A statistical analysis of the differences between rainfall estimated by Chennai DWR and conventional rainfall data on monthly and seasonal scales during the Indian northeast monsoon season

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सार - भारत मौसम विज्ञान विभाग में पहला डॉप्लर मौसम रेडार वर्ष 2002 से चे नै में काम कर रहा है और विभिन्न प्रकार के मौसम वैज्ञानिक और जल विज्ञान से संबंधित उत्पाद प्रदान कर रहा है। 2002 से 2013 तक की 12 वर्ष की अवधि के अक्तूबर-जनवरी -दिसम्बर की उत्तर पूर्वी मॉनसून (NEM) ऋतु के समय चेन्नै के 100 कि. मी. की परिधि के अंदर अर्द्धवृत्ताकार में स्थित 34 स्टेशनों के वर्षा मापी से मापी गई वर्षा (RGRF, y) के साथ डी डब्ल्यू आर से आकलित वर्षा (RE RF, x) के सत्यापन और जमीन पर ऑकड़ों के विश्लेषण किए गए। 333 m × 333 m के उच्च विभेदन पर उपलब्ध 1.42 लाख से अधिक विविक्त प्रतिदिन के RERF मान का उपयोग करते हुए मासिक और ऋतुनिष्ठ ऑकड़े प्राप्त किए गए हैं।

इस शोध पत्र का मुख्य उद्देश्य मासिक और ऋत्निष्ठ पैमाने पर उनके बीच एक निश्चित अनुमान तैयार करते हए x और y के विभिन्न सांख्यिकीय प्राचलों का परिकलन करना है। औसत गणना की तीन भिन्न - भिन्न विधियों के उपयोग के दवारा इसका विश्लेषण किया है। चेन्नै डॉप्लर वेदर रेडार के 100 कि. मी. की त्रिज्या में अक्तूबर-नवम्बर-दिसम्बर महिनों में x और y का वार्षिक औसत दोनों ही कालों में धनात्मक और ऋणात्मक बताया गया है जिसकी औसत निरपेक्ष विचलन (MAD) 11 सें. मी. (जो औसत का 17% है) आकलित की गई है। अक्तूबर, नवम्बर, दिसम्बर में अल्पावधि नॉर्मल्स CDLR100 में x के लिए 274.9, 262.6, 96.5 तथा 629.8 मि.मी. तथा y के लिए 243.8, 254.6, 128.0 एवं 627.4 मि. मी. प्राप्त किया गया है और अक्तुबर, नवम्बर, दिसम्बर में क्रमश: 31.2, -8.0, 31.5 एवं -2.4 मि. मी. के पूर्वग्रही मान ज्ञात हुए हैं। अक्तूबर, नवम्बर, दिसम्बर की वर्षा के लिए औसत निरपेक्ष विचलन (MAD) की गणना 12 × 34 के सभी मानों को मिलाकर की गई है जो 19 सें. मी. (औसत का 30%) के लगभग है और काफी वास्तविक है। सहसंबंध विश्लेषण पद्धति का उपयोग करते हए प्रत्येक माह की/पूर्वोत्तर मॉनसून ऋतू में वर्षा की प्रवृत्ति केन्द्रों के भौगोलिक स्थितियों से स्वतंत्र बताई गई है। CDLR100 के 1.42 लाख ग्रिड प्वाइंट्स पर x के अपरिष्कृत मानों तथा समानुपाती संशोधन तकनीक के आधार पर y (y) का आकलित मान ज्ञात किया गया इससे तीन महीनों का तथा अक्तूबर, नवम्बर, दिसम्बर महीनों के लिए स्थानिक औसत प्राप्त किया गया जो क्रमश: x के लिए 273.3, 262.2, 92.5 एवं 628.4 मि. मी. और (y) के लिए 243.4, 254.3, 122.9 एवं 622.1 मि. मी. था । इस शोध पत्र में x की सही व्याख्या के दवारा पूर्वग्रह की आकृति के महत्व पर प्रकाश डाला गया है। इस पूर्वग्रह की प्रकृति कम करने के लिए सांख्यिकीय विचारों के आधार पर कुछ सुझाव दिए गए हैं।

ABSTRACT. The first Doppler Weather Radar (DWR) of India Meteorological Department has been functional at Chennai since the year 2002 providing various meteorological and hydrological products. Validation and statistical analysis of the DWR estimated rainfall (RERF, x) data with rain gauge measured rainfall (RGRF, y) of 34 land based stations located in the semi-circular land area within 100 km radius of Chennai DWR (CDLR100) has been performed for the northeast monsoon (NEM) season of October-November-December (OND) for the 12 year period 2002-13. The monthly and seasonal data have been derived using more than 1.42 lakh discrete daily RERF values available at a high resolution of 333 m \times 333 m.

The major objective of the study is to compute the various statistical parameters of x and y including the bias between them on monthly and seasonal scales and to draw certain inferences. The analysis was done using three different types of averaging. The yearly means of x and y for OND over CDLR100 manifested both positive and negative epochs with the mean absolute deviation (MAD) computed as 11 cm (17% of mean). The short term normals over CDLR100 are derived as 274.9, 262.6, 96.5 and 629.8 mm for x and 243.8, 254.6, 128.0 and 627.4 mm for y for OND work pocember, December and OND yielding bias values of -31.2, -8.0, 31.5 and -2.4 mm respectively. The MAD for OND for one for each month / NEM season is shown to be independent of the geographical locations of the stations using correlation

analysis. Based on the raw values of x and a proportional correction technique, estimated values of $y(\hat{y})$ at the 1.42 lakh grid points of CDLR100 were derived yielding spatial means of 273.3, 262.2, 92.5 and 628.4 mm for x and 243.4, 254.3, 122.9 and 622.1 mm for \hat{y} for the three months and OND respectively. The importance of size of the bias in the correct interpretation of x has been discussed. A few suggestions based on certain statistical considerations have been putforth for decreasing the bias.

Keywords – Indian northeast monsoon, Doppler weather radar, Chennai, Rainfall, Reflectivity, Precipitation accumulation, *Z-R* relation, Marshall-Palmer relation, Validation, Bias, Estimation.

1. Introduction

India Meteorological Department (IMD) is one of the few National Meteorological Services which adopted radar technology for meteorological purposes as early as in the late 1940s with the acquisition of surplus radar equipments after the Second World War. Since then, X- and S-band radars have been inducted into IMD's operational network for monitoring and tracking weather events / systems. Under the modernisation programme of IMD which includes upgradation of its observing systems and the associated hardware, the analogue radars are being replaced by digital Doppler Weather Radars (DWR). The analogue radars provided output in a very basic form of photographic images but played a crucial role in IMD's service delivery in the areas of cyclone warning and aviation meteorology, for several decades. In contrast, the digital DWR is a remote sensing marvel which can be operated round the clock and throughout the year to map the time evolution of weather events in the neighbourhood of its installation. DWRs record high resolution data in digital form and generate sophisticated meteorological and hydrological products for off-line visualisation from multiple perspectives. Continuous weather surveillance enables estimation of the rate of precipitation and its accumulation.

The first *S*-band DWR commissioned by IMD functions at Chennai city located on the southern east coast of India, also called Coromandel coast bordering the Bay of Bengal (BoB). This DWR which is sited atop the Port Trust building, Chennai at an altitude of 53 metres above m.s.l. was put into operational use w.e.f. 20 February, 2002. The technical specifications and salient features of Chennai DWR have been described in Bhatnagar *et al.* (2003) and Rajesh Rao *et al.* (2004). Details of scan strategy and data acquisition are provided in Amudha *et al.* (2014 and 2016).

2. Radar as a tool for rainfall estimation

2.1. Principles of radar-based rainfall estimation

Radar works by transmitting pulses of radio energy, which are focused by the antenna into a narrow beam. When the beam intercepts a target such as rainfall (RF), some of its energy is scattered back to the antenna and detected by the radar receiver as echo power. Received echo power, a function of many factors, is converted into an independent characteristic of RF, viz., reflectivity factor Z, using the famous Probert-Jones Radar Equation (1962). For a raindrop of diameter D, the echo power is proportional to D^6 whereas the water content is proportional to D^3 . Z is converted to rain rate R as both are functions of D. Any type of RF contains millions of drops, tiny droplets to large ones, with highly varying size *vs* Number distribution. Hence, the Z-R relation $Z = AR^{b}$ according to Marshall-Palmer (1948) is a varying function of drop-size-distribution (DSD) where Z is in mm^6/m^3 and R is in mm/hr. A and b are numerical constants attaining values depending on the DSD. When the DSD details are unavailable and the type of precipitation is predominantly stratiform, A = 200 and b = 1.6 are the most commonly used values.

Surface Rainfall Intensity (SRI) is one of the important products generated by a DWR using the Z-R relation. SRI distribution is derived by Chennai DWR at every 10 minutes interval yielding a total of approximately 144 products in a day. Time integration of these 144 sets of SRI data provides another product called Precipitation Accumulation (PAC) which is the 24 hours cumulated radar estimated rainfall (RERF, notation x) data matched to Indian meteorological convention from 0300 UTC of previous day to 0300 UTC of the current day. Fig. 1 is a sample of the daily PAC image generated for the 24 hours period ending at 0300 UTC of 13 November, 2006. SRI and PAC products generated up to a range of 100 km from the DWR location are considered as reliable and accurate. Obtaining correct values of RERF is a challenging task due to various physical, engineering and instrumental aspects (Rinehart, 1991). There is a limit to the minimum altitude that can be observed at longer ranges due to the curvature of the earth. Both the beam width and the size of the sampled volume increase as the range from the radar location increases. Hence the radar based estimates of RF at ground level are less reliable beyond 100 km range though they can be derived up to a range of 250 km albeit with reduced accuracy.

2.2. Validation of RERF

RERF data on various time scales can be gainfully used in several areas associated with human development



Fig. 1. A sample image of the 24 hours PAC product (in dBA) for the period ending at 0300 UTC of 13 November, 2006 generated by DWR Chennai

such as agriculture, hydrology, town planning etc. Instantaneous values are used in nowcasting, aviation, RERF run-off modelling for flash flood forecasting and other applications. RERF from PAC product is used as an input for RF forecasting and its verification. Hence, it is desirable and necessary to conduct studies on validation of RERF with reference to a standard set of measurements for all seasons and locations to have an understanding of the extent of under- or overestimation of RF by radar. The RF data based on conventional rain gauges (RGRF, notation y) is generally taken as reference standard (Raghavan, 2003) for validation of RERF. Conceptual differences in the retrieval of RERF and RGRF exist and hence it is accepted by meteorologists that highly accurate, one-to-one match between the two types of measurements is difficult to achieve and both are generally seen as complementary to each other with the advantages outweighing the limitations. Ability to capture granularity of spatial distribution of RF unavailable from other modes of observation is the key factor in the utility of RERF.

Numerous studies have been undertaken by radar scientists to validate RERF by comparing it with RGRF. A few of them are: Zawadski *et al.* (1986); Austin (1987); Chandrasekar & Bringi (1987) and Kitchen and Blackall (1992). In India, Raghavan and Sivaramakrishnan (1982) and Raghavan *et al.* (1987) used digitised products of the old analogue radar of Madras (now Chennai) to derive *Z-R* relation for the southwest and northeast monsoon (NEM) seasons. Sen *et al.* (2009) and Pradhan and Talukdar (2011) analysed the *Z-R* relation utilising data generated by DWRs at Gadanki and Kolkata respectively. Suresh *et al.* (2005) and Amudha *et al.* (2014) conducted validation studies on RERF obtained from Chennai DWR. Vanaja *et al.* (2014) validated RERF for the Jal cyclone period of



DWR Chennai and the CDLR100 region (' indicates stations with long term normals)

4-9 November, 2010, utilising RF data from RGs in the neighbourhood of Adyar watershed, Chennai.

2.3. Objective of the study

A detailed validation analysis of the monthly and NEM (1 October-31 December) seasonal RERF for the land region within 100 km from Chennai DWR location (henceforth called CDLR100) whose geographical extent is depicted in Fig. 2 has been performed. With the BoB coast in the neighbourhood of Chennai being almost oriented north-south, the CDLR100 region is semi-circular in shape. This region receives an annual normal RF of 90-140 cm (IMD, 2010) which is highest along the coast and decreases westward. Coastal areas of CDLR100 receive RF close to 85-90 cm while the westernmost stations get 40-42cm (~61% and ~43% of annual normal respectively) during NEM. Both convective and stratiform types of RF activity are observed during the NEM season which comprises of active / light / dry spells interspersed with heavy to very heavy RF occurrences including days of cyclonic disturbances as well.

NEM sets in normally over southeast peninsular India (SPI) in the second half of October after the withdrawal of southwest monsoon (SWM) from most parts of India. Various characteristic features of NEM have been widely researched and the list is exhaustive. IMD (1973), Geetha (2011) and Raj (2012) are some of the detailed studies. The normal date of onset (DO) and date of withdrawal (DW) of NEM over coastal Tamil Nadu (CTN) re-determined based on RF data of the period 1901-2000 are 20 October and 30 December respectively (Geetha and Raj, 2015). Tamil Nadu (TN) is the major beneficiary of NEM receiving 438 mm (48%) of its annual RF of 914 mm during OND [IMD (2010) & Raj (loc.cit.)].

Considering these climatological aspects, the validation analysis has been performed by computing means of RERF, RGRF and differences or biases (d = y-x)between the two on monthly (October, November, December) and seasonal (OND) scales. Correlation coefficient (CC) between x and y, mean deviation (MD) and mean absolute deviation (MAD) have been computed and interpreted. In the relevant cases, standard deviation (SD), coefficient of variation (CV) and regression equation of y on x also have been derived. A proportional correction factor (PCF) has been computed and utilised for estimation of RGRF. The various analyses have been performed separately yearwise, stationwise and also by grouping the entire data as a set aggregate. Details of the data used for the analysis have been mentioned in Section 3. The methodologies of analysis and computations on monthly and seasonal scales are explained in Section 4. The techniques through which values of y can be estimated based on x over numerous grid points have also been elaborated. Section 5 contains general discussions while the summary and conclusions are presented in Section 6.

3. Data used

3.1. PAC product derived grid point RERF(x) values of high spatial resolution (333 m \times 333 m) in text format in the form of a 600×600 matrix for each day of the NEM season from 1 October to 31 December for the 12 year period 2002-13 (92 days \times 12 years = 1104 days altogether) have been obtained from the archives of Chennai DWR for the area bounded by 79.3616-81.2083° E and 12.1719-13.9705° N with the centre as the location coordinates of Chennai DWR (80.2883° E / 13.0728° N). For this validation study, PAC data at the grid points of only the CDLR100 area has been considered. Data outside CDLR100, even if available have not been taken into account in view of lesser reliability of RERF data beyond 100 km range. Out of the 2.83 lakhs / day of grid point values (333 m \times 333 m) in the circle of radius 100 km which includes ocean area as well, RF data within CDLR100, processed for the analysis is about half at around 1.42 lakhs/day.

3.2. For the period of the present study, the daily RF (DRF) data of 34 land based rain gauge (RG) stations located in CTN and parts of coastal Andhra Pradesh

(CAP), well representing CDLR100 was obtained from IMD and Public Works and Statistics Departments of the Government of Tamil Nadu. The various sets of DRF data were subjected to thorough quality checks before inclusion. Using the DRF data, the monthly and seasonal RGRF (*y*) data for October, November, December and OND for the 12 year period 2002-13 were generated. Spatial distribution of the 34 RG stations and the location of Chennai DWR along with the CDLR100 region are shown in Fig. 2. Geo-coordinates, three letter station identification (SID), geometric distance from Chennai DWR and elevation above mean sea level (in metres) of the 34 stations under consideration are given in Table 1.

3.3. The daily RERF data at the 34 grid points corresponding to the Long. / Lat. of the 34 RG stations (Table 1) was utilised to derive the monthly cumulative values of RERF for the three months (October, November & December) and NEM (OND) season for each year of 2002-13. A set of 4 matrices of RERF values each of 34 (stations) \times 12 (years) dimension was generated. Similarly, 4 matrices of the same dimension containing RGRF values corresponding to October, November, December and OND were constructed. Thus in all, 8 matrices each 34 \times 12, 4 for RERF(*x*) and 4 for RGRF(*y*) were used. Most of the analysis of the study is based on the data contained in these matrices only.

3.4. Though the total number of RF observations supposed to be available in a matrix is $34 \times 12 = 408$, the actual frequencies are 401, 399, 388 and 386 for October, November, December and OND respectively due mainly to missing RGRF data in some of the cells. The seasonal total cannot be computed even if data for one of the three months is not available and hence the observed lowest frequency for OND. In the case of Chennai Nungambakkam which lies on the periphery of the cone of silence area of the DWR, RERF data is available only for 8 years viz., 2002 and 2007-13 due to the processing methodologies adopted. Save for this, there is no missing RERF data. However, whenever x data was not available for a cell, the y data of the corresponding cell was also not considered for computations even if available and vice versa, to maintain homogeneity.

4. Methodology of computations, results and inferences

4.1. For clarity, we use the notations x_{ts} and y_{ts} , where, t and s denote temporal and spatial variation respectively, of x and y. Thus, x_{ts} is the x value at t-th year and s-th station, t varying from 1 to m (m = 12 is the number of

TABLE 1

Metadata of stations considered in the study

S. No.	Station	State	Station ID	Longitude (°E)	Latitude (°N)	Distance(<i>l</i>) (km) from DWR, Chennai	Elevation a.m.s.l. (metres)
1.	Arakonam	TN	ARK	79.7	13.1	68	51
2.	Chembarambakkam	TN	CMB	80.1	13.0	25	28
3.	Chengalpattu	TN	CGP	80.0	12.7	53	20
4.	Chennai_Meenambakkam	TN	MBK	80.2	13.0	15	16
5.	Chennai_Nungambakkam	TN	NBK	80.2	13.1	4	6
6.	Cheyyar	TN	CYR	79.9	12.7	59	26
7.	Cheyyur	TN	CHY	80.0	12.4	86	3
8.	Chozhavaram	TN	CLV	80.2	13.2	22	12
9.	Kalavai	TN	KLV	79.4	12.8	96	136
10.	Kancheepuram	TN	KCP	79.7	12.8	69	84
11.	Kaveripakkam	TN	KVP	79.5	12.9	91	147
12.	Kelambakkam	TN	KLB	80.2	12.8	33	19
13.	Kovilur_Anaicut	TN	KVL	79.8	12.6	76	107
14.	Madurantakam	TN	MDK	79.9	12.5	76	27
15.	Mamallapuram	TN	MML	80.2	12.6	52	3
16.	Palar	TN	PLR	79.7	12.9	71	155
17.	Panapakkam	TN	PNP	79.6	12.9	80	59
18.	Ponneri	TN	PNR	80.2	13.3	30	11
19.	Poondi	TN	PND	79.9	13.2	47	64
20.	Puttur	AP	PTR	79.6	13.4	89	150
21.	Ramakrishnapet	TN	RKP	79.4	13.2	93	85
22.	Redhills	TN	RDH	80.2	13.2	17	16
23.	Satyavedu	AP	STV	80.0	13.4	54	50
24.	Sholingur	TN	SLG	79.4	13.1	94	127
25.	Sriharikota	AP	SHR	80.2	13.7	73	6
26.	Srikalahasti	AP	SKH	79.7	13.7	98	70
27.	Sriperumbudur	TN	SPP	79.9	13.0	39	41
28.	Sulurpet	AP	SLP	80.0	13.7	76	10
29.	Tada	AP	TDA	80.0	13.6	65	8
30.	Tamaraipakkam	TN	TMP	80.0	13.2	33	41
31.	Tambaram	TN	TBM	80.1	12.9	26	29
32.	Thiruvallur	TN	TVL	79.9	13.1	42	13
33.	Tirutanni	TN	TTN	79.6	13.2	76	85
34.	Uthiramerur	TN	UTM	80.1	12.6	54	26

TN : Tamil Nadu, AP: Andhra Pradesh

TABLE 2 (a)

Yearwise monthly/seasonal means of RERF, RGRF and difference (in mm) over CDLR100 represented by 34 stations considered, 2002-13

Year		October			November		December			OND		
	$\overline{x_{t}}$	\overline{y}_t	\overline{d}_t	\overline{x}_{t}	\overline{y}_t	\overline{d}_t	\overline{x}_{t}	\overline{y}_t	\overline{d}_{t}	\overline{x}_{t}	\overline{y}_t	\overline{d}_{t}
2002	346.0	229.4	-116.6	179.3	219.7	40.4	37.9	51.2	13.2	563.2	500.2	-63.0
2003	132.5	204.5	72.0	105.5	82.8	-22.7	37.2	39.1	1.9	280.9	337.0	56.1
2004	375.0	232.7	-142.3	456.2	218.1	-238.1	2.0	9.5	7.5	877.2	503.1	-374.1
2005	694.5	506.0	-188.5	444.0	466.8	22.8	227.3	363.3	136.1	1374.4	1344.5	-29.8
2006	417.0	355.1	-61.9	225.7	205.2	-20.6	59.8	69.2	9.3	705.4	632.7	-72.7
2007	267.6	285.9	18.3	96.2	94.1	-2.1	213.0	254.5	41.5	576.8	634.6	57.8
2008	201.4	258.7	57.3	406.5	468.4	61.9	9.4	24.1	14.7	630.3	767.7	137.4
2009	55.5	67.1	11.6	428.8	421.9	-6.8	111.3	138.9	27.6	595.6	628.0	32.4
2010	131.6	159.9	28.2	251.9	279.2	27.3	193.8	223.3	29.5	577.4	662.4	85.0
2011	117.1	186.7	69.6	289.7	363.1	73.4	84.3	140.3	56.1	491.1	690.2	199.1
2012	286.3	274.2	-12.0	131.1	102.0	-29.2	119.1	146.3	27.2	536.5	522.5	-14.0
2013	311.2	169.6	-141.6	152.6	117.2	-35.4	30.5	37.5	7.0	494.4	324.3	-170.0

CDLR100 : Semi-circular land region within 100 km from Chennai DWR location

RERF / RGRF - Radar Estimated / Rain Gauge measured rainfall in mm

 \overline{x}_t and \overline{y}_t : Mean RERF/RGRF based on 34 stations (s = 1,...,34) RF values for the period 2002-13, t = 1,...,12

 $\overline{d}_t = \overline{y}_t - \overline{x}_t$ is the mean difference

TABLE 2 (b)

Monthly / Seasonal short term normals of RERF, RGRF and difference (in mm) derived from yearly means over CDLR100 represented by 34 stations

Parameter	October	November	December	OND
\overline{x}	278.0	264.0	93.8	641.9
\overline{y}	244.2	253.2	124.8	628.9
\overline{d}	-33.8 (12)	-10.8 (12)	31.0 (12)	-13.0 (12)
\overline{d} +	42.8 (6)	45.2 (5)	Nil (0)	94.6 (6)
\overline{d} –	-110.5 (6)	-50.7 (7)	31.0 (12)	-120.6 (6)
σ_x , CV_x	166.3 (60)	132.1 (50)	76.4 (81)	258.0 (40)
σ_y , \mathbf{CV}_y	105.3 (43)	139.5 (55)	104.6 (84)	251.3 (40)
MAD of \overline{d}_t	76.7 (31%)	48.4 (19%)	31.0 (25%)	107.6 (17%)
CC(x, y)	0.88*	0.84*	0.97*	0.84*

CDLR100, RERF / RGRF : As in Table 2(a).

 \overline{x} , \overline{y} , \overline{d} : Means of $\overline{x_t}$, $\overline{y_t}$ and $\overline{d_t}$ based on 12 values as given in Table 2(a)

 \overline{d} , \overline{d} , \overline{d} , \overline{d} -: Means of all, positive and negative values of \overline{d}_t with their corresponding frequencies given in brackets

For x and y : σ is SD and CV is given in percentage

MAD and its percentage of \overline{y} is provided in brackets

CC is based on 12 years RF means. *: CC significant at 0.1% Level of Significance (LS)

years) and *s* from 1 to *n* (n = 34 is the number of stations). Similar definition is assigned for y_{ts} as well. The computations to derive normal values for CDLR100 and the various other parameters derived are described in what follows:

4.1.1. Normals based on 12 yearly means

Mean values \overline{x}_t and \overline{y}_t for each of the 12 years were derived by averaging the 34 station values and are precisely defined by

$$\overline{x}_{t} = \frac{1}{n} \sum_{s=1}^{n} x_{ts} ; \overline{y}_{t} = \frac{1}{n} \sum_{s=1}^{n} y_{ts} t = 1, ..., m$$
(1)

The values of \overline{x}_t , \overline{y}_t and the MD $\overline{d}_t = \overline{y}_t - \overline{x}_t$ for the 12 years and for the months / season thus computed are presented in Table 2(a). Inter-annual variation of \overline{x}_t and \overline{y}_t of OND for the 12 years is depicted in Fig. 3. Based on the data of Table 2(a), short term normals for CDLR100 were derived by averaging the yearly means. Letting \overline{x} and \overline{y} to denote these, we have

$$\overline{x} = \frac{1}{m} \sum_{t=1}^{m} \overline{x}_{t} \quad ; \quad \overline{y} = \frac{1}{m} \sum_{t=1}^{m} \overline{y}_{t} \tag{2}$$

Table 2(b) presents \overline{x} , \overline{y} and MD $\overline{d} = \overline{y} - \overline{x}$, means of positive and negative values of \overline{d}_t denoted by $\overline{d} +$ and \overline{d} – respectively. Parameters like MAD of \overline{d}_t , SDs σ_x , σ_y of \overline{x} , \overline{y} respectively and CVs of x, y (CV_x, CV_y) have also been computed and presented.

4.1.2. Normals based on 34 station means

Mean values \overline{x}_s and \overline{y}_s for each of the 34 stations were derived by averaging the 12 yearly values and are defined by:

$$\overline{x}_{s} = \frac{1}{m} \sum_{t=1}^{m} x_{ts} \; ; \; \overline{y}_{s} = \frac{1}{m} \sum_{t=1}^{m} y_{ts} \; s = 1, ..., n \tag{3}$$

The values of \overline{x}_s , \overline{y}_s and the MD \overline{d}_s defined as $\overline{d}_s = \overline{y}_s - \overline{x}_s$ for all the 34 stations and for the months / season are presented in Table 3(a). The spatial variation of \overline{x}_s and \overline{y}_s for OND is depicted in Fig. 4. Based on the data of Table 3(a), short term normals for the entire region CDLR100 were derived by averaging the station normals. Letting \overline{x} and \overline{y} denote the region normals, we define

$$\overline{x} = \frac{1}{n} \sum_{s=1}^{n} \overline{x}_{s} \quad ; \quad \overline{y} = \frac{1}{n} \sum_{s=1}^{n} \overline{y}_{s} \tag{4}$$

Table 3(b) presents \overline{x} , \overline{y} and MD $\overline{d} = \overline{y} - \overline{x}$, PCF = $\frac{\overline{y}}{\overline{x}}$, MAD, range, SD, CV, CC and the regression coefficients (a, b) of \overline{x} and \overline{y} - the definitions of which are similar to those in the previous sub-sections.

4.1.3. Normals based on 12×34 values

All the values of x_{ts} and y_{ts} for the 12 years and 34 stations were pooled together as a single set. The normals \overline{x} and \overline{y} are defined as:

$$\overline{x} = \frac{1}{mn} \sum_{t=1}^{m} \sum_{s=1}^{n} x_{ts} \quad ; \quad \overline{y} = \frac{1}{mn} \sum_{t=1}^{m} \sum_{s=1}^{n} y_{ts} \quad (5)$$

All the parameters presented in Table 3(b) have been derived here also and the values are presented in Table 4.

4.1.4. In the equations (1) to (4), both *m* and *n* are the dividing frequencies whereas for (5) it is *mn* provided there are no missing observations. In reality, the dividing frequencies are the actual number of available observations for the respective cases. Each of the three values of \overline{x} , \overline{y} and \overline{d} computed as indicated in Sections 4.1.1, 4.1.2 and 4.1.3 and presented in Tables 2(b), 3(b) and 4 must necessarily be equal conceptually. However, missing data in the data matrices of each month as mentioned in Section 3.4 has led to minor differences amongst them. Similarly, the sum of \overline{x} and \overline{y} for the months October, November, and December must obviously be equal to the seasonal (OND) \overline{x} and \overline{y} . Minor differences in these values are evident which are also due to the same reason stated above.

4.2. The results of the computations carried out as stated in Section 4.1.1 are provided in Tables 2(a&b). The yearwise seasonal (OND) means are depicted in Fig. 3. The inferences are discussed herein below:

4.2.1. Inferences from Tables 2(a&b)

October : Normally, the onset of NEM season takes place in October. It is seen that \overline{x} , \overline{y} and \overline{d} (in mm) for October are 278.0, 244.2 and -33.8 respectively indicating slight overestimation of RF by radar. Equal number of years (6 each) have \overline{d} + and $\overline{d} - \sigma_x$ (166.3) and CV_x (60%) are greater than σ_y (105.3) and CV_y (43%)



indicating higher inter-annual variability of x when compared to y. MAD is 76.7 mm which is 31% of \overline{y} . The CC (x, y) is 0.88 significant at 0.1% level of significance (LS).

November : The rainiest month over CTN is November, which is also taken as the representative month of NEM season. In the case of November, \overline{x} , \overline{y} and \overline{d} are 264.0, 253.2 and -10.8 mm respectively indicating negligible difference. Frequencies of \overline{d} + and \overline{d} - are almost same (5 and 7). σ_x (132.1) and CV_x (50%) are almost equal to σ_y (139.5) and CV_y (55%) respectively suggestive of near identical inter-annual variability of x and y. MAD is 48.4 mm which is 19% of \overline{y} . The CC (x, y) is 0.84 significant at 0.1% LS.

December : Climatologically, December is the preferred month of withdrawal of NEM and RF activity is less compared to October and November, \overline{x} , \overline{y} and \overline{d} are 93.8, 124.8 and 31.0 mm indicating underestimation of RF by radar in December. All the 12 years have \overline{d} –. MAD is 31.0 mm which is 25% of \overline{y} . CC of 0.97 is highly significant at 0.1% LS. σ_x (76.4) is less than σ_y (104.6) but CV_x (81) and CV_y (84) are almost equal indicating similar type of variation in x and y during the month.

OND: For OND, \overline{x} , \overline{y} and \overline{d} are 641.9, 628.9 and -13.0 mm respectively. The difference is insignificant while considering the NEM season as a whole. Equal number of years (6 each) have \overline{d} + and \overline{d} –. MAD is 107.6 mm which is 17% of \overline{y} . The CC is 0.84 significant at 0.1% LS. Difference between σ_x and σ_y (258.0 and 251.3) is almost negligible while CV_x and CV_y are the



Fig. 4. Distribution of mean RERF (x) and RGRF(y) (in cm) for OND, 2002-13 at the grid points of 34 stations plotted as per legends 'x y 'and 'x y' (Refer Fig. 2 for station ID)

same at 40% indicating an almost identical inter-annual seasonal variability of *x* and *y*.

The above analysis shows that the MAD ranges from 19-31% for the months and is 107.6 mm or 17% for the season. The interpretation is that the seasonal total values of x and y for the entire area differ by nearly 11 cm on year-to-year basis. When we peruse the inter-annual variation of spatial means of x and y presented in Table 2(a), it is evident that negative and positive biases display even epochal behaviour despite the shorter period of the entire study. It is seen from Table 2(a) that while values of \overline{d} are both positive and negative for October and November, \overline{d} is positive for December in all the 12 years, ranging from 1.9 to 136.1 mm. For OND, the period 2004-06 has negative values of \overline{d} , substantially so in 2004 when there is an overestimation of OND RF with a d value of -374.1 mm. The 5 year period 2007-11 of consecutive excess RF years for the CDLR100 region manifests positive bias in all the years of OND with significant underestimation in both 2008 and 2011. The years 2004-06 display a negative bias and hence overestimation by radar (Fig. 3).

A study on validation of daily RERF data generated by Chennai DWR for the pre-monsoon (March-May) and the NEM (OND) seasons for the five year period 2006-10, using daily RGRF data of 16 stations over the CDLR100 region was undertaken by Amudha *et al.* (2014). A

TABLE 3 (a)

Stationwise monthly/seasonal short term means of RERF, RGRF and difference (in mm) at 34 stations within CDLR100

		October		November		December			OND				
S. No.	SID	\overline{x}_s	\overline{y}_s	\overline{d}_s									
1.	ARK	211.6	162.0	-49.6	212.2	211.5	-0.7	86.8	106.4	19.7	510.6	481.7	-29.0
2.	CMB	329.2	308.3	-21.0	329.5	334.4	5.0	94.6	147.1	52.5	736.6	802.7	66.1
3.	CGP	317.2	284.3	-33.0	265.0	313.3	48.3	79.4	169.7	90.3	661.6	767.4	105.7
4.	MBK	354.1	356.7	2.5	302.5	333.4	30.9	95.2	147.5	52.3	751.8	837.5	85.7
5.	NBK	187.1	266.9	79.8	311.5	345.1	33.6	93.4	152.1	58.7	592.1	764.1	172.1
6.	CYR	292.7	179.8	-112.9	285.0	182.9	-102.1	97.3	116.8	19.5	652.8	479.5	-173.3
7.	CHY	277.8	226.8	-51.0	232.3	242.2	10.0	80.9	119.3	38.3	591.0	588.3	-2.7
8.	CLV	330.8	262.9	-67.9	323.5	293.1	-30.4	131.4	135.7	4.3	785.7	691.7	-94.1
9.	KLV	208.5	164.9	-43.6	179.9	180.8	0.8	68.7	98.1	29.5	470.2	458.7	-11.6
10.	KCP	218.2	199.5	-18.7	194.0	227.8	33.8	72.6	124.6	52.0	484.8	551.9	67.1
11.	KVP	187.9	160.3	-27.6	172.8	145.0	-27.8	60.0	90.6	30.5	426.0	402.8	-23.1
12.	KLB	341.8	281.6	-60.1	256.4	307.7	51.2	95.9	151.9	55.9	694.1	741.2	47.0
13.	KVL	278.6	215.7	-62.9	285.6	230.8	-54.8	103.8	136.1	32.3	634.1	601.7	-32.4
14.	MDK	261.5	252.3	-9.2	279.4	267.5	-12.0	89.3	158.5	69.2	630.2	678.2	48.0
15.	MML	354.5	334.3	-20.2	263.0	361.8	98.8	91.6	168.9	77.3	709.2	865.1	155.9
16.	PLR	210.0	145.0	-65.0	203.6	142.2	-61.4	79.7	88.3	8.6	505.4	386.9	-118.5
17.	PNP	222.0	193.0	-29.0	173.1	197.8	24.7	68.8	113.3	44.5	449.0	500.1	51.1
18.	PNR	327.1	313.3	-13.8	292.9	306.1	13.2	120.5	134.1	13.6	740.5	753.6	13.0
19.	PND	274.7	253.1	-21.6	284.9	212.8	-72.1	116.7	120.9	4.1	676.3	586.8	-89.5
20.	PTR	224.3	190.4	-33.9	190.0	171.9	-18.1	83.2	91.4	8.2	497.5	453.7	-43.8
21.	RKP	205.8	132.6	-73.2	182.5	129.6	-53.0	71.8	72.0	0.1	475.8	327.2	-148.7
22.	RDH	392.1	301.9	-90.2	339.3	294.8	-44.5	125.7	137.6	12.0	857.0	734.3	-122.7
23.	STV	268.9	264.9	-4.0	292.5	276.1	-16.5	115.7	135.8	20.1	677.1	676.8	-0.4
24.	SLG	215.7	153.5	-62.2	179.8	141.7	-38.1	63.1	75.3	12.2	458.7	370.6	-88.1
25.	SHR	323.6	355.3	-31.7	375.2	327.9	-47.3	151.1	136.4	-14.7	849.9	819.6	-30.3
26.	SKH	265.5	272.6	7.1	305.2	336.8	31.6	94.0	127.3	33.4	664.6	736.7	72.0
27.	SPP	271.1	244.6	-26.5	257.7	258.3	0.5	72.8	140.9	68.1	601.6	643.8	42.2
28.	SLP	321.3	295.0	-26.3	352.2	286.7	-65.5	139.7	152.6	12.9	752.1	746.8	-5.3
29.	TDA	250.0	309.0	58.9	267.2	344.9	77.7	139.7	172.6	32.8	657.0	826.5	169.5
30.	TMP	289.3	254.6	-34.7	302.3	273.4	-28.8	132.7	125.4	-7.3	724.3	653.4	-70.9
31.	TBM	331.5	301.4	-30.2	260.8	286.1	25.3	90.1	136.0	45.9	682.4	723.4	41.0
32.	TVL	277.4	227.3	-50.1	290.5	251.7	-38.7	133.5	148.3	14.9	701.3	627.3	-74.0
33.	TTN	208.8	184.0	-24.9	211.8	169.6	-42.3	63.4	81.6	18.1	443.9	401.7	-42.2
34.	UTM	316.3	240.1	-76.3	272.9	271.0	-1.9	78.0	139.7	61.6	667.3	650.8	-16.5

SID - Station ID : As in Table 1, CDLR100, RERF/RGRF : As in Table 2(a)

 \bar{x}_s , \bar{y}_s : Mean RERF/RGRF for each station based on 12 years RF for the period 2002-13

 $\overline{d}_s = \overline{y}_s - \overline{x}_s$ is the difference in RF

TABLE 3 (b)

Monthly / Seasonal short term normals of RERF, RGRF and difference (in mm) over CDLR100 derived from 34 station means

Parameter	October	November	December	OND
\overline{x}	274.9	262.6	96.5	629.8
\overline{y}	243.8	254.6	128.0	627.4
$\overline{y} / \overline{x}$	0.89	0.97	1.33	0.99
\overline{d}	-31.2 (34)	-8.0 (34)	31.5 (34)	-2.4 (34)
\overline{d} +	36.0 (5)	32.4 (15)	34.2 (32)	81.2 (14)
<u>d</u> -	-42.7 (29)	-39.8 (19)	-11.0 (2)	-60.8 (20)
$\sigma_x CV_x$	54.6 (20)	54.9 (21)	25.0 (26)	118.0 (19)
$\sigma_y CV_y$	61.4 (25)	67.3 (26)	26.7 (21)	151.0 (24)
Min & Max_ \overline{y}_s	187.1, 392.1	172.8, 375.2	60.0, 151.1	426.0, 857.0
Min & Max_ \overline{y}_s	132.6, 356.7	129.6, 361.8	72.0, 172.6	327.2, 865.1
MAD of \overline{d}_s	41.7 (17%)	36.5 (14%)	32.8 (26%)	69.2 (11%)
CC(x, y)	0.79*	0.76*	0.52**	0.82*
a,b	0.88, 0.07	0.93, 10.86	0.56, 74.22	1.055, -37.0
CDLR100 : As in Table	2(a)			

 \overline{x} , \overline{y} , \overline{d} : Means of \overline{x}_s , $\overline{y}_s \& \overline{d}_s$ based on 34 stations values as given in Table 3(a)

 \vec{a} , $\vec{a} + \vec{a}$ - : Means of all, positive and negative values of \vec{a}_{s} with their corresponding frequencies given in brackets

σ, CV : As given in Table 2(b); Min, Max : Minimum and Maximum

MAD as the percentage of \overline{y} is provided in brackets;

CC(x, y) : Based on 34 stations RF means.

a, b are the regression coefficients; $\overline{y}_s = a \overline{x}_s + b$ is the regression equation

* / **: CC significant at 0.1/1 % LS

total of 2055 pairs when both RERF and RGRF had reported at least 1 mm daily RF were used out of which Mar-May accounted for 13% or 267 pairs only. As such the results of the study can be taken as almost fully valid during the OND season. The MAD ranged from 8.7 mm to 16.4 mm while the MD between RERF and RGRF was -6.8 mm indicating underestimation of RF by the radar during the period of study. The CC between RERF and RGRF was 0.80 indicating a high degree of relationship. The results from Amudha et al. (2014) indicating underestimation of DRF by radar during 2006-10 are consistent with the results of the present analysis.

4.2.2. Inferences from Tables 3(a&b)

The outcome of the computations carried out as stated in Section 4.1.2 is presented in Tables 3(a&b) while the seasonal (OND) means for the 34 stations are depicted in Fig. 4. The inferences are as under:

October : It is seen from Table 3(b) that $\overline{x}, \overline{y}$ and \overline{d} (in mm) are respectively 274.9, 243.8 and -31.2 mm indicating slight overestimation of RF by radar. Stations with \overline{d} +(5) are substantially less than those with \overline{d} -(29). MAD is 41.7 mm which is 17% of \overline{y} . CC between x and y is 0.79 significant at 0.1% LS.

November : $\overline{x}, \overline{y}$ and \overline{d} are 262.6, 254.6 and -8.0 mm respectively indicating negligible difference. Frequency of stations which have reported \overline{d} +(15) is slightly less than those that reported \overline{d} -(19). MAD is 36.5mm which is 14% of \overline{y} . CC between x and y is 0.76 significant at 0.1% LS.

December : Underestimation of RF by radar is evident from the values of \overline{x} , \overline{y} and \overline{d} which are 96.5, 128.0 and 31.5 mm respectively. Frequency of stations with \overline{d} +(32) is much greater than those of \overline{d} -(2). MAD is 32.8 mm which is 26% of \overline{y} . CC between x and y is 0.52 significant at 1% LS.

OND : For OND, $\overline{x}, \overline{y}$ and \overline{d} are 629.8, 627.4 and -2.4 mm respectively indicating almost negligible difference. Frequency of stations having d t = (14) is less than \overline{d} - (20). MAD is 69.2 mm which is 11% of \overline{y} .



Fig. 5. Monthly short term normals of RERF and RGRF (in mm) over CDLR 100 derived by pooling in all the values of 12 years (2002-13) for 34 stations

CC at 0.82 between \overline{x} and \overline{y} is highly significant at 0.1% LS.

It is observed that $\sigma_x < \sigma_y$ for all the three months. For OND, σ_x and CV_x (118.0, 19%) are less than σ_y and CV_y (151.0, 24%) respectively. The range of *x* (426.0-857.0) is also much lower than the range of *y* (327.2-865.1). These figures clearly indicate that within the CDLR100 region, the spatial variation of *y* has not been fully captured by *x*.

4.2.3. Inferences from Table 4

The results of the computations carried out as stated in Section 4.1.3 are presented in Table 4. The values of $\overline{x}, \overline{y}$ and \overline{d} (in mm) thus obtained by pooling all the data of 12 years for 34 stations for October, November, December and OND are (276.0, 243.4, -32.5), (262.6, 254.1, -84.0), (96.9, 128.4, 131.5) and (634.3, 631.4, -2.9) respectively. The variations in values of \overline{x} and \overline{y} for October, November and December are pictorially depicted in Fig. 5. The frequency of \overline{d} + is less than that of \overline{d} - for October, more in November and December and nearly equal in OND. The CV_x and CV_y of OND indicate nearly identical variability of around 50%. The statistics of Table 4 have been derived using raw values for a location for a month / season. They are not temporally or spatially averaged values like those used in the derivation of Tables 2(b) and 3(b). As such, the ranges of x and y of individual values depicted in Table 4 are much higher than the ranges presented in Tables 2(b) and 3(b). The MAD values of Table 4 for October, November, December and which are 111.4, 99.4, 57.8 and 188.8 mm OND respectively, clearly indicate the size of the individual

differences between x and y. For OND with a negligible \overline{d} of -2.9 mm, the MAD is 30% of \overline{y} which again clearly shows that the deviations are sizeable though \overline{x} and \overline{y} are nearly identical. When all the pairs are pooled, the CCs for October, November, December and OND are by and large lower than the corresponding values of Tables 2(b) and 3(b) and vary from 0.64 to 0.75 though highly significant at 0.1% LS and more stable due to the larger sample size.

4.3. Estimation of RGRF based on RERF

In the previous sections, temporal, spatial and stationwise differences between x and y in monthly / seasonal scales were analysed. The results presented in Tables 2-4 and discussed above, provide an insight on the quantum of error / bias when both x and y are compared. The measurement of y, i.e., RGRF is a simple and straight-forward exercise with minimum scope for error if the various requirements of maintaining a RG and taking observations are followed correctly. However, x, *i.e.*, RERF is remote sensed data derived from an empirical relation as explained in Section 2. It is perfectly reasonable to take y as the actual data and x as an estimated data of y. However, in the present study area over CDLR100, y at 34 specific locations and x at a very high resolution of nearly 1.42 lakh grid points are available. It would be advantageous to generate an estimate of y (notation : \hat{y}) based on x at each grid point and then use the generated distribution of \hat{y} for further analysis and use. The estimate \hat{y} should provide a correction to x and hence reduce the bias inherent in x to some extent.

The well-known form of the regression equation of y on x is

$$(y - \overline{y}) = r \frac{\sigma_y}{\sigma_x} (x - \overline{x})$$
(6)

with symbols having the usual meanings. In the first instance, the tendency is to use the regression equation as an estimation equation which is perfectly correct from the statistical point of view. However, while endeavouring to estimate *y* based on *x*, for the present study, a closer analysis reveals that the regression equation might lead to misleading estimates in some cases. To demonstrate such a deficiency, we consider regression equation of *y* on *x* for OND given in Table 4. Here, all the 12 × 34 values of *x* and *y* have been pooled in to derive the regression equation which is y = 0.76x + 149.74 with a CC of 0.72. The spatial distribution of short term normals of OND for

TABLE 4

Parameter	October	November	December	OND
\overline{x}	276.0	262.6	96.9	634.3
\overline{y}	243.4	254.1	128.4	631.4
$\overline{y} / \overline{x}$	0.88	0.97	1.32	0.99
\overline{d}	-32.5 (401)	-8.4 (399)	31.5 (388)	-2.9 (386)
\overline{d} +	87.9 (180)	86.4 (210)	66.4 (261)	184.1 (195)
\overline{d} -	-130.6 (221)	-113.8 (189)	-40.2 (127)	-193.7 (191)
$\sigma_x \operatorname{CV}_x$	199.1 (72)	165.4 (63)	90.7 (94)	310.8 (49)
$\sigma_y CV_y$	147.4 (61)	176.7 (70)	121.7 (95)	325.7 (52)
Min & Max_ x_{ts}	30.6, 1237.5	14.0, 824.7	0.0, 412.2	76.5, 2107.7
Min & Max_ y_{ts}	5.0, 1077.8	3.6, 820.0	0.0, 727.5	110.0, 1862.5
MAD of d_{ts}	111.4 (46%)	99.4 (39%)	57.8 (45%)	188.8 (30%)
CC(x, y)	0.64*	0.68*	0.75*	0.72*
a, b	0.47, 112.85	0.73, 61.97	1.01, 30.66	0.76, 149.74

Monthly / Seasonal short term normals of RERF, RGRF and difference (in mm) over CDLR100 derived by pooling in all the values of 12 years (2002-2013) for 34 stations

CDLR100 : As in Table 2(a).

 \overline{x} , \overline{y} , \overline{d} : Means of x_{ts} , y_{ts} , d_{ts} based on 12 (years) x 34 (stations) values of RF

 \vec{a} , \vec{a} +, \vec{a} -: Means of all, positive and negative values of \vec{a}_{μ} with their corresponding frequencies given in brackets.

 σ , CV, Min, Max : As in Table 2(b); MAD of \overline{d}_{μ} and its percentage w.r.t. \overline{y} is provided in brackets

CC(x, y): Based on 12(years) × 34(stations) values of RF

a, *b* : Regression coefficients; $\overline{y}_{ts} = a\overline{x}_{ts} + b$ is the regression equation.

*: CC significant at 0.1 % LS

TABLE 5

Monthly / Seasonal mean RERF, estimated RGRF, differences and error of estimate (in mm) over the CDLR100 region

Parameter	Number of grid points	October	November	December	OND
\overline{x}	1.4 lakhs	273.3	262.2	92.5	628.4
$\overline{\hat{Y}}$	1.4 lakhs	243.4	254.3	122.9	622.1
\overline{D}	1.4 lakhs	-29.9	-7.9	+30.4	-6.3
$\left \overline{D}\right $	1.4 lakhs	29.9	7.9	30.4	6.3
$\left \overline{e}\right $	34	28.4	36.2	25.2	68.9

CDLR100 : As in Table 2(a). For 1.4 lakh grid points; \overline{X} : Mean of RERF (X); $\overline{\hat{Y}}$: Mean of ERGRF (\hat{Y}); $\overline{D} = \overline{\hat{Y}} - \overline{X}$;

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$\left| \overline{D} \right|$ = Absolute of \overline{D} ; For 34 stations $\left| \overline{e} \right| = \left| y - \hat{y} \right|$

both x and y for 34 stations given in Table 3(a) are presented in Fig. 4. It is clear from the spatial variation of y that OND RF is higher close to the coast and decreases westward away from the coast, a climatological feature of NEM RF which is well-known (Raj, 2012). The distribution of x as seen from Fig. 4 also expectedly manifests similar pattern.

Suppose for example, x values are taken as 900 and 1000 mm. Then, the corresponding \hat{y} values derived from the regression equation y = 0.76x + 149.74 are 833.2 and 909.1 mm respectively, making x an overestimation of y. If x values are taken as 400 and 500 mm, then \hat{y} values are 453.5 and 529.4 mm respectively making x an underestimation of y. An easy computation, *i.e.*, (considering y = 0.76x + 149.74 > x and obtaining x = 623.9) reveals that the regression equation y = 0.76x + 149.74 overestimates / underestimates y when x < / > 623.9. In view of the climatological feature of east to west decrease of OND RF, the above relation means that x is an underestimation of y closer to the coast and overestimation in the western parts of CDLR100 leading to systematic geographical bias.

To overcome such an incongruity, two very simple methodologies are proposed as alternatives for a case when the regression equation introduces systematic bias in the estimate. These are in terms of the equations:

$$\hat{y} = x + \overline{y} - \overline{x} \tag{7}$$

and
$$\hat{y} = \frac{\overline{y}}{\overline{x}}x$$
 (8)

In equation (7) above, the mean error *viz.*, $\overline{d} = \overline{y} - \overline{x}$ is merely added to each value of *x* and so \hat{y} retains the spatial gradient of *x*. However, the equation can lead to erroneous estimates in some instances. For example, for the month of December, $\overline{y} - \overline{x}$ is 31.5 mm. In some years, if x = 0 at some locations, \hat{y} would be 31.5 mm (Table 4). The regression equation for December is y = 1.01x+30.66 which also yields $\hat{y} = 30.7$ mm when x = 0 which has been realised in a few instances (Table 4). Thus, for December, equation (7) can return high values of *y* when *x* is zero and hence not appropriate.

Equation (8) above proposed is just a proportional correction based on \overline{x} and \overline{y} . Here, if x = 0, $\hat{y} = 0$ thus eliminating the possibility of return of significant non-zero values of \hat{y} when x = 0. As RF is measured in absolute values, computing a ratio of two RF values is fully justified. But then, both \overline{x} and \overline{y} should not be

close to zero so that the ratio remains stable. Thus, this technique will work for monthly / seasonal RF but may not be suitable when we deal with daily / weekly RF or a season of sparse RF. When we consider the regression equation of Table 3(b), for October, we find that the equation y = 0.88x + 0.07 is almost same as that obtained using equation (8) and the PCF here is 0.89. By and large, similar is the case for November, as well. However for December, the regression equation (7) is unsuitable for reasons explained above. Similarly, in all the cases of Table 4 also, neither the regression equations of each month nor equation (7) are appropriate. In both these instances, the estimate based on PCF of equation (8) appears to be the most suitable option. An almost similar concept has been used by Vanaja *et al.* (2014).

4.4. *CC between l and d*

Another question arises as to whether \overline{d} values in Tables 3(a) and 4 (\overline{d} of 34 stations for 12 years) indicate any systematic bias with reference to distance of a station (1) from Chennai DWR. To investigate this, CCs between *l* and \overline{d} / $|\overline{d}|$ were computed. The values of *l* and \overline{d} are given in Tables 1 and 3(a) respectively. For October, November, December and OND, the CCs between l and \overline{d} are -0.17, -0.20, 0.20 and -0.21 while the CCs are -0.09, 0.02, -0.21 and -0.26 between l and $|\overline{d}|$ respectively. Clearly, all the above CCs which are insignificant indicate absence of relationship between l and \overline{d} of the concerned station. The analysis was extended by pooling and compositing all the 12 \times 34 values for October, November, December and OND and computing the CCs between l and $\overline{d} / |\overline{d}|$. The CCs between l and \overline{d} then are -0.02, -0.06, -0.05 and -0.06 while the CCs between *l* and $|\overline{d}|$ are -0.10, -0.08, -0.05 and -0.11 respectively which are also insignificant despite the large sample size varying between 386 and 401 (Table 4). Thus, it can be concluded that the difference $\overline{d}_s = \overline{y}_s - \overline{x}_s$ [Table 3(a)] is by and large free from any geographical bias.

4.5. Correct usage of Tables 2-4

Tables 2-4 which present in detail the various statistics of the study can be gainfully utilised to correct the bias inherent in x and to derive estimates of y. If for a given year and month / season, x is available over a specific location say P, then y can be estimated based on PCF and equation (8). Suppose the year is 2008 and season is OND. We have $\overline{x} = 630.3$, $\overline{y} = 767.7$ mm from

Table 2(a) and so PCF value is 1.218. If x = 1000 mm then $\hat{y} = 1218$ mm. The assumption here is that the PCF value of 1.218 is representative of the entire region. Suppose the PCF values for the year 2008, OND season and the 34 stations separately computed show varying pattern, a PCF value based on a few stations lying in the neighbourhood of P would yield a better estimate of y. Table 3(b) could be used to estimate the short term normal RF of any grid point within the CDLR100 area. Again, it is possible to derive PCF for a sub-region, for example, a district, catchment area of a dam etc., centred near P and use this value to compute \hat{y} from the extensive list of derived parameters given in Table 3(b). In Table 4, all the values have been pooled together for a given month / season and the parameters derived are more stable due to the large sample size. Here again, equation (8) based on the PCF concept appears to be the most suitable. The statistics of Table 4 could be used to derive \hat{y} when the given value of x is hypothetical and is not associated with any specific year. For, if the year is known, we can always use the PCF values from Table 2(a) for estimation.

It is thus evident that the estimation techniques for y based on x though basically very simple have to be assessed for their suitability, lack of systematic bias, returning acceptable estimates etc. The same technique may not be suitable in all cases. The regression equation concept may work well in certain cases. It must be stated that the standard error (SE) and also the MAD (which is approximately 0.8 SE) of \hat{y} would be lower if we used the regression equation concept and that equation (7) and (8) would invariably return higher SEs. However, lower SE alone need not be the criterion for selection of a suitable methodology for estimation as elaborated in our earlier discussions.

4.6. Mean spatial distribution of RERF and estimated RGRF over CDLR100

The spatial distribution of short term normals of RERF (denoted by X in this section) over the circular area of 100 km centred at DWR Chennai location, which includes the oceanic areas as well, for the period 2002-13 for the months of October, November, December and the season OND has been generated and depicted in pictorial format in Amudha *et al.* (loc.cit). The spatial distributions of X thus obtained for the three months and season, but confined only to the semi-circular land region, *viz.*, CDLR100 and based on nearly 1.42 lakh observations are presented in Fig. 6. The RF distributions over the region are detailed and are of much higher resolution than what is provided by the distribution of RGRF (y) based only on 34 stations data of OND (Fig. 4) and corresponding similar

distributions for individual months. However, as shown in Section 4.3, it is possible to derive a better RF distribution by correcting the inherent bias in *X* to the extent possible and generate distributions of estimated value of RGRF (Y, in this section) viz., \hat{Y} based on X for the entire region. This exercise was carried out and the short term normals of \hat{Y} at approximately 1.42 lakh grid points have been derived using the PCF ratios for October, November, December and OND based on statistics presented in Table 3(b) for the respective months/season and the results are given in Table 5. The resulting spatial distributions of \hat{Y} which are also presented in Fig. 6 should provide a more realistic depiction of actual RF over the entire region than the raw distributions of Xpresented alongside. Though the spatial patterns and geometry of the distributions of both X and \hat{Y} are similar for a given period, X values have been proportionally altered yielding values closer to Y which in fact is taken as the standard.

From the spatial distributions of X and \hat{Y} , it is now possible to derive their means by spatial averaging yielding short term normals based on 1.42 lakh grid points data over the CDLR100 region. The mean values of X, \hat{Y} and difference $D = \hat{Y} - X$, denoted respectively by $\overline{X}, \overline{\hat{Y}}$ and \overline{D} along with the absolute value of \overline{D} (in mm) for October, November, December and OND presented in Table 5 are 273.3, 262.2, 92.5, 628.4 for X, 243.4, 254.3, 122.9 and 622.1 for \hat{Y} , -29.9, -7.9, 30.4 and -6.3 for D respectively. The values of Table 5 are comparable with those for the same set of parameters (i.e., x and y) given in Tables 2(b), 3(b) and 4 based on data of 34 stations. The OND seasonal normal values for x are 641.9, 629.8, 634.3 and 628.4 mm and for y are 628.9, 627.4, 631.4 and 622.1 mm respectively corresponding to Tables 2(b), 3(b), 4 and 5. It is seen that the latter values are slightly lower than the corresponding former values. In Table 5, \overline{D} and $|\overline{D}|$ are the MD and MAD between X and \hat{Y} which are parameters of interest and relevance as they provide an index of reliability of the estimate.

The parameter 'e' defined as e=y- \hat{y} is the difference between actual (y) and estimated value of y (\hat{y}) and is called error of estimate. Table 5 also presents the values of $|\vec{e}|$ which are 28.4, 36.2, 25.2 and 68.9 mm respectively for the 4 periods and can be interpreted as the error inherent in the estimate of y, *i.e.* \hat{y} . Though based on only 34 observations, these values can be taken as representative for the entire region for the given month/season. Obviously, the mean error of estimation \vec{e} is zero.



Fig. 6. Short term normals of RERF and estimated RGRF (both in cm) based on 1.42 lakhs of grid point data over CDLR100 for October, November, December and OND for the period 2002-13

5. Discussions

In the foregoing sections, various results derived in the validation study to determine the bias inherent in RERF vis-à-vis RGRF for the CDLR100 region were presented. No doubt RERF data is a very handy input to meteorologists and hydrologists, available as it is on a continuous basis at numerous grid points which perhaps no land based rain measuring system could match. However, the bias in RERF must be known to the user for the correct interpretation and application of the data. Such an analysis on the extent of bias on monthly and seasonal scales has been attempted for Chennai DWR for the rainiest season for the region under study. The normal values of RERF, RGRF and the bias have all been determined for different periods with different methods of averaging. Though bias is obviously significant for a given station, temporal and spatial averaging does bring down its size substantially as shown by the negligible normal bias for OND. When the normal bias itself is significant, that the MAD would also be higher is evident but even when normal bias is negligible MAD could still be on the higher side. That the seasonal OND RERF carries a MAD of 107.6 mm (derived as average for 12 years) for the CDLR100 region [Table 2(b)] and 188.8 mm at a station (Table 4) despite negligible normal biases clearly reveals the size of individual bias values. This is an important aspect which should be noted by the users. When the RERF data is used for verification of monthly and seasonal forecasts of RF, preferably the size of MAD should appropriately be factored into the verification scheme.

There is some evidence of systematic over-/underestimation of actual RF by radar in the monthly scales (October and December) though absent in November and the all important seasonal scale. As shown, there have been epochs of both patterns when we consider OND RF and mixed patterns of both over-/under-estimation when we consider monthly RF. The pattern of bias is likely to differ slightly if intra-seasonal RERF for shorter durations, say daily/weekly is considered. Obviously there would be more 'noise' in the data if the period is shorter and the area smaller.

The present study is an attempt on the analysis of RERF bias for the NEM season of OND only. A similar study on RERF generated by Chennai DWR over CDLR100 for other seasons also is worth undertaking for a complete and exhaustive evaluation of DWR performance, which the authors hope will fructify in due course. The study on the various aspects of NEM based on RERF of Chennai DWR (Amudha *et al.*, loc.cit.) brought out several new features of NEM. In this analysis, plenty of RERF statistics over the region have been generated and presented in monthly/seasonal scales. The NEM RERF climatology of the CDLR100 region based on just 12 years short term data is thus available, though there is still ample scope for further research work on this topic.

IMD has installed DWRs covering many coastal and interior locations in India. As of now, RERF data is available for periods ranging from 6-13 years from such DWRs. Studies on RF climatology and biases based on RERF thus generated would be a pre-requisite for the correct interpretation of RERF in all scales *viz.*, daily, monthly and seasonal. The method of analysis adopted and used in this study which is basic, straight-forward, easy to understand and with results simple to interpret could be of help to researchers who attempt such studies.

Finally, another important aspect that needs to be addressed is whether it is possible to get the size of the biases reduced over a period of time. Hardware and software upgradation of the DWR, utilising the technology of the dual polarimetric measurements in particular, should conceptually lead to such a development. The values of A = 267 and b = 1.345 are used in the Z-R relationship by Chennai DWR. Any modification of the values would need data on drop sizes of RF measured usually by a disdrometer. Varying Z-R relations based on the type of RF such as convective, shower type etc. derived using data from a few strategically positioned disdrometers in the CDLR100 might lead to better estimates of RF.

Aside from such an approach, it must also be possible to effect some improvement based purely on statistical considerations. In a quantitative precipitation forecasting (QPF) scheme for DRF of Madras (Raj *et al.*, 1996) the liquid water content added to the atmosphere in 24 hrs (LWC24) is multiplied by a ratio called precipitation efficiency (PE) to obtain forecast DRF. Frequently PE has been taken as 0.3 but in the above QPF study, PE was computed based on previous few days of data of LWC24 and actual DRF realised which led to better performance of the model. In analogy with this technique, for DWR based RF, re-deriving *A* and *b* based on both RERF and RGRF data for few previous days and using such values in the generation of RERF for the day under consideration might lead to better estimates.

Whether it is possible to derive better ERGRF values which are closer to RGRF thereby reducing the standard / mean error of estimate at the same time yielding consistent estimates is yet another issue which could be addressed. One possible methodology is by deriving ERGRF for all the grid points based on the data of RERF and RGRF for the fixed number of stations for a given day and repeating this exercise for all the rainy days. This method invokes the reasonable assumption that rain drop sizes on a given rainy day do not display much spatial variation. The errors inherent in deriving RERF using the same values of A and b when rain drop sizes might change from day-to-day due to different types of RF realised are statistically corrected to some extent by the derivation of ERGRF also on a day-to-day basis using different equations of estimation. However, the task of deriving ERGRF for DRF would be different from that for months/ seasons. Depending on the quantum of RF, different techniques may have to be adopted for each day. If such a scheme is perfected and daily ERGRF distribution could be generated on near real time basis, the resultant estimated spatial distribution of RF could be taken as the corrected RERF distribution and also made available to the users on real time basis.

The two techniques described above involve deriving RERF / ERGRF on a daily basis and then cumulating to generate monthly values. Adopting the right type of estimation free from any type of bias for each and every day will be a considerably involved exercise. Nevertheless, this is another potential area of research on the correct interpretation and utilisation of DWR derived RF.

6. Summary

The results of the study are summarised below:

(*i*) The inter-annual variability of RERF(x) and RGRF(y) of NEM season (OND) over CDLR100 displays epochal behaviour with the presence of both positive and negative biases. The SD and CV of the OND seasonal RF for 2002-13 depict almost identical variability of x and y. The MADs for the months of October, November, and December vary between 19 and 31% and that for the OND season is 107.6 mm or 17 % of mean y, on a year-to-year basis.

(*ii*) The short term normals based on station means over CDLR100 for October, November, December and OND, have been derived as 274.9, 262.6, 96.5 and 629.8 mm for x and 243.8, 254.6, 128.0 and 627.4 mm for y yielding bias values of -31.2, -8.0, 31.5 and -2.4 mm respectively. The values of SD, CV and range of x are lower than that of y which shows that spatial variation of y has not been fully captured by x within the CDLR100 region.

(*iii*) When all the 12 (years) \times 34 (stations) raw values are pooled, the MAD for OND is substantial at 188.8 mm which is 30% of the mean *y*, despite negligible difference of only -2.9 mm between the two normals.

(*iv*) The short term normals for x and y computed by the three methodologies of averaging adopted are by and large identical. In all the three sets of normals, over- / underestimation of y by x have been observed for October / December respectively while for November / OND both the normals are nearly identical.

(v) For OND season, CC(x, y) is 0.84 for yearly means over the region, 0.82 for station means and 0.72 when all the values are pooled. All these CCs are significant at 0.1% LS.

(vi) The insignificant CCs between distance of the concerned station from DWR Chennai and the RF bias observed for each month / season clearly show that the error / bias in RERF is independent of the geographical locations of stations in the CDLR100 region.

(*vii*) To obtain estimates of $y(\hat{y})$ based on the raw values of x at nearly 1.42 lakh grid points of the CDLR100 region, three different estimation techniques were tested among which the technique based on PCF concept emerged as the best possible one and has been used. The spatial distributions of \hat{y} based on x generated using PCF concept for October, November, December and OND offered better RF estimates than the raw distribution of x.

(*viii*) The normal values computed by spatial averaging of nearly 1.42 lakh grid point data over the CDLR100 area are 273.3, 262.2, 92.5 and 628.4 mm for RERF and 243.4, 254.3, 122.9 and 622.1 mm for estimated RGRF for October, November and December and OND respectively. The normal values of RERF are by and large lower with negligible differences compared to the normals based on 34 stations data. The error of estimate based on 34 observations but representative of CDLR100 is obtained as 28.4, 36.2, 25.2 and 68.9 mm respectively for the four periods.

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