MAUSAM, **75**, 3 (July 2024), 801-814

MAUSAM

DOI : https://doi.org/10.54302/mausam.v75i3.6236
Homepage: https://mausamjournal.imd.gov.in/index.php/MAUSAM

UDC No. 551.577: 551.509.314

Assessment of uncertainty in estimation of rainfall using EV1 distribution with reference to data length

N. VIVEKANANDAN

Central Water and Power Research Station, Pune – 411 024, India

(*Received 6 April 2023, Accepted 23 November 2023*)

e-mail : vivek.n@cwprs.gov.in

सार – जलीय मॉडल के लिए प्रत्यागमन के लिए वर्षा का आकलन एक आवश्यक इनपुट माना जाता है जिसका उपयोग अभिकल्पनाविसर्जन का आकलन करने के लिए किया जाता है, जो सिविल इंजीनियरिंग बुनियादी ढांचा परियोजनाओं जैसे कि सड़कों, पुलों, हरित राजमार्गों आदि की योजना और अभिकल्पना के लिए आवश्यक है। इसका आकलन किसी क्षेत्र के कई स्टेशनों पर दर्ज वर्षा डेटा का उपयोग करके किया जा सकता है। अभिकल्पना वर्षा आकलनों में अनिश्चितताएं विभिन्न स्रोतों जैसे डेटा त्रुटि, प्रतिचयन त्रुटि, क्षेत्रीयकरण त्रुटि, मॉडल त्रुटि इत्यादि से उत्पन्न होती हैं। इसके अलावा, मॉडल त्रुटि में, डेटा की अधिकता वर्षा के आकलन में त्रुटि की अनिश्चितता का निर्धारण करने में सीधा प्रभाव डाल रही है। इस शोध का उद्देश्य महाराष्ट्र के पुणे और वडगांव मावल स्थलों के वर्षा आकलनों में अनिश्चितता का आकलन करना है, जिसे वार्षिक 1-दिवसीय वर्षा की अधिकतम श्रृंखला में चरम मान प्रकार -1 (EV1) वितरण को फिट करके चरम मान विश्लेषण (EVA) के माध्यम से किया गया। EV1 के प्राचलों को क्षणों की विधि (MoM), अधिकतम संभावना विधि (MLM), कम से कम वर्गों की विधि (MLS), संभाव्यता भारित क्षणों (PWM) और L-क्षणों की विधि (LMO) द्वारा निर्धारित किया गया, और वर्षा के लौटने की अलग-अलग अवधियों के आकलन के लिए उपयोग किया गया। वर्षा के EVA में उपयोग की जाने वाली विभिन्न डेटा लंबाई के साथ डेटा श्रृंखला की विशेषताओं की जाँच सांख्यिकीय परीक्षणों यथा: वाल्ड-वोल्फोवित्ज़ का प्रयोग यादृच्छिकता के लिए, मैन-व्हिटनी यू-परीक्षण का प्रयोग एकरूपता के लिए और ग्रब्स परीक्षण का प्रयोग डेटा श्रृंखला में बाहरी कारकों की पहचान के लिए किया गया है। वर्षा अनुमान में EV1 वितरण की पर्याप्तता का मूल्यांकन गुडनेस-ऑफ-फिट (जैसे, एंडरसन-डार्लिंग और कोलमोगोरोव-स्मिरनोव) परीक्षणों द्वारा किया गया। इस शोधपत्र में डेटा लंबाई के संबंध में वर्षा आकलन में अनिश्चितता के परिणाम प्रस्तुत किए गए, जिसका अध्ययन महाराष्ट्र के पुणे और वडगांव मावल वर्षामापी स्थलों के लिए EV1 के विभिन्न प्राचल अनुमान तरीकों को लागू करके किया गया। पुणे और वडगांव मावल के EVA परिणामों ने संकेत दिया कि *(i)* डेटा की लंबाई बढ़ने पर अनुमानित वर्षा बढ़ जाती है; *(ii)* डेटा की लंबाई बढ़ने पर अनुमानित वर्षा में मानक त्रुटि कम हो जाती है; और *(iii)* MLM द्वारा दिए गए वर्षा आकलनों में मानक त्रुटि MoM, MLS, PWM और LMO के मानों से कम है।

ABSTRACT. Estimation of rainfall for a given return period is considered as an essential input to a hydrologic model that is used to estimate design discharge, which is needed for the planning and design of civil engineering infrastructure projects, *viz*., roads, bridges, green highways, *etc*. This can be estimated by using the recorded rainfall data over many stations in a given region. Uncertainties in design rainfall estimates arise from various sources such as data error, sampling error, regionalization error, model error, *etc*. Further, in model error, the data length is having direct impact in assessing the uncertainty of error in estimation of rainfall. This paper aims to assess the uncertainty in rainfall estimates of Pune and Vadgaon Maval sites of Maharashtra, which was carried out through extreme value analysis (EVA) by fitting Extreme Value Type-1 (EV1) distribution to the series of annual 1-day maximum rainfall. The parameters of EV1 were determined by Method of Moments (MoM), Maximum Likelihood Method (MLM), Method of Least Squares (MLS), Probability Weighted Moments (PWM) and Method of L-Moments (LMO), and are used for estimation of rainfall for different return periods. The characteristics of data series with different data length considered in EVA of rainfall was examined through statistical tests, *viz*., Wald-Wolfowitz runs test for randomness, Mann-Whitney U-test for homogeneity and Grubbs' test for identifying the outliers. The adequacy of EV1 distribution adopted in rainfall estimation was evaluated by Goodness-of-Fit (*viz*., Anderson-Darling and Kolmogorov-Smirnov) tests. This paper presented the results of uncertainty in rainfall estimation with respect to data length, which was studied by applying different parameter estimation methods of EV1 for Pune and Vadgaon Maval rain-gauge sites of Maharashtra. The EVA results of Pune and Vadgaon Maval indicated that (*i*) the estimated rainfall is in increasing order when data length increases; (*ii*) the standard error in the estimated rainfall is in decreasing order when the data length increases; and (*iii*) the standard error in rainfall estimates given by MLM is less than those values of MoM, MLS, PWM and LMO.

Key words – Anderson-Darling, Extreme value type-1, Kolmogorov-Smirnov, Maximum likelihood method, Rainfall.

1. Introduction

Design rainfall is one of the hydrology quantities that can be used for computing the design flood. The design flood can be used for planning and design of civil engineering structures, *viz*., roads, bridges, green highways, *etc.* (IAEA, 2003). Generally, the hourly rainfall data is considered for designing the storm water drainage around the site and the daily rainfall data is used for generating design basis flood water level at inland sites, which are often situated near a river course. Analysis of rainfall and determination of annual 1-day maximum rainfall would also enhance the management of water resources projects as well as the effective utilization of water resources (CWC, 2010). Moreover, design rainfall depth is an important element in the hydraulic modelling of urban drainage systems, as it directly contributes to runoff. Hence, any inaccuracies caused by poor measurement or inappropriate use of rainfall data will have a direct consequence on the outputs from the hydraulic and hydrologic modelling process. This can be estimated by using the recorded rainfall data over many stations in a given region, which may have some uncertainty that has traditionally been estimated by adopting probability distribution model (Montanari and Brath, 2004).

Number of attempts has been made by various researchers to study the different types of uncertainties in estimation of rainfall (Kavetski *et al.*, 2006; Renard *et al.*, 2010; Wu *et al.*, 2011; Hailegeorgis *et al.*, 2013; Tung and Wong, 2014; Radi *et al.*, 2015; Notaro *et al.*, 2015; Tfwala *et al.*, 2017; Khan *et al.*, 2020; Ibrahim, 2022). Generally, the uncertainty in hydrological modelling can be classified as (*i*) data and sampling errors and (*ii*) modelling or structural errors (Haddad and Rahman, 2014). The data uncertainty is originated from the measurements errors resulting from instrumental and human errors and also due to inadequate representativeness of data sample due to temporal and spatial variability of the data. The use of limited quantity of rainfall data (*i.e.*, data of short record length) in the frequency analysis introduces sampling uncertainty. Also, the estimates of higher order moments (skewness and kurtosis) become unstable, in particular due to the presence of extremes or outliers in data series. The uncertainty in the model error is attributed to inability in accurately quantifying the input parameters for a model. Hailegeorgis *et al.* (2013) described that the regional frequency analysis of extreme rainfall events and the derivation of intensity – duration - frequency curves is subject to the major uncertainties of different sources that includes (*i*) data series used: data quality (*viz*., stationary and independent), sampling of data related to the time period and length of data series and the sampling type, *e.g*., annual maximum series (AMS) or partial duration series; (*ii*) selection of frequency distribution to describe the data; (*iii*) parameter estimation; and (*iv*) regionalization and quantile estimation.

Number of probability distributions include Extreme Value Type-1 (EV1), Extreme Value Type-2, 2-parameter Log Normal, Log Pearson Type-3, Generalized Gamma, Generalized Pareto, *etc*., are generally available for modelling of extreme events such as rainfall, peak discharge, temperature. However, in hydrological studies, the EV1 (Singh, 1998) is one of the most popularly used distributions for frequency analysis of extreme values of meteorological or climatic and hydrologic variables such as floods, rainfall, droughts, *etc*. In light of the above, in this paper, an assessment on uncertainty in rainfall estimates obtained from Extreme Value Type-1 (EV1) distribution was discussed. Standard analytical procedures such as Method of Moments (MoM), Maximum Likelihood Method (MLM), Method of Least Squares (MLS), Probability Weighted Moments (PWM) and Method of L-Moments (LMO) are applied for determination of parameters of EVI (commonly known as Gumbel) (Phien, 1987). Number of studies has been carried out by different researchers on analyzing the characteristics of the parameter estimation methods of EV1 (Bhagat, 2017; Kumar, 2019; Sultan Bhat *et al*., 2019; Opere and Njogu, 2020; Manohar Reddy, 2022). Research reports indicated that MoM is a natural and relatively easy parameter estimation method (Ranyal and Salas, 1986). MLM is considered the most efficient method, since it provides the smallest sampling variance of the estimated parameters and hence of the estimated quantile compared to other methods (Prabhu *et al*., 2016). But, the method has the disadvantage of frequently giving biased estimates and often failed to give the desired accuracy in estimating the extremes from hydrological data. It may not produce good estimators in small samples, especially when the random variable is restricted to an interval that depends on the parameters (Celik, 2004). The LMO was first suggested by Hosking (1990) that has been applied to determine the parameters of various probability distributions.

TABLE 1

Determination of the parameters of EV1 by different methods

Parameter estimation method	Location parameter (α)	Scale parameter (β)
MoM	$\mu(x)-(0.5772157)\beta$	$\frac{\sqrt{6}}{x}\left[\frac{1}{N-1}\sum_{i=1}^{N}[x(i)-\mu(x)]^2\right]^{0.5}$
MLM	$-\beta\ln\left\{\frac{\sum_{i=1}^{N}\exp\left[\frac{-x(i)}{\beta}\right]}{N}\right\}$	$\bar{x}=\frac{\sum_{i=1}^{N}x(i)\exp\left[\frac{-x(i)}{\beta}\right]}{\sum_{i=1}^{N}\exp\left[\frac{-x(i)}{\beta}\right]}$
MLS	$\mu(x)+\frac{\sum_{i=1}^{N}\ln\left\{-\ln\left[\frac{(i-0.44)}{(N+0.12)}\right]\right\}\beta}{N}$	$\frac{\left[\left(\sum_{i=1}^{N}x(i)\right)^2-\left(N\sum_{i=1}^{N}x(i)^2\right)\right]}{\left\{N\sum_{i=1}^{N}x(i)\left\{\ln\left[-\ln\left(\frac{i-0.44}{N+0.12}\right)\right]\right\}\right\}-\left\{\left[\sum_{i=1}^{N}x(i)\right]\left[\sum_{i=1}^{N}\ln\left[-\ln\left(\frac{i-0.44}{N+0.12}\right)\right]\right]\right\}}$
PWM	$M(100)-(0.5772157)\beta$	$\frac{M(100)-2M(101)}{\ln(2)}$
LMO	$\lambda(1)-(0.5772157)\beta$	$\frac{\lambda(2)}{\ln(2)}$

He also described that LMOs are linear combinations of the PWM tend to share similar characteristics with PWM and MLS and also the computations are simpler. However, there was no general agreement in applying particular method for a region because of the characteristics of the estimators of EV1. In light of the above, the MoM, MLM, MLS, PWM and LMO were applied in determining the parameters of EV1 distribution for rainfall estimation. The adequacy of fitting of EV1 model to the data series was evaluated by Goodness-of-Fit (GoF) tests, *viz*., Anderson-Darling (*AD*) and Kolmogorov-Smirnov (*KS*). The characteristics of the data used in EVA was evaluated by using Wald-Wolfowitz runs test for independence, Mann-Whitney U-test for homogeneity and Grubbs' test for identifying the outliers. This paper presented the methodology adopted in assessing the uncertainty in rainfall estimates given by five methods of EV1 with reference to data length with an illustrative example and the results obtained thereon.

2. Methodology

The Probability Distribution Function [PDF; *f*(*x*)] and Cumulative Distribution Function [CDF; *F*(*x*)] of EV1 distribution (Gumbel, 1995) is given by:

$$f(x)=\frac{1}{\beta}\exp\left\{-\left[\frac{x-\alpha}{\beta}\right]-\exp\left[\left(\frac{x-\alpha}{\beta}\right)\right]\right\},\ x,\ \beta>0\ \text{and}$$

$$F(x)=\exp\left\{-\exp\left[-\left(\frac{x-\alpha}{\beta}\right)\right]\right\} \quad (1)$$

where, α and β are the location and scale parameters of EV1. The parameters were computed by MoM, MLM, MLS, PWM and LMO; and also used to estimate the rainfall (*x (T)*) for different return periods from:

$$\mathrm{x(T)}=\alpha+\mathrm{Y(T)}\,\beta \quad (2)$$

Wherein *Y*(*T*) is a reduced variate that is defined by $Y(T)=-\ln\left\{-\ln\left[1-\left(\frac{1}{T}\right)\right]\right\}$. Table 1 presents the equations used in determining the parameters of EV1 by MoM, MLM, MLS, PWM and LMO (Arora and Singh, 1987; Sai Krishna and Veerendra, 2015). In LMO and PWM,

$$\lambda(1)=b(0)=M(100)=\mu(x)=\frac{\sum_{i=1}^{N}x(i)}{N} \quad (3)$$

TABLE 2

Coefficients used in computation of standard error

Parameter estimation method	Coefficients used in computation of standard error		
	A	*B*	*C*
MoM, MLS	1.1589	0.1919	1.1000
MLM	1.1087	0.5140	0.6079
PWM, LMO	1.1128	0.4574	0.8046

$$M(101) = \frac{\sum_{i=1}^{N} x(i)(N-i)}{N(N-1)} \quad (4)$$

$$\lambda(2) = b(0) - 2b(1) = \lambda(1) - 2\left\{\frac{\sum_{i=1}^{N} x(i)(i-1)}{N(N-1)}\right\} \quad (5)$$

Here, $x(i)$ is the variable (*i.e.*, rainfall) of i^{th} sample, 'i' is the rank assigned to each sample [$x(i)$] arranged in ascending order, N is the number of sample values and $\mu(x)$ is the average of observed data.

2.1. *Computation of Standard Error*

By using MoM, MLM, MLS, PWM and LMO estimators of EV1, the standard error (*SE)* in rainfall estimation was computed from Eq. (2), which is given as below:

$$SE = \frac{\beta}{\sqrt{N}}\left[A + BY(T) + CY(T)^2\right]^{0.5} \quad (6)$$

Table 2 gives the coefficients (Vivekanandan *et al.*, 2012) used in computation of *SE*.

2.2. *Estimation of Confidence Limits*

The Lower and Upper Confidence Limits (*LCL* and *UCL*) (Aydin, 2018) of the estimated extreme rainfall (*i.e.*, the observed 1-day maximum rainfall in 366 days (for leap year) or 365 days (for non-leap year) at 95% level are computed from

$$LCL = x(T)-1.96(SE) \text{ and } UCL=x(T)+1.96(SE) \quad (7)$$

The relative difference (*RD*) of the confidence limits from the estimated rainfall [$x(T)$] for a given return period (T) was computed from the following relations:

$$RD \text{ in } LCL\,(\%) = \frac{x(T) - LCL}{x(T)} * 100 \quad (8)$$

$$RD \text{ in } UCL\,(\%) = \frac{UCL - x(T)}{x(T)} * 100 \quad (9)$$

2.3. *Goodness-of-Fit Tests*

The *AD* and *KS* tests statistic (Zhang, 2002) is defined by:

$$AD = (-N) - (1/N) \sum_{i=1}^{N} \{(2i-1)\ln[Z(i)] + (2N+1-2i)\ln[1-Z(i)]\} \quad (10)$$

$$KS = \underset{i=1}{\overset{N}{Max}}\{F_e[x(i)] - F_D[x(i)]\} \quad (11)$$

where, $Z(i) = F[x(i)]$ for I = 1, 2, 3,…, N and $x(1)<x(2)<$ …. $x(N)$ wherein $x(1)$ and $x(N)$ indicates the lowest and highest values in the series of observed data. Also, $F_e[x(i)] = \frac{i}{N+1}$ and $F_D[x(i)]$ are the empirical and computed CDF of $x(i)$. If the computed values of GoF tests statistic given by the method are not greater than its theoretical values at the desired significance level then the method is considered as adequate for rainfall estimation.

3. Application

This paper presented a study on assessment of uncertainty in rainfall estimates with reference to data

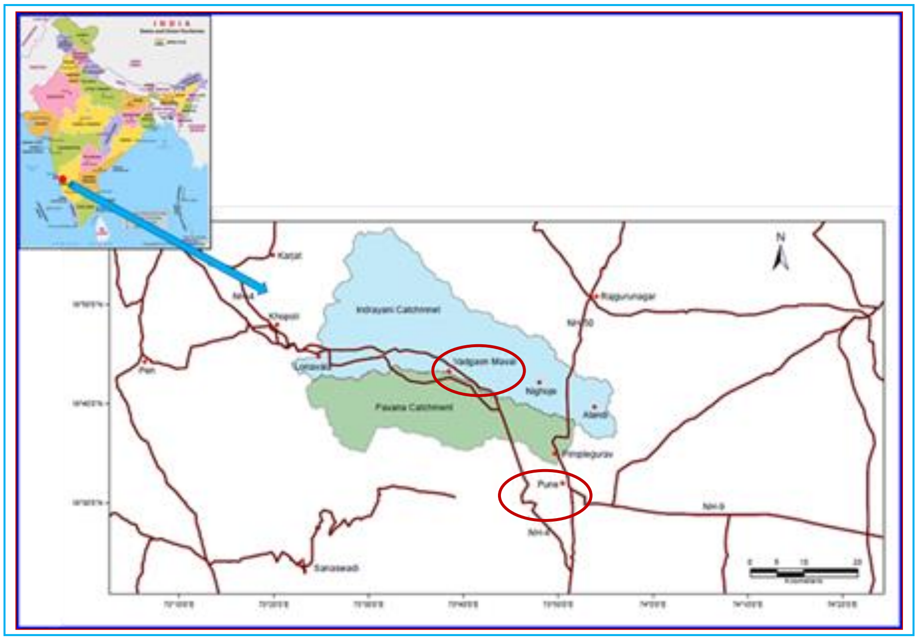


Fig. 1. Index map of the study area with locations of rain-gauge stations

TABLE 3

Descriptive statistics of AMR series for Pune and Vadgaon Maval

Data series	Pune				Vadgaon Maval			
	Average(mm)	SD (mm)	C_s	C_k	Average(mm)	SD (mm)	C_s	C_k
DS1	69.8 (4.196)	22.2 (0.321)	0.695 (-0.184)	0.458 (-0.013)	89.3 (4.437)	31.8 (0.330)	1.241 (0.369)	1.683 (-0.126)
DS2	70.7 (4.201)	24.6 (0.344)	0.780 (-0.052)	0.148 (-0.155)	93.9 (4.480)	35.7 (0.347)	1.315 (0.460)	1.640 (-0.124)
DS3	72.2 (4.222)	25.0 (0.342)	0.764 (-0.030)	0.098 (-0.265)	97.8 (4.512)	40.7 (0.366)	1.598 (0.601)	2.813 (0.123)
DS4	74.2 (4.249)	26.1 (0.344)	1.010 (-0.042)	1.695 (-0.014)	102.5 (4.556)	42.9 (0.377)	1.420 (0.472)	2.143 (-0.187)

SD: Standard Deviation; C_s: Coefficient of Skewness; C_k: Coefficient of Kurtosis. Numbers given within brackets indicates the descriptive statistics of log-transformed data

length for Pune and Vadgaon Maval sites. These sites are located on the western side of Deccan Plateau and are on leeward side of Sahyadri mountain range which forms a barrier from Arabian Sea. Fig. 1 presents the index map of the study area with locations of Pune and Vadgaon Maval sites. From Fig. 1, it was witnessed that the Pune IMD (India Meteorological Department) rain gauge site is located approximately between the latitude 18° 31' N and longitude 73° 51" E and whereas the Vadgaon Maval site is located within latitude 18° 42' N and longitude 73° 38" E.

In this paper, the daily rainfall observed at Pune (1901 to 2017) and Vadgaon Maval (1901 to 1965, 1968 to 1971 and 1973 to 2017) was used. By considering the importance of the hydrologic extremes, the missing data for the years 1966 to 1967 and 1972 were not considered in EVA for Vadgaon Maval. The AMR (*i.e.*, annual 1-day maximum rainfall) series is derived from the daily rainfall data and used to generate the data series (DS) with different data length (Mathew and Vivekanandan, 2009), *viz.*, DS1 (series with 50 years of data), DS2 (series with 75 years data), DS3 (series with 100 years data) and DS4 (series with entire data, *viz.*,

TABLE 4

Statistical tests results for randomness and homogeneity for Pune

Data series	Wald-Wolfowitz runs test				Mann-Whitney U-test			
	Computed	Theoretical	Significance level	Randomness	Computed	Theoretical	Significance level	Homogeneous
DS1	1.053	1.960	5 %	Yes	-1.174	1.960	5 %	Yes
DS2	2.564	2.580	1 %	Yes	0.207	1.960	5 %	Yes
DS3	1.709	1.960	5 %	Yes	-0.496	1.960	5 %	Yes
DS4	1.942	1.960	5 %	Yes	-1.325	1.960	5 %	Yes

TABLE 5

Statistical tests results for randomness and homogeneity for Vadgaon Maval

Data series	Wald-Wolfowitz runs test				Mann-Whitney U-test			
	Computed	Theoretical	Significance level	Randomness	Computed	Theoretical	Significance level	Homogeneous
DS1	-0.168	1.960	5 %	Yes	-1.261	1.960	5 %	Yes
DS2	-0.561	1.960	5 %	Yes	-2.123	2.580	1 %	Yes
DS3	-1.264	1.960	5 %	Yes	-1.872	1.960	5 %	Yes
DS4	-2.681	1.960	5 %	Yes	-2.845	1.960	5 %	Yes

117 years for Pune and 114 years for Vadgaon Maval). The generated AMR data series was used in rainfall estimation by applying five methods (*viz*., MoM, MLM, MLS, PWM and LMO) of EV1. Table 3 presents the descriptive statistics of AMR with different data length used in EVA for Pune and Vadgaon Maval. From Table 3, it could be observed that the average and standard deviation of rainfall of DS4 is higher than those values of DS1, DS2 and DS3.

4. Results and Discussion

By applying the procedures, as described above, the assessment of uncertainty in rainfall estimates using five methods (*viz*., MoM, MLM, MLS, PWM and LMO) of EV1 with reference to data length for Pune and Vadgaon Maval sites was carried out and the results are presented in the following sections.

4.1. *Analysis Based on Statistical Tests*

The data series used for EVA should satisfy certain basic assumption such as data should be independent and identically distributed with the meteorological process (rainfall). The term independent denotes that no observation in the data series has any influence on any other observation following, *i.e*., the data series are random. Similarly, homogeneity of the sample elements in the data series has to be checked to identify whether the data originates from a single population or not. The presence of outliers in a data sample has undesirable effect on frequency analysis. Therefore, the sample also needs to be checked for outliers if any. In the present study, Wald-Wolfowitz runs test and Mann-Whitney U-test were used for checking the randomness and homogeneity of AMS of rainfall. Grubbs' test was used for detection of outliers in the data series (Bonnini *et al.*, 2014). Tables 4 and 5 present the results of statistical tests applied to the AMS of rainfall of Pune and Vadgaon Maval respectively. From the statistical tests results, it was observed that the computed values of Wald-Wolfowitz runs test and Mann-Whitney U-test statistic using the data series (*viz*., DS1, DS2, DS3 and DS4) of Pune and Vadgaon Maval are not greater than its theoretical values either at 1% or 5% level, and at this level, the data series used in EVA is random as well as homogeneous. The Grubb's test results indicated that there were no outliers in the data series.

4.2. *Analysis Based on GoF Tests*

The GoF (*viz*., *AD* and *KS*) tests were applied for checking the adequacy of fitting five methods (*viz*., MoM, MLM, MLS, PWM and LMO) of EV1 to the AMS with different data length of Pune and Vadgaon Maval, and are

TABLE 6

Theoretical and computed values of GoF tests statistic by EV1 for Pune

Data series	Theoretical value	Computed values of AD				Theoretical value	Computed values of KS			
		MoM	MLM	MLS	PWM,LMO		MoM	MLM	MLS	PWM,LMO
DS1	1.038	0.255	0.209	0.218	0.223	0.219	0.044	0.051	0.048	0.052
DS2	1.038	0.244	0.230	0.228	0.229	0.180	0.042	0.041	0.043	0.041
DS3	1.038	0.238	0.206	0.202	0.198	0.157	0.041	0.038	0.037	0.036
DS4	1.038	0.160	0.152	0.150	0.152	0.148	0.039	0.040	0.040	0.041

TABLE 7

Theoretical and computed values of GoF tests statistic by EV1 for Vadgaon Maval

Data series	Theoretical value	Computed values of AD				Theoretical value	Computed values of KS			
		MoM	MLM	MLS	PWM, LMO		MoM	MLM	MLS	PWM, LMO
DS1	1.038	0.324	0.327	0.371	0.320	0.219	0.069	0.059	0.073	0.069
DS2	1.038	0.693	0.636	0.755	0.663	0.180	0.087	0.077	0.089	0.088
DS3	1.038	1.298	0.963	1.490	1.119	0.157	0.104	0.080	0.106	0.102
DS4	1.038	1.022	0.924	1.142	0.953	0.146	0.079	0.079	0.080	0.082

TABLE 8(a)

Estimated rainfall (in mm) with standard error for different return periods with reference to data length given by EV1 for Pune

Return period (year)	MoM								MLM							
	DS1		DS2		DS3		DS4		DS1		DS2		DS3		DS4	
	ER	*SE*	*ER*	*SE*	*ER*	*SE*	*ER*	*SE*	*ER*	*SE*	*ER*	*SE*	*ER*	*SE*	*ER*	*SE*
2	66.1	2.9	66.7	2.6	68.1	2.3	70.0	2.2	66.2	2.9	66.6	2.6	68.0	2.3	70.0	2.2
5	85.8	4.9	88.4	4.4	90.2	3.9	93.0	3.7	87.1	4.7	88.7	4.1	90.5	3.6	93.2	3.4
10	98.8	6.6	102.8	5.9	104.8	5.2	108.2	5.0	100.9	6.0	103.4	5.2	105.3	4.6	108.6	4.4
20	111.3	8.3	116.5	7.5	118.7	6.6	122.8	6.3	114.1	7.4	117.4	6.4	119.6	5.6	123.4	5.4
25	115.2	8.8	120.9	8.0	123.2	7.0	127.5	6.8	118.3	7.8	121.8	6.7	124.1	5.9	128.1	5.7
50	127.4	10.6	134.4	9.5	136.9	8.4	141.7	8.1	131.3	9.1	135.6	7.9	138.0	7.0	142.5	6.7
100	139.5	12.3	147.7	11.1	150.4	9.8	155.9	9.4	144.1	10.5	149.2	9.1	151.8	8.0	156.9	7.7
200	151.5	14.1	161.1	12.7	164.0	11.2	170.1	10.8	156.9	11.9	162.7	10.3	165.6	9.0	171.2	8.7
500	167.4	16.4	178.6	14.8	181.8	13.0	188.7	12.6	173.8	13.7	180.6	11.9	183.8	10.4	190.0	10.0
1000	179.5	18.2	191.9	16.4	195.3	14.4	202.8	13.9	186.5	15.1	194.1	13.1	197.5	11.5	204.2	11.0

TABLE 8(b)

Estimated rainfall (in mm) with standard error for different return periods with reference to data length given by EV1 for Pune

Return period (year)	MLS								PWM and LMO							
	DS1		DS2		DS3		DS4		DS1		DS2		DS3		DS4	
	ER	*SE*	*ER*	*SE*	*ER*	*SE*	*ER*	*SE*	*ER*	*SE*	*ER*	*SE*	*ER*	*SE*	*ER*	*SE*
2	66.2	3.0	66.7	2.7	68.1	2.3	70.0	2.2	66.0	3.0	66.6	2.7	68.0	2.4	69.9	2.2
5	86.5	5.0	89.1	4.5	90.7	3.9	93.4	3.8	86.3	4.8	88.9	4.3	90.7	3.8	93.2	3.6
10	99.9	6.8	103.9	6.1	105.7	5.3	108.9	5.1	99.8	6.3	103.7	5.7	105.7	5.0	108.7	4.7
20	112.8	8.5	118.1	7.7	120.0	6.7	123.8	6.5	112.7	7.8	117.8	7.0	120.2	6.2	123.5	5.9
25	116.8	9.1	122.6	8.2	124.5	7.2	128.5	6.9	116.8	8.3	122.3	7.5	124.7	6.6	128.2	6.3
50	129.4	10.9	136.5	9.8	138.6	8.6	143.0	8.2	129.4	9.9	136.2	8.8	138.8	7.8	142.6	7.4
100	141.9	12.7	150.3	11.5	152.5	10.0	157.4	9.6	141.9	11.4	149.9	10.2	152.8	9.0	157.0	8.6
200	154.3	14.5	164.0	13.1	166.3	11.4	171.8	11.0	154.4	13.0	163.6	11.6	166.8	10.2	171.3	9.7
500	170.7	16.9	182.1	15.3	184.6	13.3	190.8	12.8	170.8	15.0	181.7	13.4	185.2	11.9	190.2	11.3
1000	183.1	18.7	195.8	16.9	198.4	14.8	205.1	14.2	183.3	16.6	195.3	14.8	199.1	13.1	204.4	12.4

TABLE 9(a)

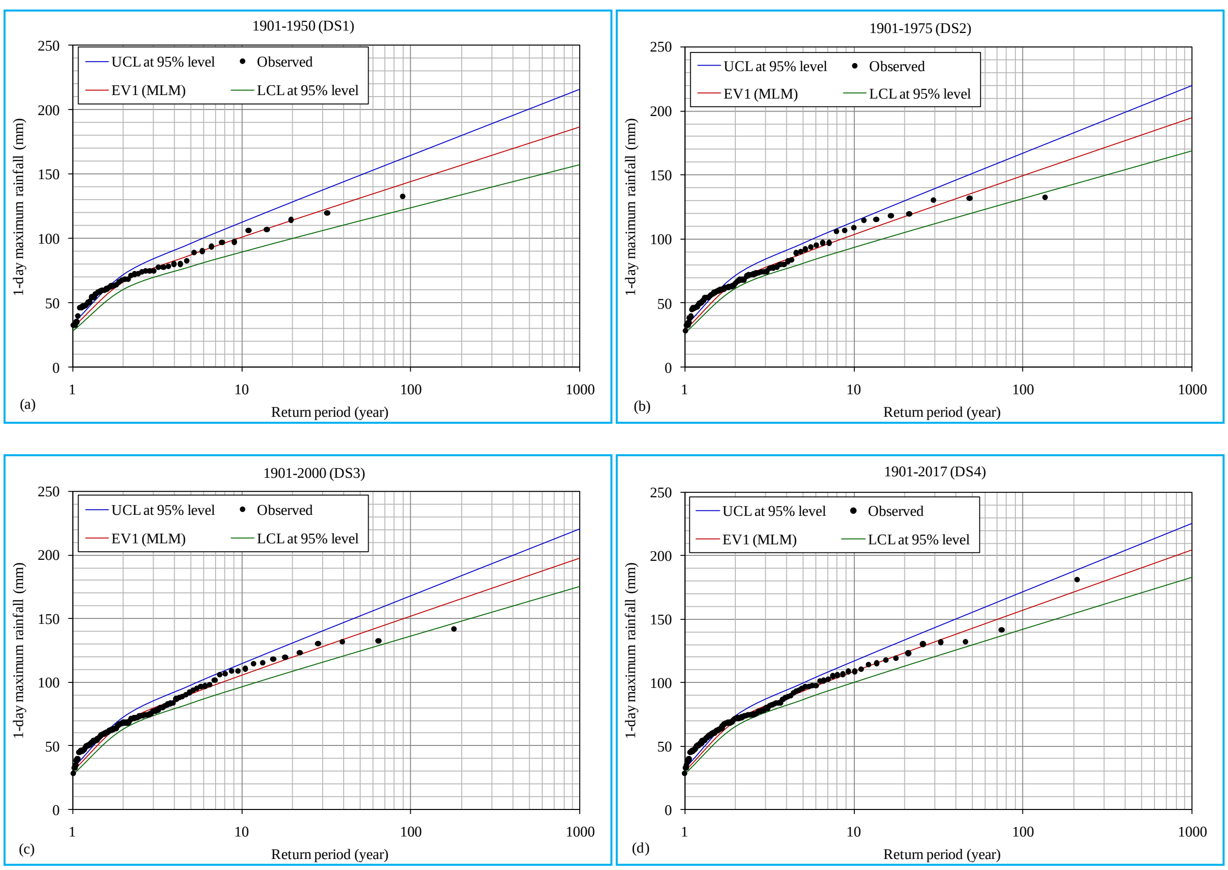
Estimated rainfall (in mm) with standard error for different return periods with reference to data length given by EV1 for Vadgaon Maval

Return period (year)	MoM								MLM							
	DS1		DS2		DS3		DS4		DS1		DS2		DS3		DS4	
	ER	*SE*	*ER*	*SE*	*ER*	*SE*	*ER*	*SE*	*ER*	*SE*	*ER*	*SE*	*ER*	*SE*	*ER*	*SE*
2	84.1	4.1	88.0	3.8	91.1	3.8	95.5	3.6	83.9	3.8	87.7	3.4	90.6	3.2	94.9	3.0
5	112.2	6.9	119.6	6.5	127.0	6.4	133.4	6.0	109.8	5.8	115.9	5.3	121.1	4.9	128.1	4.3
10	130.7	9.4	140.4	8.8	150.9	8.6	158.5	8.1	127.0	7.5	134.5	6.8	141.2	6.3	150.1	5.6
20	148.6	11.8	160.5	11.1	173.7	10.9	182.5	10.3	143.5	9.1	152.4	8.3	160.6	7.7	171.2	7.0
25	154.2	12.6	166.8	11.8	180.9	11.6	190.2	11.0	148.7	9.7	158.1	8.8	166.7	8.2	177.8	7.4
50	171.6	15.1	186.4	14.2	203.2	13.9	213.7	13.3	164.8	11.4	175.6	10.3	185.6	9.6	198.4	8.8
100	188.9	17.6	205.8	16.5	225.4	16.2	237.0	15.5	180.7	13.1	193.0	11.9	204.3	11.0	218.9	10.2
200	206.1	20.1	225.2	18.8	247.5	18.5	260.3	17.7	196.7	14.8	210.3	13.4	223.0	12.5	239.2	11.7
500	228.8	23.4	250.7	21.9	276.6	21.6	291.0	20.7	217.7	17.1	233.1	15.5	247.7	14.4	266.1	13.6
1000	246.0	25.9	270.0	24.3	298.6	23.9	314.2	22.9	233.5	18.8	250.4	17.0	266.3	15.8	286.4	15.0

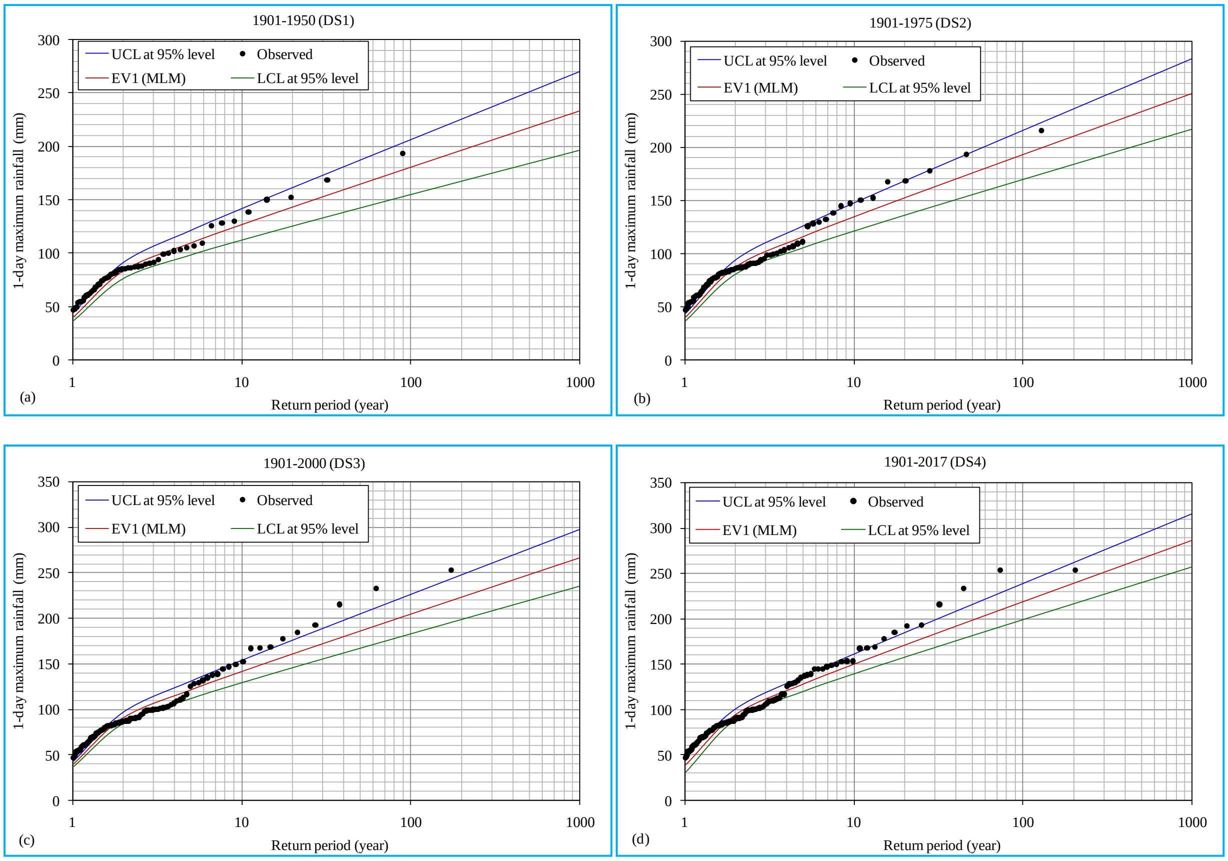
TABLE 9(b)

Estimated rainfall (in mm) with standard error for different return periods with reference to data length given by EV1 for Vadgaon Maval

Return period (year)	MLS								PWM and LMO							
	DS1		DS2		DS3		DS4		DS1		DS2		DS3		DS4	
	ER	*SE*	*ER*	*SE*	*ER*	*SE*	*ER*	*SE*	*ER*	*SE*	*ER*	*SE*	*ER*	*SE*	*ER*	*SE*
2	84.2	4.3	88.0	4.0	91.0	3.9	95.4	3.7	84.1	4.1	88.2	3.8	91.4	3.6	95.7	3.4
5	113.2	7.2	120.6	6.7	128.2	6.6	134.3	6.1	112.1	6.6	119.1	6.1	125.7	5.8	132.6	5.2
10	132.4	9.7	142.2	9.1	152.8	8.9	160.1	8.3	130.5	8.7	139.6	8.0	148.4	7.7	157.1	7.0
20	150.9	12.3	162.8	11.4	176.4	11.3	184.8	10.6	148.3	10.8	159.2	10.0	170.2	9.5	180.5	8.7
25	156.7	13.1	169.4	12.2	183.8	12.0	192.6	11.3	153.9	11.5	165.5	10.6	177.1	10.1	188.0	9.3
50	174.8	15.7	189.6	14.6	206.9	14.4	216.8	13.6	171.2	13.6	184.7	12.5	198.4	12.0	210.9	11.2
100	192.7	18.2	209.6	17.0	229.8	16.7	240.7	15.9	188.4	15.7	203.8	14.5	219.5	13.8	233.6	13.0
200	210.5	20.8	229.6	19.5	252.6	19.1	264.6	18.2	205.5	17.8	222.7	16.4	240.5	15.7	256.3	14.9
500	234.0	24.3	256.0	22.7	282.6	22.3	296.1	21.2	228.2	20.6	247.8	19.0	268.3	18.2	286.2	17.3
1000	251.8	26.9	275.9	25.1	305.4	24.7	319.9	23.5	245.2	22.7	266.7	21.0	289.3	20.1	308.8	19.2



Figs. 2(a-d). Estimated 1-day maximum rainfall by EV1 (MLM) with 95% confidence limits for different return periods using data series with different data length for Pune



Figs. 3(a-d). Estimated 1-day maximum rainfall by EV1 (MLM) with 95% confidence limits for different return periods using data series with different data length for Vadgaon Maval

presented in Tables 6 and 7. From the data analysis, it was found that the EVA and GoF tests results obtained from PWM and LMO are identical though the procedures adopted in determining the parameters using PWM and LMO are different to each other.

From Table 6, it was found that the computed values of GoF tests statistic by five methods of EV1 are lesser than its theoretical value at 1% significance level, and at this level, all methods are acceptable for EVA of rainfall for Pune. For Vadgaon Maval, the computed values of *KS* test statistic (Table 7) were compared with its theoretical value at 5% level and found that all methods of EV1 are suitable for EVA. However, for Vadgaon Maval, the *AD* test results didn't support the use of EV1 (MoM, MLS, PWM and LMO) for DS3 and EV1 (MLS) for DS4 because of the data characteristics of the series used in EVA.

4.3. *EVA of Rainfall*

The parameters of EV1 were determined by five methods and are used for estimation of rainfall at Pune and Vadgaon Maval. From the EVA results of Pune and Vadgaon Maval, as given in Tables 8(a&b) and 9(a&b), it was found that:

(*i*) The estimated rainfall is in increasing order when data length increases. The standard error in the estimated rainfall is in decreasing order when data length increases.

(*ii*) The standard error in rainfall estimates given by MLM is comparatively less than those values of MoM, MLS, PWM and LMO.

(*iii*) The estimated rainfall by MLS is higher than those values of MoM, MLM, PWM and LMO while using DS2, DS3 and DS4series in EVA for Vadgaon Maval.

(*iv*) The estimated rainfall by MLM is higher than those values of MoM, MLS, PWM and LMO while using the DS1 series in EVA for Pune.

Based on the values of standard error in the estimated rainfall, the MLM is identified as best suited method for estimation of rainfall. The estimated rainfall

TABLE 10(a)

Relative difference of the confidence limits from the estimated rainfall together with 95% confidence limits by EV1 (MLM) for Pune

Data Series		Return period (T in years)									
		2	5	10	20	25	50	100	200	500	1000
DS1	*ER*	66.2	87.1	100.9	114.1	118.3	131.3	144.1	156.9	161.0	173.8
	LCL	60.3	77.9	89.1	99.7	103.1	113.3	123.5	133.6	136.8	146.9
	UCL	72.2	96.3	112.7	128.5	133.6	149.2	164.7	180.2	185.2	200.6
	RD	± 9.0	±10.5	±11.7	±12.6	±12.9	±13.7	±14.3	±14.8	±15.0	±15.5
DS2	*ER*	66.6	88.7	103.4	117.4	121.8	135.6	149.2	162.7	167.1	180.6
	LCL	61.5	80.8	93.2	104.9	108.6	120.0	131.3	142.6	146.2	157.4
	UCL	71.8	96.7	113.6	129.9	135.1	151.1	167.0	182.9	188.0	203.9
	RD	±7.8	±9.0	±9.9	±10.6	±10.8	±11.4	±12.0	±12.4	±12.5	±12.9
DS3	*ER*	68.0	90.5	105.3	119.6	124.1	138.0	151.8	165.6	170.0	183.8
	LCL	63.5	83.5	96.3	108.6	112.5	124.4	136.1	147.9	151.6	163.3
	UCL	72.6	97.4	114.3	130.5	135.7	151.6	167.5	183.3	188.4	204.2
	RD	±6.7	±7.7	±8.5	±9.2	±9.4	±9.9	±10.3	±10.7	±10.8	±11.1
DS4	*ER*	70.0	93.2	108.6	123.4	128.1	142.5	156.9	171.2	175.8	190.0
	LCL	65.6	86.5	100.0	112.9	117.0	129.5	141.8	154.2	158.1	170.4
	UCL	74.3	99.9	117.2	133.9	139.2	155.6	171.9	188.2	193.4	209.6
	RD	±6.2	±7.2	±7.9	±8.5	±8.7	±9.2	±9.6	±9.9	±10.0	±10.3

ER: Estimated rainfall (in mm); *LCL*: Lower Confidence Limit at 95% level (in mm); *UCL*: Upper Confidence Limit at 95% level (in mm); *RD*: Relative difference of the lower confidence limit (-) and upper confidence limit (+) from the estimated rainfall (in %)

with 95% confidence limits for different return periods obtained from EV1 (MLM) using the DS1, DS2, DS3 and DS4 together with observed annual 1-day maximum rainfall for Pune and Vadgaon Maval are presented in Figs. 2(a-d) and 3(a-d). From these figures, it can be seen that the observed rainfall in the range of 30-225 mm for Pune and 31-315 mm for Vadgaon Maval are within the confidence limits of the estimated rainfall given by EV1 (MLM). Table 10 (a&b) presents the relative difference of the confidence limits from the estimated rainfall together with 95% confidence limits using EV1 (MLM). From these tables, it was found that the percentage of relative difference is in increasing order when the return period increases and in decreasing order when data length increases. These results showed that there are some uncertainty in rainfall estimation with reference to data length for Pune and Vadgaon Maval.

The study was particularly carried out to assess the uncertainty in rainfall estimation at Pune and Vadgaon Maval sites with respect to data length by adopting five different methods of EV1 distribution as a part of the research work. However, the study could be conducted with the aid of other probability distributions such as 2-parameter log-normal, 3-parameter pearson and log-pearson, extreme value type-2 (also known as Frechet), *etc* to assess the uncertainty in rainfall estimation over a time period.

TABLE 10(b)

Relative difference of the confidence limits from the estimated rainfall together with 95% confidence limits by EV1 (MLM) for Vadgaon Maval

Data Series		Return period (T in years)									
		2	5	10	20	25	50	100	200	500	1000
DS1	*ER*	83.9	109.8	127.0	143.5	148.7	164.8	180.7	196.7	201.8	217.7
	LCL	76.4	98.4	112.3	125.5	129.7	142.5	155.1	167.7	171.7	184.2
	UCL	91.3	121.2	141.6	161.4	167.7	187.1	206.4	225.6	231.8	251.1
	RD	±8.9	±10.4	±11.5	±12.5	±12.8	±13.5	±14.2	±14.7	±14.9	±15.4
DS2	*ER*	87.7	115.9	134.5	152.4	158.1	175.6	193.0	210.3	215.9	233.1
	LCL	80.9	105.5	121.3	136.2	140.9	155.4	169.8	184.0	188.6	202.9
	UCL	94.4	126.2	147.8	168.7	175.3	195.8	216.2	236.6	243.1	263.4
	RD	±7.7	±8.9	±9.9	±10.7	±10.9	±11.5	±12.0	±12.5	±12.6	±13.0
DS3	*ER*	90.6	121.1	141.2	160.6	166.7	185.6	204.3	223.0	229.0	247.7
	LCL	84.4	111.4	128.9	145.4	150.7	166.8	182.7	198.6	203.7	219.5
	UCL	96.9	130.7	153.6	175.7	182.7	204.4	225.9	247.4	254.3	275.8
	RD	±6.9	±8.0	±8.7	±9.4	±9.6	±10.1	±10.6	±10.9	±11.1	±11.4
DS4	*ER*	94.9	128.1	150.1	171.2	177.8	198.4	218.9	239.2	245.8	266.1
	LCL	89.1	119.7	139.1	157.5	163.3	181.1	198.8	216.3	222.0	239.5
	UCL	100.8	136.6	161.1	184.8	192.4	215.7	239.0	262.1	269.6	292.7
	RD	±6.2	±6.6	±7.3	±8.0	±8.2	±8.7	±9.2	±9.6	±9.7	±10.0

ER: Estimated rainfall (in mm); *LCL*: Lower Confidence Limit at 95% level (in mm); *UCL*: Upper Confidence Limit at 95% level (in mm); *RD*: Relative difference of the lower confidence limit(-) and upper confidence limit (+) from the estimated rainfall (in %)

5. Conclusions

The paper presented a study on assessment on uncertainty in rainfall estimates of Pune and Vadgaon Maval using five methods (*viz*., MoM, MLM, MLS, PWM and LMO) of EV1 distribution for three different data series, *viz*., DS1 with 50 years data, DS2 with 75 years data, DS3 with 100 years data and DS4 series with entire available data (*viz*., 117 years for Pune and 114 years for Vadgaon Maval). The results of Wald-Wolfowitz runs test and Mann-Whitney U-test indicated that the data series used in EVA was randomness as also homogeneous. Further, it was witnessed that no outliers in the data series applied in EVA. The adequacy of fitting EV1 distribution to the series of AMR was examined through *AD* and *KS* tests. The EVA results of Pune and Vadgaon Maval showed that (*i*) the estimated rainfall is in increasing order when the data length increases; (*ii*) the standard error in the estimated rainfall is in decreasing order when the data length increases; (*iii*) the MLM is better suited for rainfall estimation; and (*iv*) the relative difference of the confidence limits of the estimated rainfall witnessed that there are some uncertainty in rainfall estimation with reference to data length.

The study presented in the paper focussed on the effect of data length and its direct measure of uncertainty in rainfall estimation while estimating the design rainfall depth. Also, the results presented in the paper would be helpful to the stakeholders to use as an input for the hydraulic and hydrologic modelling. Further, the study suggested that the rainfall estimates given by EV1 (MLM) could be used as an input for planning, design and management of civil engineering infrastructure projects within the vicinity of Pune and Vadgaon Maval sites.

Acknowledgements

The author is thankful to the Director, Central Water and Power Research Station, Pune, for providing the research facilities to carry out the study. The author is also thankful to India Meteorological Department, Pune, for the supply of rainfall data.

Disclaimer: The contents and views expressed in this research paper/article are the views of the author and do not necessarily reflect the view of the organization he belong to.

References

Arora, K. and Singh, V. P., 1987, "On statistical inter-comparison of EV1 estimators by Monte Carlo simulation", *Advances in Water Resources*, **10**, 2, 87-107. doi : https://doi.org/10.1016/0309-1708(87)90013-3.

Aydin, D., 2018, "Estimation of the lower and upper quantiles of Gumbel distribution: An application to wind speed data", *Applied Ecology and Environmental Research*, **16**, 1, 1-15. doi : http://dx.doi.org/10.15666/aeer/1601_001015.

Bhagat, N., 2017, "Flood frequency analysis using Gumbel's distribution method: a case study of lower Mahi Basin, India", *Journal of Water Resources and Ocean Science*, **6**, 4, 51-54. doi : https://doi.org/10.11648/j.wros.20170604.11.

Bonnini, S., Corain, L., Marozzi, M. and Salmaso, L., 2014, "Nonparametric hypothesis testing: Rank and permutation methods with applications in R", Book series in Probability and Statistics, John Wiley & Sons Limited. doi : https://doi.org/10.1002/9781118763490.

Celik, A. N., 2004, "On the distributional parameters used in assessment of the suitability of wind speed probability density functions", *Energy Conversion and Management*, **45**, 11-12, 1735-1747. doi : https://doi.org/10.1016/j.enconman.2003.09.027.

CWC, 2010, "Development of hydrological design aids (surface water) under Hydrology Project II: State of the art report, Consulting Engineering Services (India) in Association with HR Wallingford, Central Water Commission (CWC), New Delhi.

Gumbel, E. J., 1995, "Statistic of Extremes", Columbia University Press, New York.

Haddad, K. and Rahman, A., 2014, "Design rainfall estimation and changes, In Handbook of Engineering Hydrology: Modelling", *Climate Change and Variability*, Edited by SaeidEslamian, CRC Press, 173-190.

Hailegeorgis, T. T., Thorolfsson, S. T. and Alfredsen, K., 2013, "Regional frequency analysis of extreme precipitation with consideration to uncertainties to update IDF curves for the city of Trondheim", *Journal of Hydrology*, **498**, 1, 305-318. doi : https://doi.org/10.1016/ j.jhydrol.2013.06.019.

Hosking, J. R. M., 1990, "L-moments: Analysis and estimation of distributions using linear combinations of order statistics", *Journal of Royal Statistics Society: Series B (Statistical Methodology)*, **52**, 1, 105-124. doi : https://doi.org/10.1111/ j.2517-6161.1990.tb01775.x.

Ibrahim, M. N., 2022, "Assessment of the uncertainty associated with statistical modeling of precipitation extremes for hydrologic engineering applications in Amman, Jordan", *Sustainability*, Paper ID. 17052, **14**, 24, 1-20. doi : https://doi.org/ 10.3390/su142417052.

International Atomic Energy Agency (IAEA), 2003, "Meteorological events in site evaluation for nuclear power plants-IAEA Safety Guide No.Ns-G-3.4", International Atomic Energy Agency, Vienna.

Kavetski, D., Kuczera, G. and Franks, S., 2006, "Bayesian analysis of input uncertainty in hydrological modeling : 1. Theory", *Water Resources Research*, Paper ID. W03407, **42**, 3, 1-9. doi : https://doi.org/10.1029/2005WR004368.

Khan, S., Ali, Q. S. W. and Wesley, C. J., 2020, "Rainfall estimation for drought analysis using gumbel's distribution method for Lucknow district", *International Journal of Agriculture Sciences*, **12**, 18, 10214-10222. Google Scholar.

Kumar, R., 2019, "Flood frequency analysis of the Tapti river basin using log Pearson type-III and Gumbel extreme value-1 methods", *Journal of the Geological Society of India*, **94**, 5, 480-484. doi : https://doi.org/10.1007/s12594-019-1344-0.

Manohar Reddy, V., 2022, "Flood frequency analysis using Gumbel's distribution method and Log-Pearson Type III distribution method in Krishna River, Andhra Pradesh", Proceedings of Recent Developments in Sustainable Infrastructure (ICRDSI-2020) and Part of the Lecture Notes in Civil Engineering Book Series, Volume **207**, doi : https://doi.org/ 10.1007/978-981-16-7509-6_36.

Mathew, F. T. and Vivekanandan, N., 2009, "Effect of data length on water resources assessment", ISH Journal of Hydraulic Engineering, **15**, 3, 27-39. doi : https://doi.org/10.1080/ 09715010.2009.10514957.

Montanari, A. and Brath, A., 2004, "A stochastic approach for assessing the uncertainty of rainfall-runoff simulations", *Water Resources Research*, Paper ID. W0110640, **1**, 1-11. doi : https://doi.org /10.1029/2003WR002540

Notaro, V., Liuzzo, L., Freni, G. and Loggia, G. L., 2015, "Uncertainty analysis in the evaluation of extreme rainfall trends and its implications on urban drainage system design", *Water*, **7**, 12, 6931-6945. doi : https://doi.org/10.3390/w7126667.

Opere, A. O. and Njogu, A. K., 2020, "Using extreme value theory to estimate available water in the upper Awach-Kibuon catchment in Nyamira County, Kenya", *American Journal of Water Resources*, **8**, 4, 200-210. doi : https://doi.org/10.12691/ajwr-8-4-6.

Phien, H.N., 1987, "A review of methods of parameter estimation for the extreme value type-1 distribution", *Journal of Hydrology*, **90**, 3-4, 251-268. doi : https://doi.org/10.1016/ 0022-1694(87)90070-9.

Prabhu, J., George, T., Vijayakumar, B. and Ravi, P.M., 2016, "Extreme value statistical analysis of meteorological parameters observed at Kudankulam site during 2004-2014", *Radiation Protection*

Environment, **39**, 2, 107-112. doi : https://doi.org/10.4103/0972-0464. 190388.

Radi, N., Zakaria, R. and Azman, M., 2015, “Estimation of missing rainfall data using spatial interpolation and imputation methods”, *AIP Conference Proceedings*, **1643**, 42-48.

Ranyal, J.A. and Salas, J.D., 1986, “Estimation procedures for the type-1 extreme value distribution”, *Journal of Hydrology*, **87**, 3-4, 315-336. doi : https://doi.org/10.1016/0022-1694(86) 90022-3.

Renard, B., Kavetski, D., Kuczera, G., Thyer, M. and Franks, S. W., 2010, “Understanding predictive uncertainty in hydrologic modelling: The challenge of identifying input and structural errors”, *Water Resources Research*, Paper ID. W05521, **46**, 5, 1-22. doi : https://doi.org/ 10.1029/2009WR008328.

Sai Krishna, G. and Veerendra, G. T. N., 2015, “Flood frequency analysis of Prakasam barrage reservoir Krishna District, Andhra Pradesh using Weibull, Gringorten and L-Moments Formula”, *International Journal of Civil, Structural, Environmental and Infrastructure Engineering Research and Development*, **5**, 2, 57-62. Google Scholar.

Singh, V. P., 1998, “Extreme value type 1 distribution. In: entropy-based parameter estimation in hydrology”, *Water Science and Technology Library*, **30**, Springer, Dordrecht. doi : https://doi.org/10.1007/978-94-017-1431-0_8.

Sultan Bhat, M., Alam, A., Ahmad, B., Kotlia, B.S., Farooq, H., Taloor, A. K. and Ahmad, S., 2019, “Flood frequency analysis of river Jhelum in Kashmir basin”, *Quaternary International*, **507**, 1, 288-294. doi : https://doi.org/10.1016/j.quaint.2018.09.039

Tfwala, C. M., Van Rensburg, L. D., Schall, R., Mosia, S. M. and Dlamini, P. 2017, “Precipitation intensity-duration-frequency curves and their uncertainties for Ghaap plateau”, *Climate Risk Management*, **16**, 1-9. doi : https://doi.org/10.1016/ j.crm.2017.04.004.

Tung, Y. and Wong, C., 2014, “Assessment of design rainfall uncertainty for hydrologic engineering applications in Hong Kong”, *Stochastic Environmental Research and Risk Assessment*, **28**, 3, 583-592. doi : https://doi.org/10.1007/s00477-013-0774-2

Vivekanandan, N., Mathew, F. T. and Roy, S. K., 2012, “Modelling of wind speed data using probabilistic approach”, *Journal of Power and River Valley Development*, **62**, 3-4, 42-45.

Wu, S. J., Yang, J. C. and Tung, Y. K., 2011, “Risk analysis for flood control structure under consideration of uncertainties in design flood”, *Natural Hazards*, **58**, 1, 117-140. doi : https://doi.org/ 10.1007/s11069-010-9653-z.

Zhang, J., 2002, “Powerful goodness-of-fit tests based on the likelihood ratio”, *Journal of Royal Statistical Society: Series B (Statistical Methodology)*, **64**, 2, 281-294. doi : https://doi.org/ 10.1111/1467-9868.00337.