# **Modeling and forecasting of metrological factors using ARCH process under different errors distribution specification**

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सार – विभिन्न मौसम की घटनाओं का उच्च परिशुद्धता के साथ मॉडल तैयार करना और पूर्वान्**मान कर**ना कठिन होता है। इस अध्ययन ने विभिन्न प्राचलों अर्थात् अधिकतम और न्यूनतम तापमान, सुबह और शाम की सापेक्ष आर्द्रता को प्राचलिक मॉडल अर्थात् स्वसमाश्रयी समेकित गतिमान माध्य (ARIMA) और सामान्यीकृत स्वसमाश्रयी कंडीशनल हेट्रोसेकेडसिटी (GARCH) मॉडल का उपयोग करके पूर्वान्**मानित किया है। आंकड़ों में जनवरी, 1996** से नवंबर, 2019 तक के मध्य प्रदेश के होशंगाबाद जिले के दैनिक समय श्रृंखला आँकड़े शामिल थे। AIC और BIC कसौटी का उपयोग प्रतियोगी मॉडलों में चयन के लिए किया गया था। वर्तमान जांच से पता चला है कि ARIMA-GARCH मॉडल न्यनू तम तापमान**,** अधधकतम तापमान औि सापेक्ष आर्द्ाता के पिू ाानमु ान के भलए अधधक उपयक्ुत हैं।

**ABSTRACT**. Various weather phenomenon are difficult to model and forecast with high precision. This study has modelled and forecasted the various parameter namely maximum and minimum temperature, morning and evening relative humidity using parametric models namely Autoregressive Integrated Moving Average (ARIMA) and Generalized Autoregressive conditional heteroskedasticity (GARCH)) models. The data consisted of daily time series data for Hoshangabad district of Madhya Pradesh from January, 1996 to November, 2019. The AIC and BIC criterion were used to select among competing models. Present investigation has revealed that ARIMA-GARCH models are more suitable for forecasting of minimum temperature, maximum temperature and relative humidity.

**Key words** – Weather, Time series, Error distribution, ARCH, GARCH.

### **1. Introduction**

Weather factors play vital role in production of crops; if weather factors (*viz*., rainfall, maximum temperature, minimum temperature, etc.) are not favorable to the crop, then crop cannot be grown well (Kaul, 2006). The associated impacts of high temperatures, changing patterns of precipitation and possible increased frequency of extreme events such as drought and floods would combine to reduce yields and increase risks in agricultural production in several parts of the globe. In India, agricultural production is often determined by the whims of nature. Information about seasonal forecast of weather allows farmers to make planning and management strategies for improvement in production of crops. Humidity is important to make photosynthesis possible and temperature a plant on a sunny day is mainly regulated by cooling through evaporation. The changes in the average temperature of the earth can translate into large shifts in the climate and weather. Martínez *et al.* (2012) estimated and forecasted temperature and humidity by using artificial neural network (ANN) for tobacco dry processing. Smith *et al.* (2009) predicted the air temperature by using time series model. Shukla *et al.*  (2014) used GARCH model to forecast the climate parameters temperature, maximum temperature and relative humidity.

The key in forecasting nowadays is to understand the different forecasting methods and their relative merits and so be able to choose which method to apply in a particular situation. Though modeling and forecasting of phenomena has a long history, its application, especially in the field of weather forecasting become substantially visible during the latter half of the last century. It got further boost with the introduction of Box - Jenkins methodology, GARCH and ARCH is particularly in time series forecasting, followed by availability and wide use of computer softwares. Several researchers have applied SARIMA model to rainfall data of various locations and found satisfactory results (Akpanta *et al*., 2015; Wanga *et al*., 2013; Murthy *et al*., 2018; Dabral and Murry, 2017). The next paragraph discusses the noteworthy applications of Box-Jenkins and ARCH family models to other fields. Mishra *et al.* (2017 & 2018) compared time series models (ARIMA, GARCH and ARCH) and used for forecasting Time series data is usually fraught with non stationarity and volatility. Keeping in view the wide popularity of these methods of forecasting in predicting weather and non-weather data, this study was carried out with following specific objectives:

(*i*) Identification and comparison of suitable forecasting models based on model selection criteria for weather parameters.

(*ii*) Developing a local scale statistical model enabling to predict certain surface meteorological parameters, crucial for agricultural operation and validation of the models.

### **2. Methods and data**

The time series data on monthly maximum and minimum temperature, morning and evening relative humidity in percent (R.H.) for Hoshangabad district of Madhya Pradesh from January, 2017 to November, 2019 daily data was used for the present study. Time series data was collected from Gramin Krishi Mausam Seva (GKMS) from ZARS, Powarkhada, J.N.K.V.V. The series contains 8491 data points, out of which 8401 data points (January to December 1996-2018) were used for model building and 90 points (January to December 2019) were kept for validation. An area comes under Central Narmada Valley Agro Climatic Zone [Fig. 1(a)]. Average rainfall is 1243 mm during 2017-18.The on set of monsoon started on  $1<sup>st</sup>$  week of June and decreased on  $30<sup>th</sup>$  September. Maximum rain was received in the month of July and August.

Since the work of Mandelbrot (1963) and Fama (1965), it is known that the "good" statistical assumptions retained in traditional models are sometimes caught in

default in finance. Friedman and Vandersteel (1982); Bollerslev (1987); Baillie and Bollerslev (1989) and more recently Chan and Grant (2016); Silva *et al.* (2019); Peter et al.,(1991);Francq and Zakoian (2019) have shown that the assumption of normality and homoscedasticity of the error term relating to models on financial assets is frequently rejected.

Weiss (1984) proposed ARMA models with ARCH errors. This approach is adopted and extended by many researchers for modeling economic time series (Hauser and Kunst, 1998; Karanasos, 2001). In the field of geosciences, Tol (1996) fitted a GARCH model for the conditional variance and the conditional standard deviation, in conjunction with an AR (2) model for the mean, to model daily mean temperature. In this study, ARMA-GARCH error (or, for notation convenience, called ARMA-GARCH) model was selected for modeling daily streamflow processes.

In the basic  $AR(1)$  model,

$$
y_t = \alpha + \phi_1 y_{t-1} + \varepsilon_t
$$
; where,  $\varepsilon_t \sim N(0, \sigma_{\varepsilon}^2)$ 

where, it is assumed that  $\sigma_{\varepsilon}^2$  was constant. This is a restrictive approach. In finance and financial engineering,  $\sigma_{\varepsilon}^2$  (standard deviation) is a central variable that corresponds to the concept of volatility. In general,  $\sigma_{\varepsilon}^2$  is assumed to be constant, which is not always satisfactory. Experience has shown that volatility can fluctuate significantly. In practice, we estimate  $\sigma_{\varepsilon}^2$  over short periods, which make it possible to indirectly incorporate a form of change. Following the pioneer work of (Engle, 1982), through it proceed to a modification of the AR (1) model to take account of a volatility which changes over time according to a well-defined approach.

With a brief rappel, the mean of an (1), is :  $(y_t)$  $1 - \phi_1$  $E(y_t) = \mu = \frac{\alpha}{1 - \phi_1}$  and the variance is :  $(y_t) = \frac{U_{\varepsilon}}{2}$ .  $1 - \phi_1^2$ 2  $\phi_1$  $\sigma_{\varepsilon}$  $V(y_t) = \frac{\sigma_{\varepsilon}}{1 - \phi_1^2}$ . Furthermore, by supposing  $y_{t-1}$  is known,

the conditional mean of this process is given by :  $E(y_t / y_{t-1}) = \alpha + \phi_1 y_{t-1}$ , which clearly depends on the previous information in time  $t - 1$ , so it's not necessarily constant over time; in contrast the conditional variance was considered constant (does not depends of time scale),

$$
Var[y_t | y_{t-1}] = E\{ [y_t - E(y_t | y_{t-1})]^2 | y_{t-1} \}
$$
  
= 
$$
E[(\varepsilon_t)^2 | y_{t-1}] = \sigma_{\varepsilon}^2
$$



**Fig. 1(a).** Area comes under central Narmada valley agro climatic zone



Fig. 1(b). Pattern evolution of the four time series over the period of study

In fact, this result is generated by the hypothesis  $\varepsilon_t \sim N(0, \sigma_{\varepsilon}^2)$ ; which is a more restricted one, it needs another procedure that take in consideration the dependence of the conditional variance, the ARCH family models is a better approach for this situation.

For the mean equation, the ARIMA (*p*, *q*) process had been used, where in identification step, the study used Akaike's information criterion is written as follows:

$$
AIC = 2k - 2 \ln(k)
$$

where,  $\boldsymbol{k}$  is the number of parameters to be estimated from the model and  $\bf{k}$  is the maximum of the likelihood function of the model. Whereas, Burnham & Anderson (2002) strongly recommend the use of the AICc instead of the AIC if  $\boldsymbol{n}$  is small and / or  $\boldsymbol{k}$  large,

$$
AICc = AIC + \frac{2k(k+1)}{n-k+1}
$$

It is to be noted further that the AICc tends towards the AIC when *n* becomes large. The AICc was first proposed by Hurvich and Tsai (1989); Brockwell & Davis (1991) recommend using the AICc as the primary criterion for the selection of ARMA time series models. McQuarrie & Tsai (1998) confirmed the interest of AICc using a large number of regression and time series simulations.

Definition 2.1 an  $ARCH(q)$  process is given as :

$$
\sigma_t^2 = \beta_0 + \sum_{i=1}^q \beta_i \varepsilon_{t-i}^2 = \beta_0 + \beta(L)\varepsilon_t^2
$$

where,  $\beta_0 > 0$  and  $\beta_i \ge 0 \forall i$ . to guarantee that the unconditional variance be positive. The ARCH (*q*) model makes it possible to take into account the volatility clustering, (Lu & Marchesi, 1998), *i.e*., the strong (respectively the weak) variation of  $(y_t)$  are followed by other strong (weak) variations, this notion is firstly developed by Mandelbrot (1963).

### **3. Estimation of ARCH (***p***) model**

Estimates can be obtained from the maximum likelihood method, Engle (1982); Pantula (1985**)**, or by the generalized moments method. The identification of the order of the ARCH (*q*) model can also be obtained using the traditional tests used for the identification of AR processes by proceeding in two steps.

It is to be noted that the conditional likelihood function of an AR $(1)$  is given by :

$$
\ln L(\phi, \sigma^2 | y) = -\frac{T-1}{2} \ln(2\pi) - \frac{T-1}{2} \ln \sigma^2 - \sum_{t=2}^T \frac{(y_t - \phi_{t-1})}{2\sigma^2}
$$

In the case of ARCH errors, we have and  $\epsilon_t \sim N(0, \sigma_{\varepsilon}^2)$  and the likelihood function can easily be changed by replacing  $\sigma_{\varepsilon}^2$  with  $s_t$ .

$$
\ln L(\phi, \alpha_0, \alpha_1 | y) = -\frac{T - 2}{2} \ln(2\pi) - \frac{1}{2} \sum_{t=3}^{T} \ln h_t
$$

$$
- \sum_{t=3}^{T} \frac{(y_t - \phi y_{t-1})}{2h_t}
$$

### **4. The errors' distributions**

Early achievements in financial time series modeling by ARCH / GARCH were limited to the case of Normal errors, for which an explicit conditional probability function is readily available to promote parameter estimation in the model, Hall and Yao (2003). Investigation of non-Normal cases has been partly driven by empirical evidence that financial time series can be very heavytailed [Mittnik *et al*. (1998); Mittnik and Rachev (2000)]. It should specify the form of the conditional distribution for the errors, most frequently used distribution (or the default) is Normal (Gaussian), the Student's *t*, the Generalized Error (GED), the Student's *t* with fixed d.f., or the GED with fixed parameter.

### **5. Results and discussion**

The data description is shown in Table 1, where a summary statistics of the central tendency (mean, median), the dispersion and the form data distribution were mentioned into. Over the study period, in average the minimum temperature and the maximum temperature were (respectively) 19.47 °C and 31.7 °C. Relating to the evaporations measures in morning and evening, in average the estimation was about 72.67% and 49.59% (respectively). For the dispersion statistics, we see that the min and max temperature have weak variations compared to the evaporations measures. The form of the data distribution for all time series did not show any normally patterns (according to the Skewness, Kurtosis statistics).

From Fig. 1(b) it can be observed that all the four time series are stationary, after testing the hypothesis of unit roots by using the Augmented Dickey Fuller (ADF), Phillips-Perron (PP) and KPSS unites roots tests, it was confirmed that the time series are stationary except the Morning evaporation time series, which is integrated of first order; detailed results are presented in Table 1. There was no trend effect in all the four time series (and



# **Fig. 2.** The first difference of the time series plots over the study period. *Source* : plots using Eviews program

### **TABLE 1**

#### **Summary statistics of data**



*Notes* : SD : Sample Standard deviation. According to skewness, kurtosis and Jarque-Bera statistic test, we reject the normality of data distribution for all time series



### **Results of Augmented-Dickey-Fuller (ADF), Phillips-Perron (PP) and KPSS unites roots tests**

*Source* : Own estimation using Eviews and R programs. (\*\*) We use the Augmented Dickey-Fuller (ADF) to take account the autocorrelation n::n error terms. (\*\*): The b KPSS (1992) test differs from the other unit root tests by assuming trend-stationary time series under the null hypothesis (so, if the according *p.value* is greater than the choosing significance levels (1%, 5%, 10 %...), the decision is: the time series is stationary; which is the case for this study

#### **TABLE 3**

#### **ARIMA (2, 0, 3) – ARCH (1) models results for the maximum temperature time Series**



*Source* : Own estimation using Eviews software. Here we selected the normal distribution for the conditional errors process. Before estimated variance equation, the estimate coefficient of the  $MA(2)$  was statistically significant. Note:  $\mathbf{\varepsilon}_{t-1}^2$  is for the **ARCH**(1) estimate coefficient

this was well validated by the unit root tests where "trend" component is not significant for these time series (Table 2).

## **6. ARCH models results**

### 6.1. *Identification of the mean equation using ARIMA process*

To highlight some of the difficulties of ARCH models given in this section. Starting from an initial ARMA representation (Tables 3-6), the study has gradually introduced into the more complex model with hypotheses taking into account the heteroskedasticity of the residues or their non-normality. Heteroscedasticity is taken into account by the introduction of an ARCH effect (Variance equation), while non-normality is incorporated using a thick-tailed distribution, the Student's *t*-distribution. After identification of mean equations of the time series, study proceeded to test for the presence of ARCH effect by using the LM test, it is the test of significance of the coefficients of the regression  $\varepsilon_t^2$  on  $\varepsilon_{t-q}^2$ , which allows to determine the order **q** of the ARCH process knowing that an ARCH process of order 3 seems



**Fig. 3.** Conditional variance plots of the four time series [*Source* : Plots using Eviews software (10 Version)]

a maximum, beyond that, the model will be justifiable from a GARCH (Generalized ARCH) type process. We cite also the test developed by Breusch and Pagan (1979)'s LM test (BP) and the White (1980)'s general test, the tests statistic are calculated under the null hypothesis (H0: *No* heteroscedasticity); furthermore, these tests are based on OLS residuals.

For our study, because the *p*-value is <0.05, we reject the null hypothesis and conclude the presence of ARCH (1) effects.

Through our selection of distributions, we observed that the coefficients in the equation of the mean have decreased slightly but remain very close to those in Tables (3-6). The estimates of ARCH (*q*) process with selected distributions are presented in Tables (3-6). In general, all estimates of parameters are significantly different from zero at five percent level of significance level in all models and their magnitudes are fairly close across different models. To compare the models, both log likelihood and Akaike Information Criterion (AIC) are presented.

### 6.2. *Validation of estimated models and forecasting results*

For the validation step, the following three aspects of the residuals from fitted GARCH model should be tested :

The standardized residuals from the GARCH models should approach normal distribution (if we assumed the conditional distribution of errors terms as normal distribution). For this point, this study used Shapiro-Wilk (S-W) test and Jarque-Bera normality test. Histogram of the residuals and quantilequantile (Q-Q) plots were also used as visual tool to check normality.

In the present case, the residuals didn't follow a normal distribution because the p-values of Shapiro-Wilk (S-W) test and Jarque-Bera were close to zero:  $p$  – value = 0.000; both below the level of significance  $\alpha = 5\%$ ; furthermore, it is observed here clearly that normality assumption was rejected based on the *Q-Q* plot (Fig. 5).

A *second step* for validation of GARCH modeling is to check that the standardized squared residuals



**Fig. 4.** ACF plots of estimated models (for detecting the remaining of auto-correlation in the Squared Standardized Residuals). *Source* : Plotting from R program

**ARIMA (2, 1, 1) – GARCH (1, 2) models results with a** *t***-student errors distribution for the evening evaporation time series**

Variable	Coefficient	Std. Error	z-Statistic	Prob.			
<b>Mean Equation</b>							
AR(1)	1.569021	0.033112	47.38499	0.0000			
AR(2)	$-0.570978$	0.032911	$-17.34918$	0.0000			
MA(1)	$-0.941486$	0.036274	$-25.95465$	0.0000			
MA(2)	0.107544	0.021307	5.047261	0.8062			
<b>Variance Equation</b>							
$\mathcal{C}$	0.869745	0.16758	5.190032	0.0000			
$e_{t-1}^2$	0.358683	0.033595	10.67653				
$e_{i-2}^2$	$-0.192248$	0.03158	$-6.087642$				
GARCH(1)	0.864372	0.008788	98.35275	0.0000			
<b>Fitted Adequacy</b>							
R-squared	0.832814	Log likelihood	$-29944.53$				
<b>Adjusted R-squared</b>	0.832755	Durbin-Watson stat	2.08021				

*Source* : Own estimation using Eviews software. Where, we selected the *t*-student distribution for the conditional errors process. Note:  $\mathbf{\varepsilon}_{t-1}^2$  and  $\mathbf{\varepsilon}_{t-2}^2$  are for the ARCH(1) and ARCH(2) estimate coefficients



**Fig. 5.** Q.Q plots of estimated models (for detecting the normality assumption of the conditional distributions). *Source* : Plotting from R program

**ARIMA (2, 0, 1) – ARCH (1) models results with generalized error distribution for the minimum temperature time series**

Variable	Coefficient	Std. Error	z-Statistic	Prob.			
<b>Mean Equation</b>							
AR(1)	1.485814	9.49E-07	1565856	0.0000			
AR(2)	$-0.48598$	0.000194	$-2504.083$	0.0000			
MA(1)	$-0.784435$	0.006614	$-118.595$	0.0000			
<b>Variance Equation</b>							
$\mathsf{C}$	2.519735	0.016807	149.9252	0.0000			
$e_{i+1}^2$	0.20461	0.007925	25.81689	0.0000			
<b>Fitted Adequacy</b>							
R-squared	0.91713	Log likelihood	$-18046.1$				
Adjusted R-squared	0.91711	Durbin-Watson stat	2.173431				

*Source* : Own estimation using Eviews software. Here we selected the generalized error distribution distribution for the conditional errors process. Note:  $\mathbf{\varepsilon}_{t-1}^2$  is for the ARCH (1) estimate coefficient

should not be auto-correlated. This study has used a Box & Pierce (1970) and Ljung & Box (1978) statistics tests for this purpose. It can be observed from the ACF of squared standardized residuals (Fig. 4), which are not auto-correlated, because all autocorrelation terms are inside the confidence intervals; the same result was found for ACF of standardized residuals.



**Fig. 6.** Plots of forecasting results. *Source*: Plotted by using R program. In the X-axe the range (8300-8500) corresponds to the number of values from time series included in plot, here we include the past 200 observations. The lines in bleu are the forecasting where the dashed in grey correspond to the prediction intervals , the dark gray one is for the 80% levels and the light gray is for the 95%





*Source* : Own estimation using Eviews software. Here we selected the normal distribution for the conditional errors process. Note:  $\epsilon_{t-1}^2$  is for the ARCH(1) estimate coefficient





*Source* : Own estimation using R program.

*Note* : M.F: it's the mean forecast of the time series, *i.e.*, the ARIMA models part. L.I and U.I : are-respectively the lower and upper Intervals of the mean forecast (for risque  $\alpha = 0.05$ ). C.S.D: is the conditional standard deviation (which is the predictive volatility) generated by the GARCH models

A third step for validating GARCH model is to run the ARCH-LM test on the residuals. This test can also be conducted to check for remaining ARCH effects in the residuals; for our estimating results, there is no ARCH effects (the  $p$  – value of L-M tests for the four models is upper than 0.7), so alternate hypothesis of presence of heteroscdacity is reject. Forecasting results are presented in Table 7, we noted here that ARIMA models are mainly efficient for the short term forecasting, so we are limited our forecasting for 10 days, however the shorter the term, the more accurate forecasts.

### **7. Conclusions**

Time series models are crucial tool for future projection as well as policy implication. Present investigation has compared the forecasting performance of ARIMA, GARCH and ARCH models using weather data for Hoshangabad districts. These results shows

that predicted future value of the monitored variables and the estimated actual value of these variables using a fitting GARCH as proposed. The results revealed absence of trend in weather time series data along with volatility which is a significant finding. Considering the impetus of knowing about the behavior of weather in advance will provide right path for farmers in deciding right management about their strategies for improvement in production of crops and government about their interventions. This result also highlights the inherent volatility which is important for climate change studies. This result would enable the local research station to better capture the volatility in weather while predicting the weather situation. The enhanced accuracy from weather prediction would also help farmers in better decision making on farm.

**Conflict of interest** : The authors declare that they have no conflict of interest.

**Disclaimer :** The contents and views expressed in this research paper/article are the views of the authors and do not necessarily reflect the views of the organizations they belong to.

#### **Reference**

- Akpanta, A. C., Okorie, I. E. and Okoye, N. N., 2015, "SARIMA modelling of the frequency of monthly rainfall in Umuahia, Abia state of Nigeria", *Amj. Math. Stat*., **5**, 82-87.
- Baillée, R. T. and Bollerslev, T. 1991, "Intra Day and Inter Market Volatility in Foreign Exchange Rates", *R. of Economies Studies*, 565-585.
- Bollerslev, T., 1987, "Generalized autoregressive Conditional Heteroskedasticity", *Journal of Econometrics,* **31**, 307-327.
- Box, G. E. and Pierce, D. A., 1970, "Distribution of residual autocorrelations in autoregressive-integrated moving average time series models", *Journal of the American statistical Association*, **65**, 332, 1509-1526.
- Burnham, K. P. and Anderson, D. R., 2002, "Model Selection and Multimodel Inference : A Practical Information-Theoretic Approach", Springer-Verlag, 2002 [\(ISBN](https://fr.wikipedia.org/wiki/International_Standard_Book_Number) [0-387-95364-7\)](https://fr.wikipedia.org/wiki/Sp%C3%A9cial:Ouvrages_de_r%C3%A9f%C3%A9rence/0-387-95364-7).
- Chan, J. C. and Grant, A. L., 2016, "Modeling energy price dynamics: GARCH versus stochastic volatility", *Energy Economics*, **54**, 182-189.
- Dabral, P. P. and Murry, M. Z., 2017, "Modelling and forecasting of rainfall time series using SARIMA", *Environ. Proces*., **4**, 399-419.
- Engle, R. F., 1982, "Autoregressive Conditional Heteroscedasticity with Estimates of the Variance ofUnited Kingdom Inflation", *Econometrica*, **50**, 987-1008.
- Fama, E., 1965, "The Behaviour of Stock Market Priées", *J. of Business*, 34-105.
- Francq, C. and Zakoian, J. M., 2019, "GARCH models: structure, statistical inference and financial applications", John Wiley & Sons.
- Friedman, D. and Vandersteel, S., 1982, "Short-Run Fluctuations in Foreign Exchange Rates : Evidence firm Data 1973-1979", *J. of International Economies*, 171-186.
- Hall, P. and Yao, Q., 2003, "Inference in ARCH and GARCH Models with Heavy - Tailed Errors", *Econometrica*, **71**, 285-317. do[i:10.1111/1468-0262.00396.](https://doi.org/10.1111/1468-0262.00396)
- Hauser, M. A. and Kunst, R. M., 2001, "Forecasting high*-*frequency financial data with the ARFIMA-ARCH model", *Journal of Forecasting*, **20**, 7, 501-518.
- Hurvich, C. M. and Tsai, C. L., 1989, "Regression and time series model selection in small samples", *Biometrika*, **76**, 2, 297-307.
- Karanasos, M., 2001, "Prediction in ARMA models with GARCH in mean effects", *Journal of Time Series Analysis*, **22**, 5, 555-576.
- Kaul, S., 2006, "Economic Analysis of Productivity of Rice Production-State-wise Analysis", presented at the 14<sup>th</sup> Annual Conference of Agricultural Economics Research Association, held during September 27-28, 2006 at G. B. Pant University of Agriculture.
- Ljung, G. M. and Box, G. E., 1978, "On a measure of lack of fit in time series models", *Biometrika*, **65**, 2, 297-303.
- Mandelbrot, B., 1963, "The Variation of Certain Speculative Prices", *The Journal of Business*, **36**, 4, 394-419.
- Martínez Martínez, V., Baladrón, C., Gomez Gil, J., Ruiz Ruiz, G., Navas Gracia, L. M., Aguiar, J. M. and Carro, B. 2012, "Temperature and relative humidity estimation and prediction in the tobacco drying process using artificial neural networks", *Sensors*, **12**, 10, 14004-14021.
- McQuarrie, A. D. and Tsai, C. L., 1998, "Regression and time series model selection", *World Scientific*, https://doi.org/10.1142/3573.
- Mishra, P., Padmanaban, K. and Dwivedi, Meenakshee, 2017, "Modelling and Forecasting of Black Pepper Production in India", *Indian Journal of Ecology*, **44**, 4, 741-745.
- Mishra, P., Padmanaban, K., Dhekale, B. S. and Tailor, A. K., 2018, "Statistical investigation of production performance of cumin in India", *Economic Affairs*, **63**, 2, 547-555.
- Mittnik, S. and Rachev, S. T., 2000, "Stable Paretian Models in Finance", New York : Wiley.
- Mittnik, S., Rachev, S. T. and Paolella, M. S., 1998, "Stable Paretian Modeling In Finance : Some Empirical and Theoretical Aspects", In a Practical Guide to Heavy Tails, Bosotn: Birkhäuser, 79-93.
- Murthy, K. V. N., Saravana, R. and Vijaya, K., 2018, "Modeling and forecasting rainfall patterns of southwest monsoons in North-East India as a SARIMA process", *Meteorol. Atmos. Phys*., **130**, 99-106.
- Pantula, S. G., 1985, "Estimation for autoregressive processes with several unit roots", North Carolina State University, Deptt. of Statistics.
- Peter, J. Brockwell and Richard, A. Davis, 1991, "Time Series : Theory and Methods", Springer, [\(ISBN](https://fr.wikipedia.org/wiki/International_Standard_Book_Number) [0387974296\)](https://fr.wikipedia.org/wiki/Sp%C3%A9cial:Ouvrages_de_r%C3%A9f%C3%A9rence/0387974296).
- Shukla, A. K., Garde, Y. A. and Jain, I., 2014, "Foreca**s**t of weather parameters using time series data", *Mausam*, **65**, 4, 509-520.
- Silva, P. V. J. D. G., Klotzle, M. C., Pinto, A. C. F. and Gomes, L. L., 2019, "Volatility estimation for cryptocurrencies using Markovswitching GARCH models", *International Journal of Financial Markets and Derivatives*, **7**, 1, 1-14.
- Smith, B. A., Hoogenboom, G. and McClendon, R. W., 2009, "Artificial neural networks for automated year-round temperature prediction", *Computers and Electronics in Agriculture*, **68**, 1, 52-61.
- Tol, R. S., 1996, "The damage costs of climate change towards a dynamic representation", *Ecological Economics*, **19**, 1, 67-90.
- Wanga, S., Fenga, J. and Liu, G., 2013, "Application of seasonal time series model in the precipitation forecast", *Math. Comput Model.*, **58**, 677-683.
- Weiss, A. A., 1984, "ARMA models with ARCH errors", *Journal of time series analysis*, **5**, 2, 129-143.