parcel, displaced vertically, undergoes buoyancy oscillation which gives rise to internal gravity waves (IGW). Now these IGW can propagate vertically to a great distance carrying energy and momentum to higher levels in the atmosphere. Sometimes, they are associated with the

formation of clear air turbulence (CAT). The information about standing waves, which under favourable meteorological conditions form on the lee side of the mountain barrier, is very important for the safety of aviation. Many aircraft accidents reported in mountainous areas are often attributed to the vertical velocities of large magnitude associated with the lee waves. Hence the studies on lee waves associated with air flow across an orographic barrier have an important bearing on the safety of aviation.

The studies of the effect of orographic barrier on air-stream may broadly be divided into two categories. In one category, the orographic barrier has been assumed to have

an infinite extension in the direction normal to the basic flow, so that the flow can be considered to be essentially 2-D. The 2-D mountain wave problem was first studied Lyra (1943) and subsequently by Quency (1947, 1948); Scorer (1949); Sawyer (1960); Sarker (1965, 1966, 1967); De (1973); Sinha Ray (1988); Kumar *et al.* (1998) etc. In another type of theoretical studies, the orographic barrier has been assumed to have finite extension in both directions, *viz.*, along and across the basic flow, due to which the perturbations are essentially of 3-dimensional. The 3-D mountain wave problem was first addressed by Scorer and Wilkinson (1956) and subsequently Wurtele (1957); Crapper (1959); Sawyer (1962); Das (1964); Onishi (1969); Smith (1979, 1980); Dutta *et al.* (2002); Dutta (2003, 2005); Naresh (2007); Das *et al.* (2013, 2016) etc.

In some of the above studies, wind and stability were assumed to be either invariant with height or assumed to have some analytical behaviour with height. Solutions for such studies were essentially obtained by analytical method. In other studies, realistic vertical variation of wind and stability were considered and the solution obtained using quasi numerical or numerical method.

In India studies on the effect of an orographic barrier on airflow have been addressed by Das (1964); Sarker (1965, 1966, 1967); Sarker *et al.* (1978); De (1971, 1973); Farooqui and De (1974); SinhaRay (1988); Dutta (2002, 2005); Naresh *et al.* (2005) etc.

In all the studies the barrier (2-D) or the major ridge axis (3-D) of the barrier has been assumed to be extended broadly either in the East-West (EW) direction or in North-South (NS) direction.

In India in the northeast region, the Khasi-Jayantia hill (KJH) is broadly East-West oriented whereas the Assam-Burma hill (ABH) is broadly North-South oriented and they meet at almost right angle forming a mountain corner to the northeast. It is believed that weather and climate in that region are neither controlled by KJ hill alone nor it is controlled by AB hill alone, rather they may be controlled by their combined effect. To address the problem of this combined effect, one has to investigate the effect of the above mountain corner on airflow and rainfall in that region.

From the foregoing discussion it appears that hardly any study has been made to address the problem on the effect of such a mountain corner on airflow and rainfall in that region.

The objective of the present study is to develop a 3-D dynamical model for lee wave across a meso-scale

mountain corner for a mean flow with realistic vertical variation of wind and temperature and solving it quasi-numerically, similar to Dutta (2005), for the corner effect of KJ hills & AB hills, which has not been addressed so far.

2. Data

Guwahati (26.19° N Latitude and 91.73° E Longitude) is the only Radio-Sonde station to the upstream of ABH. Accordingly for the present study we have used the average of 0000 UTC and 1200 UTC RS/RW data of Guwahati for 30th September, 2001, 26th September, 2003 and 1st June, 2004, have been obtained from Archive of India Meteorological Department (IMD), Pune, India.

3. Methodology

In this section, we shall develop a 3-D dynamical model for lee wave across a meso-scale mountain corner. The proposed model considers a steady state, adiabatic, inviscid, non-rotating and Boussinesq mean flow ($U, V, 0$) across a meso-scale mountain corner. The realistic vertical variation of mean flow has been considered here. Under these assumptions the linearized governing equations are simplified to:

$$U \frac{\partial u'}{\partial x} + V \frac{\partial u'}{\partial y} + w' \frac{dU}{dz} = -\frac{1}{\rho_0} \frac{\partial p'}{\partial x} \quad (1)$$

$$U \frac{\partial v'}{\partial x} + V \frac{\partial v'}{\partial y} + w' \frac{dV}{dz} = -\frac{1}{\rho_0} \frac{\partial p'}{\partial y} \quad (2)$$

$$U \frac{\partial w'}{\partial x} + V \frac{\partial w'}{\partial y} = -\frac{1}{\rho_0} \frac{\partial p'}{\partial z} + g \frac{\theta'}{\theta_0} \quad (3)$$

$$\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} + \frac{\partial w'}{\partial z} = 0 \quad (4)$$

$$U \frac{\partial \theta'}{\partial x} + V \frac{\partial \theta'}{\partial y} + w' \frac{d\theta_0}{dz} = 0 \quad (5)$$

where, U, V, ρ_0, θ_0 , are respectively zonal component and meridional component, density and potential temperature of basic flow and u', v', w', p', θ' are respectively the perturbation part of zonal wind, meridional wind, vertical wind, pressure, density and potential temperature. Since the perturbation quantities u', v', w', p', θ' etc are all continuous functions of

x, y, z hence their horizontal variation may be represented by a double Fourier integral, such as,

$$u'(x, y, z) = \frac{1}{4\pi^2} \int_{-\infty-\infty}^{\infty} \int_{-\infty-\infty}^{\infty} \hat{u}(k, l, z) e^{i(kx+ly)} dk dl$$

where,

$$\hat{u}(k, l, z) = \int_{-\infty-\infty}^{\infty} \int_{-\infty-\infty}^{\infty} u'(x, y, z) e^{-i(kx+ly)} dx dy$$

is the double Fourier transform of $u'(x, y, z)$.

Performing double Fourier transform from (1) to (5) we get,

$$i(kU + lV)\hat{u} + \hat{w} \frac{dU}{dz} = -ik \frac{\hat{p}}{\rho_0} \quad (6)$$

$$i(kU + lV)\hat{v} + \hat{w} \frac{dV}{dz} = -il \frac{\hat{p}}{\rho_0} \quad (7)$$

$$i(kU + lV)\hat{w} = -\frac{1}{\rho_0} \frac{\partial \hat{p}}{\partial z} + g \frac{\hat{\theta}}{\theta_0} \quad (8)$$

$$ik\hat{u} + il\hat{v} + \frac{\partial \hat{w}}{\partial z} = 0 \quad (9)$$

$$i(kU + lV)\hat{\theta} + \hat{w} \frac{d\theta_0}{dz} = 0 \quad (10)$$

Eliminating $\hat{u}, \hat{v}, \hat{p}, \hat{\theta}$ from the equations (6) to (10) we get,

$$\frac{\partial^2 \hat{w}}{\partial z^2} + \frac{1}{\rho_0} \frac{d\rho_0}{dz} \frac{\partial \hat{w}}{\partial z} + \left\{ \begin{array}{l} \frac{N^2(k^2 + l^2)}{(kU + lV)^2} \\ - \left(\frac{k \frac{dU}{dz} + l \frac{dV}{dz}}{kU + lV} \right) \frac{1}{\rho_0} \frac{d\rho_0}{dz} \\ - \left(\frac{k \frac{d^2U}{dz^2} + l \frac{d^2V}{dz^2}}{kU + lV} \right) \\ - (k^2 + l^2) \end{array} \right\} \hat{w} = 0 \quad (11)$$

where, $N = \sqrt{\frac{g}{\theta_0} \frac{d\theta_0}{dz}}$ is the Brunt-Vaisala frequency.

Now putting $\hat{w}(k, l, z) = \sqrt{\frac{\rho_0(0)}{\rho_0(z)}} \hat{w}_1(k, l, z)$ in equation (11), we get vertical structure equation

$$\frac{\partial^2 \hat{w}_1}{\partial z^2} + \{f(k, l, z) - \kappa^2\} \hat{w}_1 = 0 \quad (12)$$

where,

$$f(k, l, z) = \frac{N^2(k^2 + l^2)}{(kU + lV)^2} - \left(\frac{k \frac{dU}{dz} + l \frac{dV}{dz}}{kU + lV} \right) \frac{1}{\rho_0} \frac{d\rho_0}{dz} - \left(\frac{k \frac{d^2U}{dz^2} + l \frac{d^2V}{dz^2}}{kU + lV} \right) + \frac{1}{4\rho_0^2} \left(\frac{d\rho_0}{dz} \right)^2 - \frac{1}{2\rho_0} \frac{d^2\rho_0}{dz^2} \quad (13)$$

and $\kappa^2 = k^2 + l^2$

It is very complicated to solve the equation (12) analytically. So, the equation (12) is solved quasi-numerically for the given wave number vectors (k, l) of all vertical levels. The direction of the zonal wind changes from north to south during winter season at all levels (De, 1973). The solution of (12) is strictly indeterminate unless the values of $f(k, l, z)$ is specified at great height. Therefore it is assumed that above the upper boundary $f(k, l, z)$ is constant. For simplicity, it is also assumed that above the upper boundary $f(k, l, z) = 0$, which is similar to Sarker (1967); Dutta (2005, 2007); Sinha Ray (1988) etc.

Therefore, the approximate solution of the equation (12) in the region $f(k, l, z) = 0$ is of the form

$$\hat{w}_1(k, l, z) = C e^{-\kappa z} \quad (14)$$

where, 'C' is an arbitrary constant. Since the pressure and vertical velocity are continuous function of z .

So, $\hat{w}_1, \frac{\partial^2 \hat{w}_1}{\partial z^2}$ are also continuous function of z in the region $f(k, l, z) = 0$.

$$\therefore \frac{\partial \hat{w}_1}{\partial z} = -\kappa \hat{w}_1 \quad (15)$$

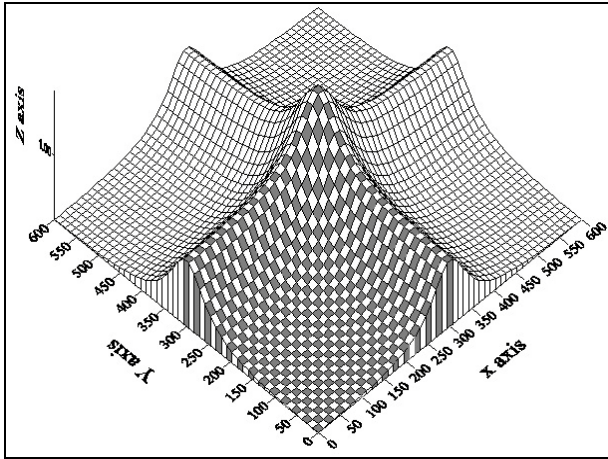


Fig. 1. The profile of corner mountain

Here, the equations (14) and (15) are the upper boundary condition of the equation (12). Now at the surface the airflow follows the contour of the corner mountain, the profile is given by :

$$h(x, y) = \frac{H}{2} \left[\frac{a^2}{a^2 + (x - x_0)^2} + \frac{b^2}{b^2 + (y - y_0)^2} \right] \quad (16)$$

Profile of (15) is given by Fig.1. The chess color of this figure is the corner mountain, which we are interested to study. Now the double Fourier transform of (16) is given by :

$$\hat{h}(k, l) = -\frac{iH}{2} \left[\frac{a}{l} e^{-ak - ikx_0} + \frac{b}{k} e^{-bk - ily_0} \right] \quad (17)$$

Now the linearized lower boundary condition (at $z = 0$) for w'_1 is given by :

$$w'_1(x, y, 0) = w'(x, y, 0) = U(0) \frac{\partial h(x, y, 0)}{\partial x} + V(0) \frac{\partial h(x, y, 0)}{\partial y}$$

Therefore,

$$\hat{w}_1(k, l, 0) = i \{kU(0) + lV(0)\} \hat{h}(k, l) \quad (18)$$

Using the above boundary conditions and following Dutta (2005, 2007) the equation (12) have been solved

quasi-numerically. Therefore, the solution for $\hat{w}_1(k, l, z)$ is given by :

$$\hat{w}_1(k, l, z) = i \{kU(0) + lV(0)\} \hat{h}(k, l) \frac{\psi(k, l, z)}{\psi(k, l, 0)} \quad (19)$$

Therefore,

$$\hat{w}(k, l, z) = \sqrt{\frac{\rho_0(0)}{\rho_0(z)}} i \{kU(0) + lV(0)\} \hat{h}(k, l) \frac{\psi(k, l, z)}{\psi(k, l, 0)} \quad (20)$$

where, $\psi(k, l, z)$ is an arbitrary function, which satisfying equations (14) and (15) and it is value above the upper boundary, $\psi(k, l, z) = 1$. Following Dutta (2005) $\psi(k, l, z)$ has been computed numerically at different vertical levels and different vertical grid points, at intervals of $d = 0.25$ km, for a given wave number vector (k, l) . Hence the perturbation vertical velocity $w'(x, y, z)$ [using inverse Fourier transformation] is given by:

$$w'(x, y, z) = \sqrt{\frac{\rho_0(0)}{\rho_0(z)}} \text{Real part of} \quad (21)$$

$$\left[i \int_0^\infty \int_0^\infty \{kU(0) + lV(0)\} \hat{h}(k, l) \frac{\psi(k, l, z)}{\psi(k, l, 0)} e^{i(kx + ly)} dk dl \right]$$

The above integration is carried numerically for the following range of the values of k and l , k varying from $4 \delta k$ to $20 \delta k$ and l varying from $-20 \delta l$ to $-4 \delta l$ and $4 \delta l$ and $20 \delta l$, where $\delta k = \delta l = 2\pi/4L_{\max}$, where L_{\max} is the maximum horizontal wave length scale. The horizontal grid size Δx or Δy has been taken as 5 km. So, the minimum horizontal wave length of the disturbance that can be resolved by the chosen grid is 30 km.

Now the value of the double integral in equation (21) depends on the character of $\psi(k, l, 0)$ in the range of the integration. For the given value of divergent wave number l , the values of k for which $\psi(k, l, 0)$, corresponding to the transverse lee wave number. Using scale analysis made by Dutta (2005 and 2007) it follows that to ignore the effect

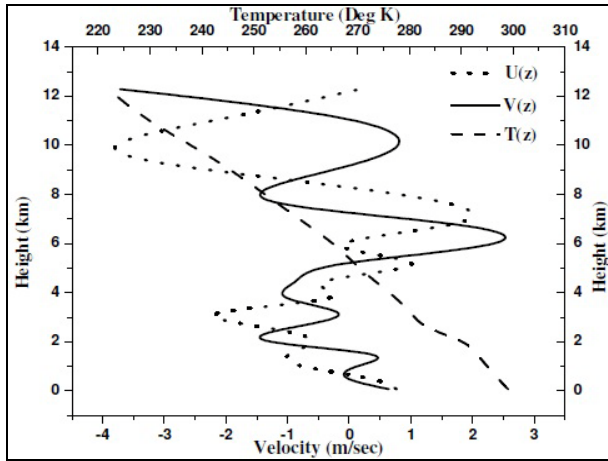


Fig. 2(a) Vertical profile of U, V and T for case 1

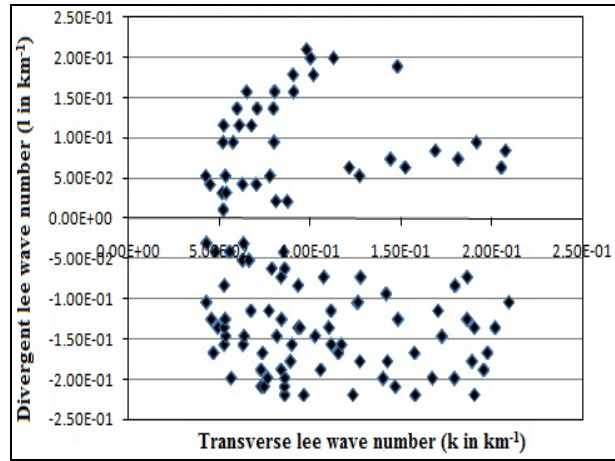


Fig. 2(b). Relation between k and l for case 1

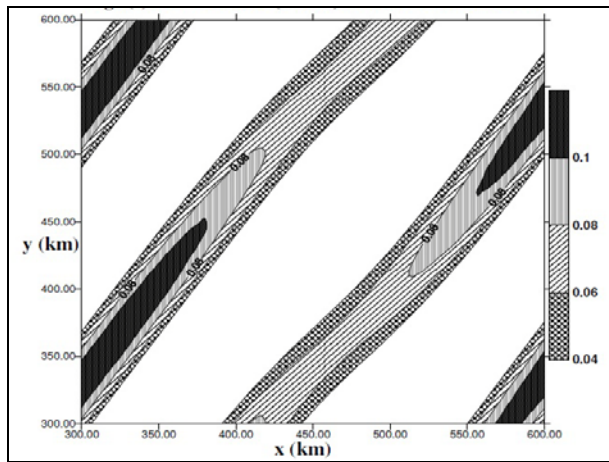


Fig. 2(c). Contour of w' (cm/sec) at 1.5 km level for case 1

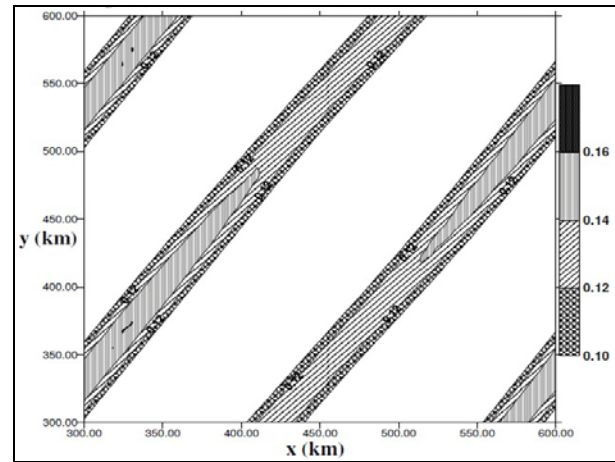


Fig. 2(d). Contour of w' (cm/sec) at 3 km level for case 1

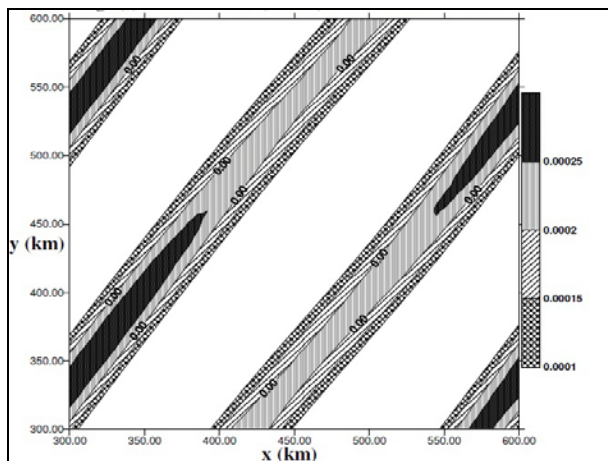


Fig. 2(e). Contour of w' (cm/sec) at 6 km level for case 1

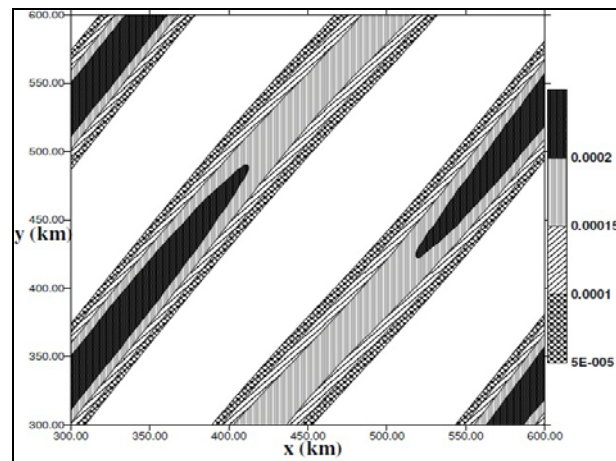


Fig. 2(f). Contour of w' (cm/sec) at 10 km level for case 1

of earth's rotation (Coriolis force), the maximum horizontal extent of the disturbance should not exceed 150 km.

The above expression in equation (21) the perturbation vertical velocity $w'(x,y,z)$ consist of two parts, one is forcing part which significant on the wind ward

side of the corner mountain and another is wave part which is significant on the lee side of corner mountain. In the present study we shall compute only lee wave part.

Now for searching the lee wave part (wave part), we consider a given divergent lee wave number $l = l_0$, the corresponding transverse lee wave number is evaluated by the following method.

Let us consider two consecutive values k_p and k_{p+1} of k , where $\psi(k, l_0, 0)$ differs in sign between k_p and k_{p+1} . Then, there exist for some k'_p ($k_p < k'_p < k_{p+1}$) such that:

$$\psi(k'_p, l_0, 0) = 0$$

Then $k = k'_p$ is the transverse lee wave number for given divergent lee wave number $l = l_0$. Now k_p and k_{p+1} are very closed to each other on k -axis in the wave number plane (k, l), the character of $\psi(k, l_0, 0)$ is approximating linear between k_p and k_{p+1} . Thus k'_p is given by :

$$k'_p = k_p - \frac{(k_{p+1} - k_p)\psi(k'_p, l_0, 0)}{\psi(k'_{p+1}, l_0, 0) - \psi(k'_p, l_0, 0)} \quad (\text{Dutta, 2005})$$

Using this way we have found out the possible transverse lee wave number for a given divergent lee wave number. So, the total contribution of all possible transverse lee waves for all divergent lee waves towards the vertical velocity $w'(x, y, z)$ can be obtained (Dutta, 2005) as:

$$\begin{aligned} w'_{lee\ wave}(x, y, z) = & -\pi H \sqrt{\frac{\rho_0(0)}{\rho_0(z)}} \sum_l \sum_{k'_p} \{k'_p U(0) + lV(0)\} \\ & \left[\frac{a}{l} \exp(-ak'_p) \cos(k'_p x + ly - k'_p x_0) \right. \\ & + \frac{b}{k'_p} \exp(-bk'_p) \cos(k'_p x + ly \\ & \left. - ly_0) \right] \left[\frac{\psi(k'_p, l_0, 0)}{\frac{\partial \psi(k, l_0, 0)}{\partial k}} \right]_{k=k'_p} \delta l \end{aligned} \quad (22)$$

4. Results and discussion

Using the formula (22), the 3-D lee wave numbers associated with vertical velocity (w') has been computed for those atmospheric condition which are favourable to give rise to lee waves across the Corner Mountain hills (CMH) during summer and rainy season. In this model three cases have been studied, where in every case, the graphical relation between the lee wave numbers (k, l) and contours pattern of perturbation vertical velocity (w') have been presented.

Case 1: 30th September, 2001 (Rainy season)

The vertical profile of $U(z)$, $V(z)$ and $T(z)$ are shown in Fig. 2(a), which are based on the average of 0000 UTC and 1200 UTC/RW data of Guwahati (26.19° N Latitude and 91.73° E Longitude) on 30th September, 2001. Dutta (2005) showed that during winter season the profile of $T(z)$ are almost constant lapse rate throughout the modal vertical domain. In the present study the profile of $T(z)$ shows an almost constant lapse rate with vertical and also have found the multi modal character in the profile of $U(z)$ and $V(z)$, which makes the air stream characteristic to be favourable for the occurrence of the lee waves across the CMH in India at that day.

Using the above profile of $U(z)$, $V(z)$ and $T(z)$ we have computed the all possible pairs of lee wave numbers (k, l). The graphical relation between the wave number vectors (k, l) as a scatter diagram has been shown in Fig. 2(b). From this figure it is seen that a given divergent lee wave number (l) can give rise to a number of transverse lee wave number (k) across the CMH in rainy season and k increase with $|l|$ and the wave length of transverse lee waves vary between 31 km to 148 km and corresponding to the range of computed wave length of divergent lee wave is 30 km to 600 km.

The maximum updraft regions of perturbation vertical velocity w' (cm/sec) at 1.5 km, 3 km, 6 km and 10 km above mean sea level, which approximately resemble to 850 hPa, 700 hPa, 500 hPa and 300 hPa respectively are shown in Fig. 2(c-f). From these figures it is seen that the magnitude of maximum updraft is more at 3 km level and have not found any specific shaped and no tilting with vertical also it decreases with vertical. Das *et al.* (2013) have showed that the updraft regions are crescent shaped and tilting with vertical across AB hills at every level during winter season.

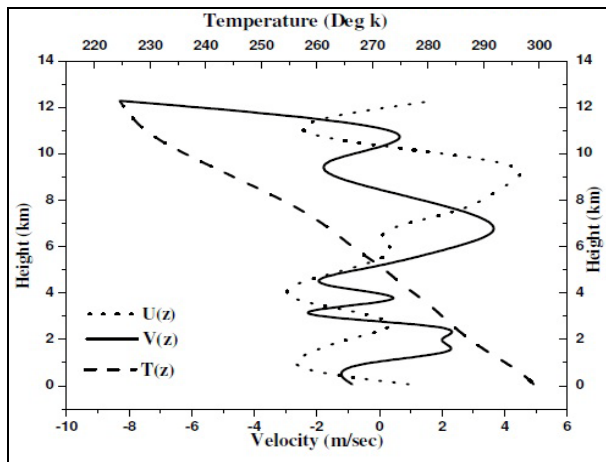


Fig. 3(a). Vertical profile of U, V and T for case 2

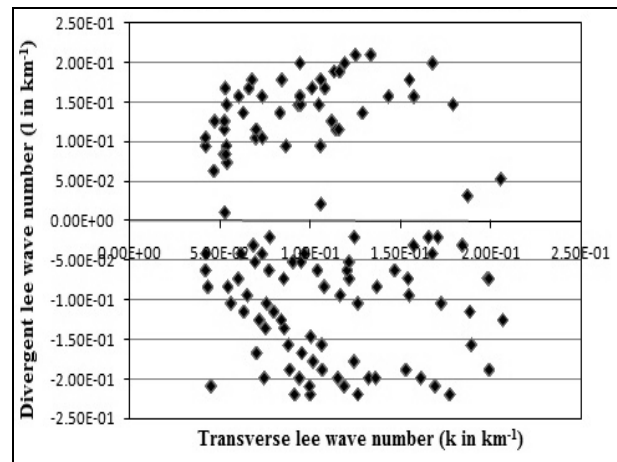


Fig. 3(b). Relation between k and l for case 2

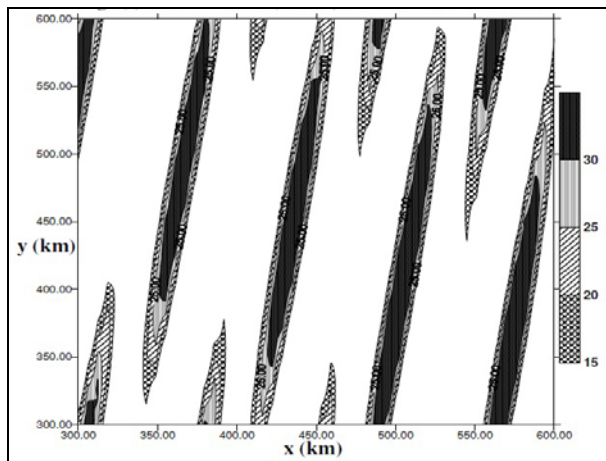


Fig. 3(c). Contour of w' (cm/sec) at 1.5 km level for case 2

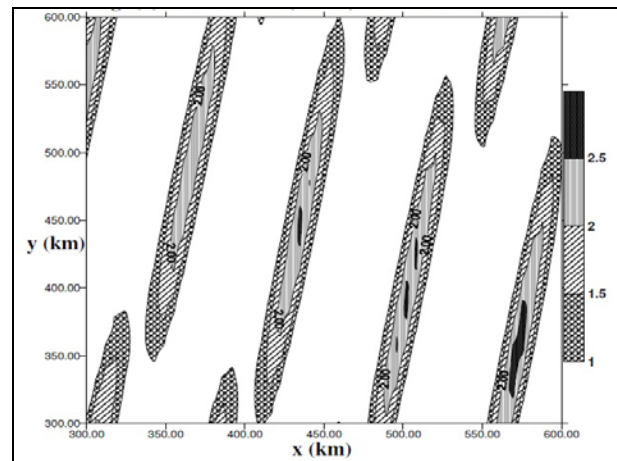


Fig. 3(d). Contour of w' (cm/sec) at 3 km level for case 2

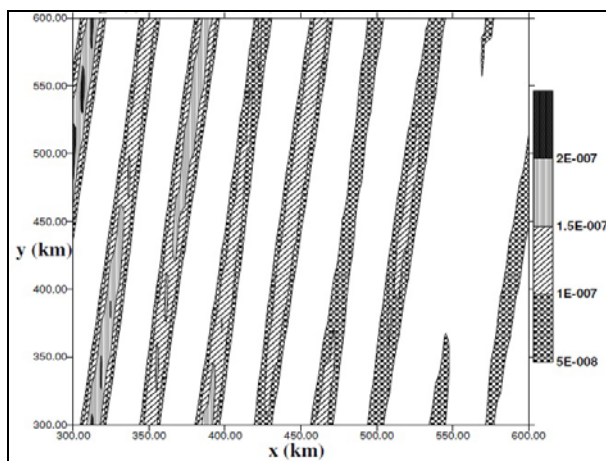


Fig. 3(e). Contour of w' (cm/sec) at 6 km level for case 2

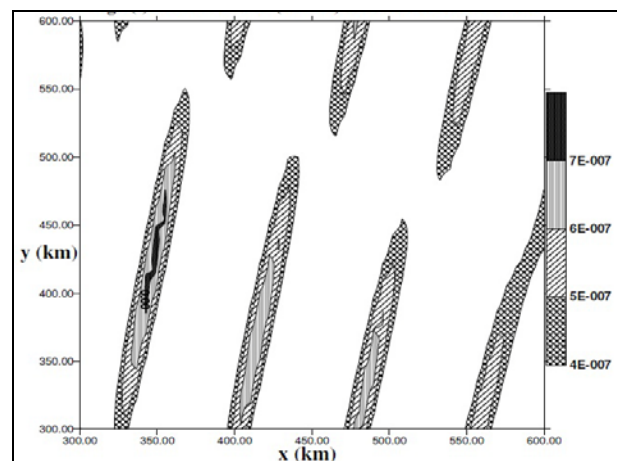


Fig. 3(f). Contour of w' (cm/sec) at 10 km level for case 2

Case 2: 26th September, 2003 (Rainy season)

The vertical profiles of $U(z)$, $V(z)$ and $T(z)$ have been shown in Fig. 3(a), which are based on average

of 0000 UTC and 1200 UTC RS/RW data of Guwahati on that date. In this case the profile of temperature $T(z)$ is constant lapse rate with height and also the temperature $T(z)$ is constant lapse rate with

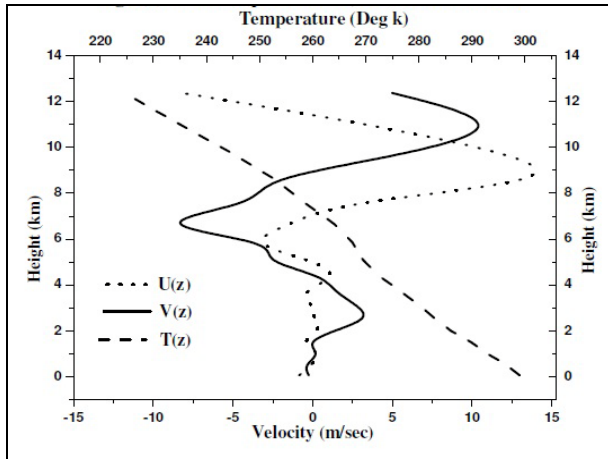


Fig. 4(a). Vertical profile of U, V and T for case 3

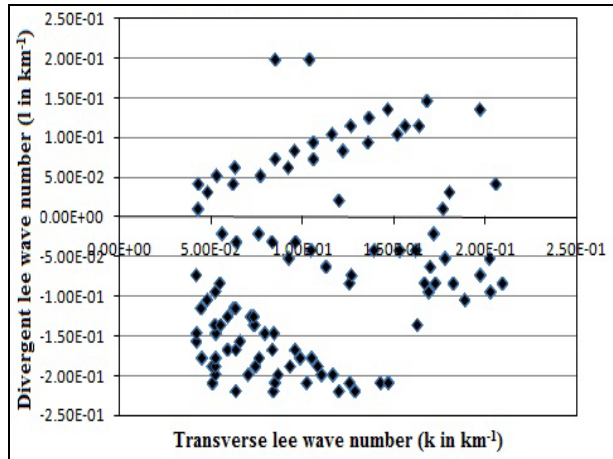


Fig. 4(b). Relation between k and l for case 3

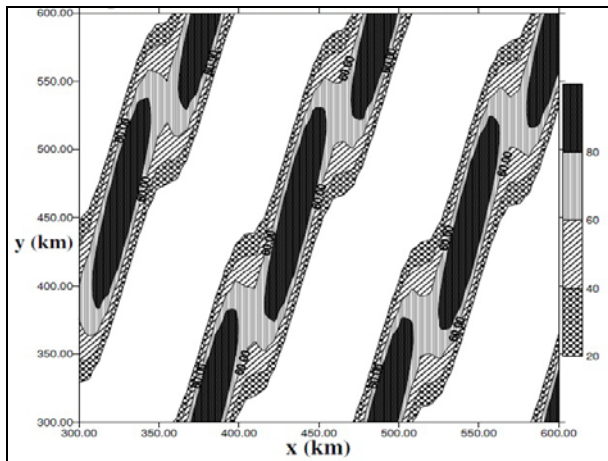


Fig. 4(c). Contour of w' (cm/sec) at 1.5 km level for case 3

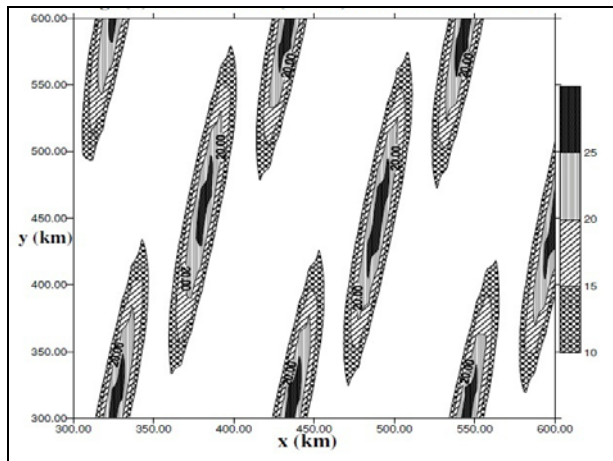


Fig. 4(d). Contour of w' (cm/sec) at 3 km level for case 3

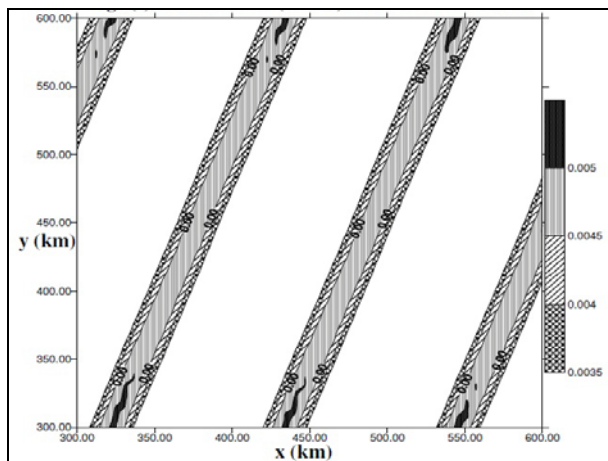


Fig. 4(e). Contour of w' (cm/sec) at 6 km level for case 3

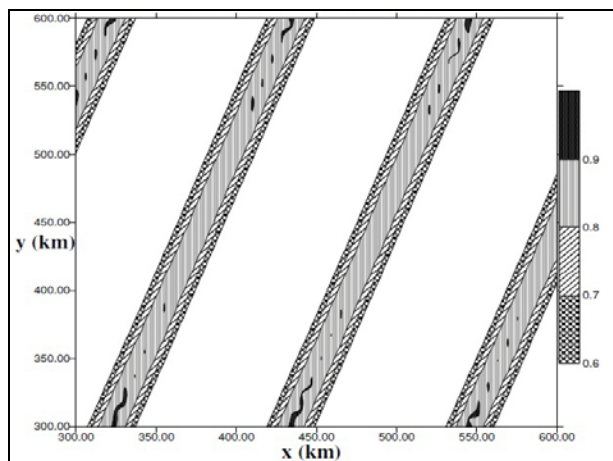


Fig. 4(f). Contour of w' (cm/sec) at 10 km level for case 3

the modal vertical domain. Dutta (2005, 2007) have showed that the vertical profile of wind is favourable for given lee wave.

The graphical relation between lee wave numbers k and l and, using the profile of $U(z)$, $V(z)$ and $T(z)$ as input, is shown in Fig. 3(b). This figure shows that a given

divergent lee wave number (l) can give rise a number of transverse lee wave number (k) across CMH on that date. This also shows k increases with $|l|$ and the wave length of transverse lee wave varies between 30 km to 150 km and corresponding to the range of computed wave length of divergent lee wave is 30 km to 600 km.

The contours of w' (cm/sec) at 1.5 km, 3 km, 6 km and 10 km above mean sea level, which approximately resemble to 850 hPa, 700 hPa, 500 hPa and 300 hPa respectively, are shown in Figs. 3(c-f). Similar to previous case, the magnitude of maximum updraft is more at 1.5 km level and it decreases with vertical and no specific shaped tilting with vertical have seen. Dutta *et al.* (2002) and Dutta (2005) showed that, the maximum updraft regions were approximately crescent shaped across Western Ghats (WG) during winter season.

Case 3: 1st June, 2004 (Summer season)

The profiles of basic flow $U(z)$, $V(z)$ and $T(z)$ in the undisturbed flow are shown in Fig. 4(a). This is based on the average of 0000 UTC and 1200 UTC RS/RW data of Guwahati on that date. The figure shows that the profile of temperature $T(z)$ is constant lapse rate with vertical and the vertical profile of two components of basic flow $U(z)$ and $V(z)$ make the air stream characteristic to be favourable for the occurrence of the lee waves across the CMH during summer season. De (1970, 1971, 1973) also investigated that the airstream characteristic across the AB hills during winter season is favourable for the occurrence of the lee waves.

The graphical relation between k and l as scatter diagram is shown in Fig. 4(b) by using the profile of $U(z)$, $V(z)$ and $T(z)$ as input. From this figure clearly it is seen that for a given divergent lee wave number (l) rises to a number of transverse lee wave (k) on that date across CMH and also k increases with increases $|l|$. Here the wave length of transverse lee waves vary from 30 km to 146 km and corresponding range of divergent wave length is from 31 km to 600 km.

Figs. 4(c-f) show the contours of perturbation vertical velocity w' (cm/sec) at 1.5 km, 3 km, 6 km and 10 km above mean sea level, which approximately resemble to 850 hPa, 700 hPa and 500 hPa respectively. Similar to case 1 and case 2, in this case also it is seen that the magnitude of maximum updraft is at 1.5 km level and it decrease with vertical and no specific tilting with vertical.

5. Conclusions

In this investigation, we have presented a mathematical model for the 3-D dynamics of lee wave across a meso-scale mountain corner following quasi numerical approach. In the sequel, we have made some interesting observation. Moreover,

(i) All the above cases the temperature $T(z)$ is constant lapse rate with vertical and the profile of $U(z)$, $V(z)$ shows the multi modal character, which makes the air stream characteristic to be favourable for the occurrence of the lee waves across the CMH during summer as well as rainy season.

(ii) During the summer and rainy season, when atmosphere is strongly stratified, a given divergent lee wave (l) gives to rise to a number of transverse lee wave numbers (k) across the Corner Mountain hills (CMH) and also k increases with $|l|$.

(iii) Strength of the updraft in all the cases found to be more at lower levels and decaying rapidly with vertical thereafter.

(iv) No specific shapes of the updraft regions have been found.

(v) No specific tilting or lateral spreading with vertical have been found.

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