# **Interacting inclined strike-slip faults in a layered medium**

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**सार** – इस शोध पत्र में भूमि के अदर श्यान प्रत्यास्थ परत में स्थित एक दूसरे पर झुके दो नतिलब सपेण भ्रंश जो श्यान प्रत्यास्थ के आधे भाग में एक दूसरे से मिलते हुए स्थित है और लिथोर्स्फेयर-एस्थेनोर्स्फेयर प्रणाली को प्रदर्शित करता है, पर विचार किया गया है। ग्रीन के फंक्शन और संपूर्ण (Integral) आकृतियों के प्रयोग से संबंधित तकनीक का उपयोग करते हुए स्थान परिवर्तन, प्रतिबलों और विकृतियों में बदलाव के कारणों का पता लगाने की तीन संभ स्थितियों - जैसे वह स्थिति जब कोई भी भ्रंश खिसक नहीं रहा है, जब एक भ्रंश खिसक रहा हो और दूसरा जकड़ा हुआ है तथा ऐसी स्थिति जब दोनों भ्रंश खिसक रहे हों; पर विचार किया गया है।

इस शोध पत्र में एक भ्रंश के अंदर या अन्य भ्रंश के निकट अचानक हलचल से अपरूपण प्रतिबल पर पड़ने वाले प्रभाव की पड़ताल की गई है। इसमें कुछ स्थितियों की पहचान की गई है जहां एक भ्रंश के अंदर अचानक हलचल के परिणाम नजदीक के अन्य भ्रंश में अपरूपण प्रतिबल छोड़ते हैं, जिसकी वजह से इसके अंदर भूकंपीय हलचल होने की संभावना कम होती है। अन्य स्थितियों की भी पहचान की गई है जिसमें एक भ्रंश में अचानक हलचल होने से भूकंपीय भ्रंश में हलचल की संभावनाएं बढ़ जाती हैं। इन भ्रंशों के मध्य बिन्द्ओं के नजदीक घटित दो लगातार भूकंपीय परिघटनाओं के विस्तृत अध्ययन से उनके बीच की समय अवधि का आकलन किया जा सकता है। ऐसी आशा की जाती है कि इस प्रकार के अध्ययन भूकंप प्रक्रियाओं की क्रियाविधि (मेकेनिज्म) को समझने के लिए उपयोगी होगी और इसे एक भूकप पूर्वगामी के रूप में जाना जा सकेगा।

**ABSTRACT.** Two inclined, interacting, strike-slip faults, both buried, situated in a viscoelastic layer, resting on and in welded contact with a viscoelastic half space, representing the lithosphere-asthenosphere system, is considered. Solutions are obtained for the displacements, stresses and strains, using a technique involving the use of Green's functions and integral transforms, for three possible cases - the case when no fault is slipping, the case when one fault is slipping and the other is locked and the case when both the faults are slipping.

The effect of sudden movement across one fault on the shear stress near the fault itself and near the other faults has been investigated. Some situations are identified where a sudden movement across one fault results in the release of shear stress near the other fault, reducing the possibility of seismic movements across it. Other situations are also identified where a sudden movement across one fault increases the possibility of seismic fault movements. A detail study may lead to an estimation of the time span between two consecutive seismic events near the mid points of the faults. It is expected that such studies may be useful in understanding the mechanism of earthquake processes and may be identified as an earthquake precursor.

**Key words** – Viscoelastic, Aseismic, Strike-slip faults, Sudden movement, Mantle convection, Stress accumulation, Earthquake precursor.

#### **1. Introduction**

Earthquakes are generated due to various types of movements across seismic faults having different geometrical features. Two consecutive seismic events are usually separated by long quasi-static aseismic period which may extend up to several years. Stresses accumulate near the faults during this aseismic period due to various tectonic reasons including mantle convection. When the accumulated stress exceed some thresholds value, movement across the fault occurs leading to an earthquake. Kayal *et al*. (2002), Mishra and Zhao (2003), Mishra *et al*. (2008), Singh *et al*. (2013), Singh and Mishra (2015) have shown that pre-existing intersecting faults having different geometrical shapes associated with fluid filled fractured material may introduce heterogeneous environment leading to differential strains and thereby bring the brittle fracture. These observations justify our theoretical approach to understand the nature of seismogenic faults in sub-surface layers having varying degree of brittle nature of the layered medium. The degree of cracks and porosity variation in the sub-surface layered media may influence the extend of rock failure.



**Fig.1.** A schematic sketch showing two interacting strike-slip faults inclined to the vertical – both buried. Section by the plane  $y_1 = 0$ 

In most of the viscoelastic theoretical papers developed so far for aseismic ground deformations a single and/or double vertical and/or inclined strike-slip faults situated in a viscoelastic half space has been considered [Rundle *et al*. (1977), Sen *et al*. (1993) and Sen *et al*. (2012)]. Some theoretical models of the lithosphere-asthenosphere system in seismically active regions during aseismic periods have been developed, for a single locked fault or a single creeping fault, by Mukhopadhyay and Mukherji (1978a), Mukhopadhyay *et al*. (1979b, 1980a) and Cohen *et al*. (1984). In some cases layered model consisting of an elastic and/or viscoelastic layer overlying a viscoelastic half space has been considered by Mukhopadhyay *et al*. (1980b) and Ghosh *et al*. (1992a, 1992b, 2011) to represent the lithosphere-asthenosphere system. It may be noted in this connection that the lithosphere rheology is assumed to be approximately 'brittle elastic', which includes the earth's crust and a part of the upper mantle. The region below it, called the asthenosphere, is assumed to be composed of relatively softer material which exhibit more viscoelastic behaviour. This enables us to suggest a layered model of lithosphere-asthenosphere system consisting of an elastic and/or viscoelastic layer overlying a viscoelastic half space as more realistic rather than a viscoelastic half space model.

It has been observed that major fault systems in different parts of the world consists of a number of neighbouring faults instead of a single fault which may interact when creep or sudden seismic fault movement occurs across one or more of them. A movement across any one of these neighbouring faults will affect the rate of stress accumulation near the other and thereby causes significant changes in the possible movement across the other. Creeping or sudden movement across a fault is generally found to reduce the rate of accumulation of shear stress near the fault. The effect of aseismic creep or slip across one fault on the shear stress near the other fault is found to depend on the distance, dimensions, relative position and other characteristics of the two faults. Some theoretical models of the lithosphere-asthenosphere system in seismically active regions during aseismic periods, with two interacting creeping/slipping faults, have been developed by Mukhopadhyay *et al*. (1978b, 1979c,), Mukhopadhyay and Mukherji (1984, 1986) and Ghosh *et al*. (1992a, 1992b, 2011).

In most of the theoretical models developed so far the faults were taken to be vertical surface breaking and/or buried. But fault system may often consist of inclined faults. The inclination of the fault may affect the nature of stress/strain accumulation near the fault. With these points

in view, in the present case we consider two inclined buried strike-slip faults in a viscoelastic layer overlying a viscoelastic half space and study the nature of aseismic accumulation of shear stress in the system. The medium is under the influence of tectonic forces due to mantle convection or some related phenomena. The faults undergo a sudden movement when the stresses in the region exceed certain threshold values.

## **2. Formulation**

We consider two long, inclined and interacting strike-slip faults  $F_1$  and  $F_2$  situated in a viscoelastic layer of thickness H. The layer rests on and is in welded contact with a viscoelastic half space. Let  $\theta_1$  and  $\theta_2$  be the inclinations of the faults  $F_1$  and  $F_2$  respectively with the horizontal. Let  $D_1$  and  $D_2$  be the widths of the faults  $F_1$ and  $F_2$  respectively and  $d_1$  and  $d_2$  be the depths of their upper edges below the free surface. Let D be the distance between the lines on the free surface vertically above the upper edges of the faults.

We introduce rectangular Cartesian coordinate system  $(y_1, y_2, y_3)$  for the fault  $F_1$  and  $(z_1, z_2, z_3)$  for the fault  $F_2$  with the free surface as  $y_3 = 0$  and  $z_3 = 0$ ,  $y_3$  and  $z_3$  axes pointing into the half-space and  $y_1$  and  $z_1$  axes being chosen along the straight lines on the planes  $y_3 = 0$ and  $z_3 = 0$  which are vertically above the upper edges of the faults for the buried faults. For convenience of analysis we introduce another set of rectangular Cartesian co-ordinate system  $(y_1, y_2, y_3)$  for the fault  $F_1$  with  $y_1'$ -axis along the upper edge of the fault and the plane of the fault as the plane  $y'_2 = 0$ ,  $y'_1$ -axis being parallel to  $y_1$ axis. Similarly for the fault  $F_2$ , we introduce another set of rectangular, Cartesian co-ordinate system  $(z_1, z_2, z_3)$  as shown in [Fig. 1]. The relations between different coordinate systems systems are given by :

$$
y'_{1} = y_{1}
$$
  
\n
$$
y'_{2} = y_{2} \sin \theta_{1} - (y_{3} - d_{1}) \cos \theta_{1}
$$
  
\n
$$
y'_{3} = y_{2} \cos \theta_{1} + (y_{3} - d_{1}) \sin \theta_{1}
$$
  
\n
$$
z'_{1} = z_{1}
$$
  
\n
$$
z'_{2} = z_{2} \sin \theta_{2} - z_{3} \cos \theta_{2}
$$
  
\n
$$
z'_{3} = z_{2} \cos \theta_{2} + z_{3} \sin \theta_{2}
$$

where,

$$
z_2 = y_2 - D, z_3 = y_3 - d_2
$$

Thus, the faults  $F_1$  and  $F_2$  are given by :

$$
F_1: (y_2' = 0, 0 \le y_3' \le D_1)
$$
  

$$
F_2: (z_2' = 0, 0 \le z_3' \le D_2)
$$

The section of this model in the plane  $y_1 = 0$  is shown in [Fig. 1].

Here, we assume that the lengths of the faults are large compared with their depths, and we take the displacements, stresses and strains to be independent of *y*<sup>1</sup> and depended on  $y_2$ ,  $y_3$  and the time *t*. With this assumption, the components of displacement, stress and strain  $u_1$ , ( $\tau_{12}, \tau_{13}$ ) and ( $e_{12}, e_{13}$ ) in the viscoelastic layer and  $u_1$ ,  $(\tau_{12}, \tau_{13})$  and  $(e_{12}, e_{13})$  in the viscoelastic half-space associated with the strike-slip movement only and are independent of the other components of displacement, stress and strain. We shall consider here the strike-slip movements only.

For the viscoelastic layer the constitutive equations are taken to be

$$
\left(\frac{1}{\eta_1} + \frac{1}{\mu_1} \frac{\partial}{\partial t}\right) \tau_{12} = \frac{\partial^2 u_1}{\partial t \partial y_2}
$$
\n
$$
\left(\frac{1}{\eta_1} + \frac{1}{\mu_1} \frac{\partial}{\partial t}\right) \tau_{13} = \frac{\partial^2 u_1}{\partial t \partial y_3}
$$
\n
$$
(0 \le y_3 \le H, -\infty \le y_2 \le \infty, t \ge 0
$$
\n(1)

where,  $\mu_1$  and  $\eta_1$  are the effective rigidity and viscosity of the viscoelastic layer respectively.

For the viscoelastic half space the constitutive equations are taken to be

$$
\left(\frac{1}{\eta_2} + \frac{1}{\mu_2} \frac{\partial}{\partial t}\right) \tau'_{12} = \frac{\partial^2 u'_1}{\partial t \partial y_2}
$$
\n
$$
\left(\frac{1}{\eta_2} + \frac{1}{\mu_2} \frac{\partial}{\partial t}\right) \tau'_{13} = \frac{\partial^2 u'_1}{\partial t \partial y_3}
$$
\n
$$
(y_3 \ge H, -\infty \le y_2 \le \infty, t \ge 0),
$$
\n(2)

where,  $\mu_2$  and  $\eta_2$  are the effective rigidity and viscosity of the viscoelastic half space respectively. The time t is being measured from a suitable instant when there is no seismic movement.

We consider slow quasi-static aseismic deformation of the system when the inertial terms in the stress equations of motion are small and can be neglected, as explained by Mukhopadhyay *et al*. (1980a). For such aseismic deformation, the stresses satisfy the relations:

$$
\frac{\partial}{\partial y_2}(\tau_{12}) + \frac{\partial}{\partial y_3}(\tau_{13}) = 0 \quad (0 \le y_3 \le H)
$$

$$
\frac{\partial}{\partial y_2}(\tau_{12}) + \frac{\partial}{\partial y_3}(\tau_{13}) = 0 \quad (y_3 \ge H)
$$
(3)

 $(-\infty \leq y_2 \leq \infty, t \geq 0)$ 

From  $(1)$ ,  $(2)$  and  $(3)$  we find that:

$$
\frac{\partial}{\partial t} (\nabla^2 u_1) = 0 \text{ and}
$$
  

$$
\frac{\partial}{\partial t} (\nabla^2 u_1') = 0 \text{ which are satisfied if,}
$$
  

$$
\nabla^2 u_1 = 0 \quad (0 \le y_3 \le H)
$$
  

$$
\nabla^2 u_1' = 0 \quad (y_3 \ge H) \tag{4}
$$
  

$$
(-\infty \le y_2 \le \infty, t \ge 0)
$$

Boundary conditions are

$$
\tau_{13} = 0 \t at y_3 = 0
$$
  
\n
$$
\tau_{13} = \tau'_{13} \t at y_3 = H
$$
  
\n
$$
u_1 = u'_1 \t at y_3 = H
$$
  
\n
$$
\tau'_{13} \to 0 \t as y_3 \to \infty
$$
  
\n
$$
(-\infty \le y_2 \le \infty, t \ge 0)
$$
\n(5)

We also assume that at a large distance from the fault plane there is a shear strain, maintained by the tectonic forces, *i.e*., we have conditions

$$
e_{12} \to (e_{12})_{0\infty} + f(t)
$$
  
\n
$$
e_{12}^{'} \to (e_{12}^{'} )_{0\infty} + f(t) \text{ as } |y_2| \to \infty \text{, for } t \ge 0 \qquad (6)
$$

where,

$$
(e_{12})_{0\infty} = \lim_{|y_2| \to \infty} (e_{12})_0 \text{ and}
$$

$$
(e'_{12})_{0\infty} = \lim_{|y_2| \to \infty} (e'_{12})_0
$$

 $(e_{12})_0$  and  $(e'_{12})_0$  are the values of  $e_{12}$  and  $e'_{12}$ respectively at  $t = 0$ , where  $f(t)$  is a continuous and

increasing function of t, such that  $f(0) = 0$ . The same function  $f(t)$  is taken for both  $e_{12}$  and  $e_{12}$  to ensure that the boundary condition  $u_1 = u'_1$  at  $y_3 = H$  is satisfied as  $|y_2| \rightarrow \infty$  and there is a uniform rate of change of shear strains  $e_{12}$  and  $e_{12}^{'}$  as  $|y_2| \rightarrow \infty$ .

## **3. Displacements and stresses in the absence of fault movement**

 We take the displacements and stresses to be continuous throughout the system. We measure the time from a suitable instant after which the conditions  $(1) - (6)$ become applicable, so that they are valid for  $t \geq 0$ . We assume that  $(u_1)_0$ ,  $(u_1')_0$ ,  $(\tau_{12})_0$ , ...,  $(e_{13}')_0$  are the values of  $u_1, u_1, \tau_{12}, \ldots, e_{13}$  at time  $t = 0$ . We take Laplace transforms of  $(1)$  -  $(6)$  with respect to t. This gives a boundary value problem. Inversion of the Laplace transform then gives the solution for the displacements and stresses. As  $(\tau_{13})_0$  and  $(\tau'_{13})_0$  satisfy (1) - (6), they have the same value at  $y_3 = H$ . Let  $T_H(y_2)$  be the common value of  $(\tau_{13})_0$  and  $(\tau'_{13})_0$  at  $y_3 = H$ , *i.e.*,

$$
T_H(y_2) = \{ (\tau_{13})_0 \}_{y_3 = H} = \{ (\tau_{13})_0 \}_{y_3 = H}
$$

In case when  $T_H(y_2) = T_H$  (a constant)  $\neq 0$ , we obtain :

$$
u_1 = (u_1)_0 + f(t).y_2 + \frac{T_H \eta_1}{\mu_1 \mu_2} \left(\frac{\mu_2}{\eta_2} - \frac{\mu_1}{\eta_1}\right) \left(1 - e^{-\frac{\mu_1}{\eta_1}t}\right).H
$$
  

$$
\tau_{12} = (\tau_{12})_0 e^{-\frac{\mu_1}{\eta_1}t} + \mu_1 \int_0^t f'(\tau) e^{-\frac{\mu_1}{\eta_1}(t-\tau)} d\tau
$$
  

$$
\tau_{13} = (\tau_{13})_0 e^{-\frac{\mu_1}{\eta_1}t}
$$
  

$$
e_{12} = (e_{12})_0 + f(t) \tag{7}
$$

for the viscoelastic layer, and

$$
u_1' = (u_1')_0 + f(t) \cdot y_2 + \frac{T_H \eta_1}{\mu_1 \mu_2} \left(\frac{\mu_2}{\eta_2} - \frac{\mu_1}{\eta_1}\right) \left(1 - e^{-\frac{\mu_1}{\eta_1}t}\right) \cdot y_3
$$

$$
\tau_{12}^{'} = (\tau_{12}^{'} )_0 e^{-\frac{\mu_2}{\eta_2}t} + \mu_2 \int_0^t f'(\tau) e^{-\frac{\mu_2}{\eta_2}t - \tau} d\tau
$$
  

$$
\tau_{13}^{'} = (\tau_{13}^{'} )_0 e^{-\frac{\mu_2}{\eta_2}t} + T_H \left( e^{-\frac{\mu_1}{\eta_1}t} - e^{-\frac{\mu_2}{\eta_2}t} \right)
$$

$$
e_{12}^{'} = (e_{12}^{'})_0 + f(t) \tag{8}
$$

for the viscoelastic half space.

From the above solution we find that the stress component  $\tau_{12}$  increases with time. We assume that the rheological properties of the layer and the half space are such that when the relevant stress components reach a certain threshold value,  $\tau_{c_1}$ (say) either of the fault  $F_1$  or  $F_2$ slips after a time (say  $T_1$ ). The stress accumulation pattern changes significantly after the movement across the fault.

## **4. Displacements and stresses after the commencement of fault movement**

We assume that the sudden movement of one of the slipping fault, say  $F_1$ , occurs at time  $t = T_1(> 0)$  while the other fault  $F_2$  remains locked. All the previous equations remains valid for  $t \geq T_1$  also, but in addition we have the following conditions which characterize the slip across  $F_1$ :

$$
[u_1]_{F_1} = U_1 \cdot g_1(y_3') \cdot H(t_1) \text{ across } F_1
$$
  
( $y_2 = 0$ ,  $0 \le y_3' \le D_1$ ),  $t_1 \ge 0$  ( $t_1 = t - T_1$ ) (9)

where  $[u_1]_{F_1}$  is the relative displacement across  $F_1$ , *i.e*.,

$$
[u_1]_{F_1} = \lim_{y_2 \to 0+0} (u_1) - \lim_{y_2 \to 0-0} (u_1),
$$
  
H(t<sub>1</sub>) = 0, for t<sub>1</sub>  $\leq$  0  
= 1, for t<sub>1</sub> > 0

 $g_1(y_3)$  give the spatial dependence of the slip movement along the fault  $F_1$ .

We assume that  $u_1$ ,  $u'_1$ ,  $\tau_{12}$ ,  $\tau_{13}$ ,  $\tau'_{12}$  and  $\tau'_{13}$  are continuous everywhere in the model. Let us consider the model after the commencement of fault slip across  $F_1$ .

Let us first consider the slip across the fault  $F_1$ after a time  $T_1$ . Then sudden movement across  $F_1$ generates disturbances in the near regions. Our constitutive equations do not remain valid when the near region is disturbed. We leave out this short duration of time. However, the disturbances gradually die out and a seismic state re-established. We re-consider our model after the restoration of aseismic state in the region. We now try to find solutions for  $u_1$ ,  $u'_1$ ,  $\tau_{12}$ ,  $\tau_{13}$ ,  $\tau'_{12}$ ,  $\tau'_{13}$  in the form :

$$
u_1 = (u_1)_1 + (u_1)_2
$$
  

$$
u'_1 = (u'_1)_1 + (u'_1)_2
$$
  

$$
\tau_{12} = (\tau_{12})_1 + (\tau_{12})_2
$$

$$
\tau_{13} = (\tau_{13})_1 + (\tau_{13})_2
$$
  
\n
$$
\tau'_{12} = (\tau'_{12})_1 + (\tau'_{12})_2
$$
  
\n
$$
\tau'_{13} = (\tau'_{13})_1 + (\tau'_{13})_2
$$
\n(10)

where,  $(u_1)_1$ ,  $(u_1')_1$ ,  $(\tau_{12})_1$ , ...,  $(\tau_{13})_1$  are continuous everywhere in the model and satisfy (1) - (6) ; they are therefore given by  $(7) - (8)$ .

We now have to find the values of  $(u_1)_2$ ,  $(u_1)_2$ ,  $(\tau_{12})_2$ , ...,  $(\tau_{13})_2$  which depend on the fault slip across  $F_1$ . The values of  $(u_1)_2$ ,  $(u_1)_2$ ,  $(\tau_{12})_2$ , ...,  $(\tau_{13})_2$  are assumed to be zero for  $t \leq T_1$ , satisfying (1) – (6). So for  $(u_1)_2$ ,  $(u_1)_2$ ,  $(\tau_{12})_2$ , ...,  $(\tau_{13})_2$  for  $t = t_1$  we have constitutive equations, equations of motion, boundary conditions  $(1) - (5)$ , equation  $(8)$  and equation given by (10) which replace by (6)

$$
\begin{aligned}\n(e_{12})_2 &\to 0 \\
(e_{12})_2 &\to 0\n\end{aligned}\n\Big\}\n\text{ as } |y_2'| \to \infty \quad (t_1 \ge 0)\n\tag{11}
$$

We apply modified form of Green's function technique developed by Maruyama (1966) and Rybicki (1971, 1973) and we obtain :

$$
(u_1)_2 = \frac{U_1 \cdot H(t - T_1)}{2\pi} \cdot \psi_1(y_2, y_3, t)
$$
  
\n
$$
(\tau_{12})_2 = \frac{\mu_1 \cdot U_1 \cdot H(t - T_1)}{2\pi} e^{-\frac{\mu_1}{\eta_1}(t - T_1)} \cdot \psi_2(y_2, y_3, t)
$$
  
\n
$$
(\tau_{13})_2 = \frac{\mu_1 \cdot U_1 \cdot H(t - T_1)}{2\pi} e^{-\frac{\mu_1}{\eta_1}(t - T_1)} \cdot \psi_3(y_2, y_3, t)
$$
  
\n
$$
(u_1')_2 = \frac{\mu_1 \cdot U_1 \cdot H(t - T_1)}{\pi(\mu_1 + \mu_2)} \left[ \frac{\eta_1(\mu_1 + \mu_2)}{\mu_1(\eta_1 + \eta_2)} + \frac{\mu_1 \eta_2 - \mu_2 \eta_1}{\mu_1(\eta_1 + \eta_2)} e^{-\frac{\mu_1 \mu_2(\eta_1 + \eta_2)}{\eta_1 \eta_2(\mu_1 + \mu_2)}(t - T_1)} \right] \cdot \phi_1(y_2, y_3, t)
$$
  
\n
$$
(\tau_{12})_2 = \frac{U_1 \cdot H(t - T_1)}{\pi} \cdot \frac{\mu_1 \mu_2}{(\mu_1 + \mu_2)} e^{-\frac{\mu_1 \mu_2(\eta_1 + \eta_2)}{\eta_1 \eta_2(\mu_1 + \mu_2)}(t - T_1)} \cdot \phi_2(y_2, y_3, t)
$$

$$
(\tau'_{13})_2 = \frac{U_1 \cdot H(t - T_1)}{\pi} \cdot \frac{\mu_1 \mu_2}{(\mu_1 + \mu_2)} e^{-\frac{\mu_1 \mu_2 (\eta_1 + \eta_2)}{\eta_1 \eta_2 (\mu_1 + \mu_2)} (t - T_1)} \cdot \phi_3(y_2, y_3, t)
$$
\n(12)

where,

$$
\psi_{1}(y_{2}, y_{3}, t) = \int_{0}^{b_{1}} g_{1}(\xi_{3}) \left[ \frac{y_{2}sin\theta_{1} - y_{3}cos\theta_{1}}{A_{1}} + \frac{y_{2}sin\theta_{1} + y_{3}cos\theta_{1}}{A_{2}} \right] d\xi_{3}' + \sum_{m=1}^{\infty} A_{m}(t) \int_{0}^{b_{1}} g_{1}(\xi_{3}')
$$
\n
$$
\left[ \frac{y_{2}sin\theta_{1} - y_{3}cos\theta_{1} - 2mHcos\theta_{1}}{A_{3}} + \frac{y_{2}sin\theta_{1} + y_{3}cos\theta_{1} - 2mHcos\theta_{1}}{A_{4}} + \frac{y_{2}sin\theta_{1} - y_{3}cos\theta_{1} + 2mHcos\theta_{1}}{A_{5}} + \frac{y_{2}sin\theta_{1} + y_{3}cos\theta_{1} + 2mHcos\theta_{1}}{A_{6}} \right] d\xi_{3}' \qquad (13)
$$

$$
\psi_2(y_2, y_3, t) = \int_0^{D_1} g_1(\xi_3')
$$
\n
$$
\left[\frac{\xi_3^{'2} sin \theta_1 + (y_3^2 - y_2^2) sin \theta_1 + 2y_2 y_3 cos \theta_1 - 2y_3 \xi_3'}{A_1^2} + \frac{A_1^2}{A_2^2}\right] d\xi_3'
$$
\n
$$
+ \sum_{m=1}^{\infty} A_m(t) \int_0^{D_1} g_1(\xi_3') \left[\frac{\psi_{21}}{A_3^2} + \frac{\psi_{22}}{A_4^2} + \frac{\psi_{23}}{A_5^2} + \frac{\psi_{24}}{A_6^2}\right] d\xi_3'
$$
\n(14)

where,

$$
\psi_{21} = \xi_3^2 \sin \theta_1 - 2\xi_3' (y_3 + 2mH) - \{y_2^2 - (y_3 + 2mH)^2\} \sin \theta_1 + 2y_2 (y_3 + 2mH) \cos \theta_1
$$

$$
\psi_{22} = \xi_3^{'2} sin\theta_1 + 2\xi_3' (y_3 + 2mH) \n-[y_2^2 - (y_3 + 2mH)^2] sin\theta_1 - 2y_2 (y_3 + 2mH) cos\theta_1
$$

$$
\psi_{23} = \xi_3^{'2} sin\theta_1 - 2\xi_3' (y_3 - 2mH) -\n\{y_2^2 - (y_3 - 2mH)^2\} sin\theta_1 + 2y_2 (y_3 - 2mH) cos\theta_1
$$

$$
\psi_{24} = \xi_3^{'2} sin\theta_1 + 2\xi_3' (y_3 - 2mH) -\n{y_2^2 - (y_3 - 2mH)^2} sin\theta_1 - 2y_2 (y_3 - 2mH) cos\theta_1
$$

$$
\psi_3(y_2, y_3, t) = \int_0^{D_1} g_1(\xi_3')
$$
\n
$$
[\frac{-\xi_3^{'2} \cos \theta_1 + 2\xi_3^{'2} y_2 - (y_2^2 - y_3^2) \cos \theta_1 - 2y_2 y_3 \sin \theta_1}{A_1^2} + \frac{\xi_3^{'2} \cos \theta_1 - 2\xi_3^{'2} y_2 + (y_2^2 - y_3^2) \cos \theta_1 - 2y_2 y_3 \sin \theta_1}{A_2^2}] d\xi_3'
$$
\n
$$
+ \sum_{m=1}^{\infty} A_m(t) \int_0^{D_1} g_1(\xi_3') \left[ \frac{\psi_{31}}{A_3^2} + \frac{\psi_{32}}{A_4^2} + \frac{\psi_{33}}{A_5^2} + \frac{\psi_{34}}{A_6^2} \right] d\xi_3'
$$
\n(15)

where,

$$
\psi_{31} = -\xi_3^{'2} \cos \theta_1 + 2\xi_3' y_2 - (y_2^2 - y_3^2) \cos \theta_1 -2y_2 y_3 \sin \theta_1 - 4mH(y_2 \sin \theta_1 - y_3 \cos \theta_1) + 4m^2H^2 \cos \theta_1
$$

$$
\psi_{32} = \xi_3^{'2} \cos \theta_1 - 2\xi_3' y_2 + (y_2^2 - y_3^2) \cos \theta_1 - 2y_2 y_3 \sin \theta_1 + 4mH (y_2 \sin \theta_1 + y_3 \cos \theta_1) -
$$
  

$$
(4m^2H^2 \cos \theta_1)
$$

$$
\begin{array}{c}\n\psi_{33}=-\xi_3^{'2}cos\theta_1+2\xi_3'y_2-(y_2^2-y_3^2)cos\theta_1-\\ \n2y_2y_3sin\theta_1+4mHy_2sin\theta_1-y_3cos\theta_1)+\\ \n(4m^2H^2cos\theta_1\n\end{array}
$$

$$
\psi_{34} = \xi_3^{'2} \cos \theta_1 - 2\xi_3' y_2 + (y_2^2 - y_3^2) \cos \theta_1 - 2y_2 y_3 \sin \theta_1 - 4mH(y_2 \sin \theta_1 + y_3 \cos \theta_1) - 4m^2H^2 \cos \theta_1
$$

$$
\phi_1(y_2, y_3, t) = \int_0^{b_1} g_1(\xi_3)
$$
\n
$$
\left[ \frac{y_2 \sin \theta_1 - y_3 \cos \theta_1}{A_1} + \frac{y_2 \sin \theta_1 + y_3 \cos \theta_1}{A_2} \right] d\xi_3' + \frac{\sum_{m=1}^{\infty} A_m(t) \int_0^{b_1} g_1(\xi_3') \left[ \frac{y_2 \sin \theta_1 - y_3 \cos \theta_1 + 2mH \cos \theta_1}{A_5} + \frac{y_2 \sin \theta_1 + y_3 \cos \theta_1 + 2mH \cos \theta_1}{A_6} \right] d\xi_3'
$$
\n(16)

$$
\phi_2(y_2, y_3, t) = \int_0^{D_1} g_1(\xi_3)
$$
\n
$$
\frac{\xi_3^{'2} \sin \theta_1 + (y_3^2 - y_2^2) \sin \theta_1 + 2y_2 y_3 \cos \theta_1 - 2y_3 \xi_3'}{A_1^2} + \frac{\xi_3^{'2} \sin \theta_1 - (y_2^2 - y_3^2) \sin \theta_1 - 2y_2 y_3 \cos \theta_1 + 2y_3 \xi_3'}{A_2^2} d\xi_3' + \sum_{m=1}^{\infty} A_m(t) \int_0^{D_1} g_1(\xi_3') \left[ \frac{\psi_{23}}{A_5^2} + \frac{\psi_{24}}{A_6^2} \right] d\xi_3' \tag{17}
$$

$$
\phi_3(y_2, y_3, t) = \int_0^{D_1} g_1(\xi_3')
$$
  
\n
$$
-\frac{\xi_3^{'2} \cos \theta_1 + 2\xi_3 y_2 - (y_2^2 - y_3^2) \cos \theta_1 - 2y_2 y_3 \sin \theta_1}{A_1^2}
$$
  
\n
$$
\frac{\xi_3^{'2} \cos \theta_1 - 2\xi_3 y_2 + (y_2^2 - y_3^2) \cos \theta_1 - 2y_2 y_3 \sin \theta_1}{A_2^2} d\xi_3' + \sum_{m=1}^{\infty} A_m(t) \int_0^{D_1} g_1(\xi_3') \left[ \frac{\psi_{33}}{A_5^2} + \frac{\psi_{34}}{A_6^2} \right] d\xi_3' \tag{18}
$$

where,

$$
A_1 = \xi_3^{'2} - 2\xi_3' (y_2 \cos \theta_1 + y_3 \sin \theta_1) + y_2^2 + y_3^2
$$
  
\n
$$
A_2 = \xi_3^{'2} - 2\xi_3' (y_2 \cos \theta_1 - y_3 \sin \theta_1) + y_2^2 + y_3^2
$$
  
\n
$$
A_3 = \xi_3^{'2} - 2\xi_3' (y_2 \cos \theta_1 + y_3 \sin \theta_1 + 2mH \sin \theta_1)
$$

$$
+y_2^2 + y_3^2 + 4y_3mH + 4m^2H^2
$$
  
\n
$$
A_4 = \xi_3^{'2} - 2\xi_3' (y_2 \cos\theta_1 - y_3 \sin\theta_1 + 2mH\sin\theta_1)
$$
  
\n
$$
+y_2^2 + y_3^2 - 4y_3mH + 4m^2H^2
$$

$$
A_5 = \xi_3^{'2} - 2\xi_3' (y_2 \cos\theta_1 + y_3 \sin\theta_1 - 2mH\sin\theta_1) + y_2^2 + y_3^2 - 4y_3 mH + 4m^2H^2
$$

$$
A_6 = \xi_3^{'2} - 2\xi_3' (y_2 \cos\theta_1 - y_3 \sin\theta_1 - 2mH\sin\theta_1) + y_2^2 + y_3^2 + 4y_3 mH + 4m^2H^2
$$

$$
A_m(t) = L^{-1} \left\{ \left( \frac{\overline{\mu_1} - \overline{\mu_2}}{\overline{\mu_1} + \overline{\mu_2}} \right)^m \right\}
$$
 (19)

where,

$$
\bar{\mu}_1 = \frac{p}{\frac{1}{\eta_1} + \frac{p}{\mu_1}}
$$
 and  $\bar{\mu}_2 = \frac{p}{\frac{1}{\eta_2} + \frac{p}{\mu_2}}$ 

where, *p* is the Laplace transform variable.

From the solution we find that the stress further accumulates due to the tectonic activities and stresses either accumulates or releases due to the movement across the fault  $F_1$ . We assume that the second fault  $F_2$  slips after a time  $T_2$  when the accumulated relevant stress near it exceeds the critical value  $\tau_{c_2}$ (say).

The slip condition is characterize by :

$$
[u_1]_{F_2} = U_2 \cdot g_2(z'_3) \cdot H(t_2) \text{ across } F_2
$$
  
(z'\_2 = 0, 0 \le z'\_3 \le D\_2), t\_2 \ge 0 (t\_2 = t - T\_2) (20)

where,  $[u_1]_{F_2}$  is the relative displacement across  $F_2$ ,

$$
[u_1]_{F_2} = \lim_{z_2 \to 0+0} (u_1) - \lim_{z_2 \to 0-0} (u_1),
$$
  
 
$$
H(t_2) = 0, \text{ for } t_2 \le 0
$$
  
= 1, \text{ for } t\_2 > 0

 $g_2(z_3)$  give the spatial dependence of the slip movement along the fault  $F_2$ .

Proceeding in a similar way, we obtain the final solution as :

$$
u_1 = (u_1)_1 + (u_1)_2 + (u_1)_3
$$
  

$$
u'_1 = (u'_1)_1 + (u'_1)_2 + (u'_1)_3
$$
  

$$
\tau_{12} = (\tau_{12})_1 + (\tau_{12})_2 + (\tau_{12})_3
$$

$$
\tau_{13} = (\tau_{13})_1 + (\tau_{13})_2 + (\tau_{13})_3
$$
  
\n
$$
\tau'_{12} = (\tau'_{12})_1 + (\tau'_{12})_2 + (\tau'_{12})_3
$$
  
\n
$$
\tau'_{13} = (\tau'_{13})_1 + (\tau'_{13})_2 + (\tau'_{13})_3
$$
\n(21)

We now have to find the values of  $(u_1)_3$ ,  $(u_1)_3$ ,  $(\tau_{12})_3$ , ...,  $(\tau_{13})_3$  which depend on the fault slip across  $F_2$ . The values of  $(u_1)_3$ ,  $(u_1)_3$ ,  $(\tau_{12})_3$ , ...,  $(\tau_{13})_3$  are assumed to be zero for  $t \leq T_2$ , satisfying (1) - (6). So for  $(u_1)_3$ ,  $(u_1)_3$ ,  $(\tau_{12})_3$ , ...,  $(\tau_{13})_3$  for  $t = t_2$  we have constitutive equations, equations of motion, boundary conditions  $(1)$  -  $(5)$ , equation  $(19)$  and equation given by (21) which replace by (6) :

$$
\begin{aligned}\n(e_{12})_3 &\to 0 \\
(e'_{12})_3 &\to 0\n\end{aligned}\n\Bigg\} \text{ as } |z'_2| \to \infty \quad (t_2 \ge 0)\n\tag{22}
$$

Proceeding as earlier we obtain:

$$
(u_1)_3 = \frac{U_2 \cdot H(t - T_2)}{2\pi} \cdot \psi_1'(z_2, z_3, t)
$$
  
\n
$$
(\tau_{12})_3 = \frac{\mu_1 \cdot U_2 \cdot H(t - T_2)}{2\pi} e^{-\frac{\mu_1}{\eta_1}(t - T_2)} \cdot \psi_2'(z_2, z_3, t)
$$
  
\n
$$
(\tau_{13})_3 = \frac{\mu_1 \cdot U_2 \cdot H(t - T_2)}{2\pi} e^{-\frac{\mu_1}{\eta_1}(t - T_2)} \cdot \psi_3'(z_2, z_3, t)
$$
  
\n
$$
(u_1')_3 = \frac{\mu_1 \cdot U_2 \cdot H(t - T_2)}{\pi(\mu_1 + \mu_2)} \left[ \frac{\eta_1(\mu_1 + \mu_2)}{\mu_1(\eta_1 + \eta_2)} + \frac{\mu_1 \eta_2 - \mu_2 \eta_1}{\mu_1(\eta_1 + \eta_2)} (t - T_2) \right] \cdot \phi_1'(z_2, z_3, t)
$$
  
\n
$$
\frac{\mu_1 \eta_2 - \mu_2 \eta_1}{\mu_1(\eta_1 + \eta_2)} e^{-\frac{\mu_1 \mu_2(\eta_1 + \eta_2)}{\eta_1 \eta_2(\mu_1 + \mu_2)} (t - T_2)}.
$$

$$
(\tau_{12})_3 = \frac{\pi}{\pi}.
$$
  

$$
\frac{\mu_1 \mu_2}{(\mu_1 + \mu_2)} e^{-\frac{\mu_1 \mu_2(\eta_1 + \eta_2)}{\eta_1 \eta_2(\mu_1 + \mu_2)}(t - T_2)} \cdot \phi_2'(z_2, z_3, t)
$$

$$
(\tau_{13})_3 = \frac{U_2 \cdot H(t - T_2)}{\pi} \cdot \frac{\mu_1 \mu_2}{(\mu_1 + \mu_2)} e^{-\frac{\mu_1 \mu_2 (\eta_1 + \eta_2)}{\eta_1 \eta_2 (\mu_1 + \mu_2)} (t - T_2)} \cdot \phi_3'(z_2, z_3, t) \tag{23}
$$

where,  $\psi'_1$ ,  $\psi'_2$ ,  $\psi'_3$ ,  $\phi'_1$ ,  $\phi'_2$ ,  $\phi'_3$  have similar expressions as those of  $\psi_1$ ,  $\psi_2$ ,  $\psi_3$ ,  $\phi_1$ ,  $\phi_2$ ,  $\phi_3$  respectively as given in  $(12) - (17)$  and can be obtained from them on replacing  $\xi'_3$ ,  $g_1(\xi'_3)$ ,  $D_1$ ,  $\theta_1$ ,  $y_2$ ,  $y_3$ ,  $t_1$  by  $\eta'_3$ ,  $g_2(\eta'_3)$ ,  $D_2$ ,  $\theta_2$ ,  $z_2$ ,  $z_3$  and  $t_2$  respectively.

Thus, the final solution for displacements and stresses for  $t > T_2$  are given by :

$$
u_1 = (u_1)_0 + f(t) \cdot y_2 + \frac{T_H \eta_1}{\mu_1 \mu_2} \left(\frac{\mu_2}{\eta_2} - \frac{\mu_1}{\eta_1}\right)
$$
  

$$
\left(1 - e^{-\frac{\mu_1}{\eta_1}t}\right) \cdot H + \frac{U_1 \cdot H(t - T_1)}{2\pi} \cdot \psi_1(y_2, y_3, t)
$$
  

$$
+ \frac{U_2 \cdot H(t - T_2)}{2\pi} \cdot \psi_1(z_2, z_3, t)
$$

$$
\tau_{12} = (\tau_{12})_0 e^{-\frac{\mu_1}{\eta_1}t} + \mu_1 \int_0^t f'(\tau) e^{-\frac{\mu_1}{\eta_1}(t-\tau)} d\tau + \frac{\mu_1 \cdot U_1 \cdot H(t-T_1)}{2\pi} e^{-\frac{\mu_1}{\eta_1}(t-T_1)} \cdot \psi_2(y_2, y_3, t) + \frac{\mu_1 \cdot U_2 \cdot H(t-T_2)}{2\pi} e^{-\frac{\mu_1}{\eta_1}(t-T_2)} \cdot \psi_2'(z_2, z_3, t)
$$

$$
\tau_{13} = (\tau_{13})_0 e^{-\frac{\mu_1}{\eta_1}t} + \frac{\mu_1 \cdot U_1 \cdot H(t - T_1)}{2\pi} e^{-\frac{\mu_1}{\eta_1}(t - T_1)}.
$$
  

$$
\psi_3(y_2, y_3, t) + \frac{\mu_1 \cdot U_2 \cdot H(t - T_2)}{2\pi} e^{-\frac{\mu_1}{\eta_1}(t - T_2)}.
$$
  

$$
\psi_3'(z_2, z_3, t)
$$

$$
u'_{1} = (u'_{1})_{0} + f(t). y_{2} + \frac{T_{H} \eta_{1}}{\mu_{1} \mu_{2}} \left(\frac{\mu_{2}}{\eta_{2}} - \frac{\mu_{1}}{\eta_{1}}\right)
$$
  
\n
$$
\left(1 - e^{-\frac{\mu_{1}}{\eta_{1}}t}\right) \cdot y_{3} + \frac{\mu_{1} \cdot U_{1} \cdot H(t - T_{1})}{\pi(\mu_{1} + \mu_{2})}
$$
  
\n
$$
\left[\frac{\eta_{1}(\mu_{1} + \mu_{2})}{\mu_{1}(\eta_{1} + \eta_{2})} + \frac{\mu_{1} \eta_{2} - \mu_{2} \eta_{1}}{\mu_{1}(\eta_{1} + \eta_{2})} e^{-\frac{\mu_{1} \mu_{2}(\eta_{1} + \eta_{2})}{\eta_{1} \eta_{2}(\mu_{1} + \mu_{2})}(t - T_{1})}\right]
$$
  
\n
$$
\phi_{1}(y_{2}, y_{3}, t) + \frac{\mu_{1} \cdot U_{2} \cdot H(t - T_{2})}{\pi(\mu_{1} + \mu_{2})}
$$
  
\n
$$
\left[\frac{\eta_{1}(\mu_{1} + \mu_{2})}{\mu_{1}(\eta_{1} + \eta_{2})} + \frac{\mu_{1} \eta_{2} - \mu_{2} \eta_{1}}{\mu_{1}(\eta_{1} + \eta_{2})} e^{-\frac{\mu_{1} \mu_{2}(\eta_{1} + \eta_{2})}{\eta_{1} \eta_{2}(\mu_{1} + \mu_{2})}(t - T_{2})}\right]
$$
  
\n
$$
\cdot \phi_{1}(z_{2}, z_{3}, t)
$$

$$
\tau_{12} = (\tau_{12})_0 e^{-\frac{\mu_2}{\eta_2}t} + \mu_2 \int_0^t f'(\tau) e^{-\frac{\mu_2}{\eta_2}(t-\tau)} d\tau + \frac{U_1 \cdot H(t-T_1)}{\pi} \cdot \frac{\mu_1 \mu_2}{(\mu_1 + \mu_2)} \cdot e^{-\frac{\mu_1 \mu_2(\eta_1 + \eta_2)}{\eta_1 \eta_2(\mu_1 + \mu_2)}(t-T_1)} \n\cdot \phi_2(y_2, y_3, t) + \frac{U_2 \cdot H(t-T_2)}{\pi} \n\frac{\mu_1 \mu_2}{(\mu_1 + \mu_2)} \cdot e^{-\frac{\mu_1 \mu_2(\eta_1 + \eta_2)}{\eta_1 \eta_2(\mu_1 + \mu_2)}(t-T_2)} \cdot \phi_2'(z_2, z_3, t)
$$

$$
\tau_{13} = (\tau_{13})_0 e^{-\frac{\mu_2}{\eta_2}t} + T_H \left( e^{-\frac{\mu_1}{\eta_1}t} - e^{-\frac{\mu_2}{\eta_2}t} \right) +
$$
\n
$$
\frac{U_1 \cdot H(t - T_1)}{\pi} \cdot \frac{\mu_1 \mu_2}{(\mu_1 + \mu_2)} \cdot e^{-\frac{\mu_1 \mu_2 (\eta_1 + \eta_2)}{\eta_1 \eta_2 (\mu_1 + \mu_2)}(t - T_1)}.
$$
\n
$$
\phi_3(y_2, y_3, t) + \frac{U_2 \cdot H(t - T_2)}{\pi} \cdot \frac{\mu_1 \mu_2}{(\mu_1 + \mu_2)}
$$
\n
$$
\cdot e^{-\frac{\mu_1 \mu_2 (\eta_1 + \eta_2)}{\eta_1 \eta_2 (\mu_1 + \mu_2)}(t - T_2)} \cdot \phi_3'(z_2, z_3, t) \tag{24}
$$

where,  $\psi_1$ ,  $\psi_2$ ,  $\psi_3$ ,  $\phi_1$ ,  $\phi_2$ ,  $\phi_3$  are given in (12) - (17) and  $\psi'_1$ ,  $\psi'_2$ ,  $\psi'_3$ ,  $\phi'_1$ ,  $\phi'_2$ ,  $\phi'_3$  have similar expressions as those of  $\psi_1$ ,  $\psi_2$ ,  $\psi_3$ ,  $\phi_1$ ,  $\phi_2$ ,  $\phi_3$  and can be obtained from

them on replacing  $\xi'_3$ ,  $g_1(\xi'_3)$ ,  $D_1$ ,  $\theta_1$ ,  $y_2$ ,  $y_3$ ,  $t_1$  by  $\eta'_3$ ,  $g_2(\eta'_3)$ ,  $D_2$ ,  $\theta_2$ ,  $z_2$ ,  $z_3$  and  $t_2$  respectively.

## **5. Numerical computations**

We consider the following values of the model parameters as suggested in different books (Cathles, 1975) and papers [Clift, P. *et al*. (2002) and Karato, (2010)] :

$$
\mu_1 = 3.5 \times 10^{11} \, \text{dyn cm}^{-2}
$$
\n
$$
\mu_2 = 4.0 \times 10^{11} \, \text{dyn cm}^{-2}
$$
\n
$$
\eta_1 = 6.0 \times 10^{20} \, \text{poise}
$$
\n
$$
\eta_2 = 2.0 \times 10^{21} \, \text{poise}
$$

 $f(t) = k \cdot t$ , with  $k = 0.6 \times 10^{-12} \text{ year}^{-1}$  (as compatible with the observational values)

$$
H = 60 \text{ km}
$$
  
\n
$$
D_1 = 10 \text{ km}
$$
  
\n
$$
D_2 = 15 \text{ km}
$$
  
\n
$$
d_1 = 10 \text{ km}
$$
  
\n
$$
d_2 = 15 \text{ km}
$$
  
\n
$$
D = 7 \text{ km}
$$
  
\n
$$
\theta_1 = \frac{\pi}{3}
$$
  
\n
$$
\theta_2 = \frac{\pi}{2}
$$
  
\n
$$
U_1 = 100 \text{ cm}
$$
  
\n
$$
U_2 = 50 \text{ cm}
$$
  
\n
$$
g_1(y'_3) = 1 - \frac{3y_3'^2}{D_1^2} + g_2(z'_3) = 1 - \frac{3z_3'^2}{D_2^2} + g_2(z'_3) = 1 - \frac{3z_3'^2}{D_2^2} + g_3(z'_3) = 1 - \frac{3z_3'^2}{D_3^2} + g_4(z'_3) = 1 - \frac{3z_3'^2}{D_4^2} + g_5(z'_4) = 1 - \frac{3z_3'^2}{D_5^2} + g_6(z'_5) = 1 - \frac{3z_3'^2}{D_6^2} + g_7(z'_6) = 1 - \frac{3z_3'^2}{D_7^2} + g_8(z'_7) = 1 - \frac{3z_3'^2}{D_8^2} + g_9(z'_8) = 1 - \frac{3z_3'^2}{D_9^2} + g_9(z'_8) = 1 - \frac{3z_3'^2}{D_9^2} + g_1(z'_9) = 1 - \frac{3z_3'^2}{D_9^2} + g_1(z'_9) = 1 - \frac{3z_3'^2}{D_9^2} + g_2(z'_9) = 1 - \frac{3z_3'^2}{D_9^2} + g_3(z'_9) = 1 - \frac{3z_3'^2}{D_9^2} + g_4(z'_9) = 1 - \frac{3z_3'^2}{D_9^2} + g_4(z'_9) = 1 - \frac{3z_3'^2}{D_9^2} + g_5(z'_9) = 1 - \frac{3z_3'^2}{D_9^2} + g_6(z'_9) = 1 - \frac{3z_3'^2}{D_9^2} + g_7
$$

We compute the following quantities numerically (taking  $m = 1$ , for  $m \ge 2$  the corresponding terms will be negligibly small) :

 $2y_3^{\prime 3}$  $D_1^3$ 

 $2z_3^{\prime 3}$  $D_2^3$ 

(a) Displacement on the free surface due to movement across both the faults  $F_1$  and  $F_2$  [Fig. 2].



**Fig.2.** A plot showing displacement on the free surface du**e** to the movement across both the faults  $F_1$  and  $F_2$ 



**Fig.3.** A plot showing strain on the free surface  $(y_2=0, y_3=0)$  against time due to the movement across the fault  $F_1$  only

- (b) Strain on the free surface due to :
- (*i*) Movement across the fault  $F_1$  only,
- ( $ii$ ) Movement across the fault  $F_2$  only,
- (*iii*) After the movement across both the faults  $F_1$  and  $F_2$ at a point  $y_2=0$ ,  $y_3=0$  against time [Figs. (3-5)].

(c) Region of stress accumulation and release in the layer due to the :

- (*i*) Movement across the fault  $F_1$  only,
- ( $ii$ ) Movement across the fault  $F_2$  only,
- (*iii*) After the movement across both the faults  $F_1$  and  $F_2$ [Figs. (6-8)].



**Fig.4.** A plot showing strain on the free surface  $(y_2=0, y_3=0)$  against time due to the movement across the fault  $F_2$  only



**Fig.5.** A plot showing strain on the free surface  $(y_2=0, y_3=0)$  against time due to the movement across both the faults  $F_1$  and  $F_2$ 

(d) Contour map showing the stress accumulation / release due to the movement across :

- (*i*) The fault  $F_1$  only,
- ( $ii$ ) The fault  $F_2$  only,
- (*iii*) Both the faults  $F_1$  and  $F_2$  [Figs. (9-12)].

The above figure [Fig. 2] show the displacement (in cms.) on the free surface ( $y_3 = 0$ ) due to the movement across both the faults  $F_1$  and  $F_2$ .

It is found that in each case, the magnitude of the strain at the free surface is of the order of 10−6 per year which is in good conformity with the observed ground deformation during the aseismic periods in seismically active regions [Figs. (3-5)].



**Fig.6.** A diagram showing region indication for stress accumulation and release due to the movement across the fault  $F_1$  only



**Fig.7.** A diagram showing region indication for stress accumulation and release due to the movement across the fault  $F_2$  only



**Fig.8.** A diagram showing region indication for stress accumulation and release due to the movement across both the faults F1 and  $F<sub>2</sub>$ 



**Fig.9.** A map showing stress accumulation / release due to the movement across the fault  $F_1$  only



**Fig.10.** A map showing stress accumulation /release due to the movement across the fault  $F_2$  only



**Fig.11.** A map showing stress accumulation / release due to the movement across both the faults  $F_1$  and  $F_2$ <br>  $(\theta_1 = \frac{\pi}{3}, \theta_2 = \frac{\pi}{2})$ 



**Fig.12.** A map showing stress accumulation / release due to the movement across both the faults F<sub>1</sub> and F<sub>2</sub> ( $\theta_1 = \frac{\pi}{3}$ ,  $\theta_2 = \frac{\pi}{4}$ )

The regions of stress accumulation and release have been clearly shown in the above figures[Fig. (6-8)] .

The contour maps in the above figures [Fig. (9-12)] show the nature of stress accumulation / release in the layer due to the fault movements across  $F_1$  and / or  $F_2$  or both.

#### **6. Conclusion and remarks**

(*i*) In the above results we find that the strain on the free surface due to the movements of the faults is of the order of 10<sup>-6</sup> per year and gradually decreases with time.

(*ii*) The region of stress accumulation and release in the layer depends on the orientation and the relative positions of the faults.

(*iii*) The magnitude of stress accumulation / release in the near region of the faults varies from -8 bars to +8 bars, which is large compared to the values  $\pm 0.2$  bar in the half space model [Debnath and Sen (2014, 2015)].

(*iv*) Interaction effect between the two faults depends significantly in the relative positions of the faults.

(*v*) This approach may help in understanding the earthquake generating process to identify possible earthquake precursor.

(*vi*) The lithosphere-asthenosphere system may be represented in a more realistic way by introducing the concept of functionally graded materials with gradual and continuous changes in their rheological behavior. Such model will involved more complicated mathematical techniques.

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#### **References**

- Cathles III, L. M., 1975, "The viscoelasticity of the Earth's mantle", *Princeton University Press, Princeton, N. J.*
- Clift, P., Lin, J. and Barckhausen, U., 2002, "Evidence of low flexural rigidity and low viscosity lower continental crust during continental break-up in the South China Sea", *Marine and Petroleum Geology*, **19**, 951-970.
- Cohen, S. C. and Kramer, M. J., 1984, "Crustal deformation, the earthquake cycle and models of viscoelastic flows in the asthenosphere", *Geophys. J. Roy . Astron. Soc.*, **78**, 735 - 750.
- Debnath, P. and Sen, S., 2014, "Creeping Movement across a Long Strike-Slip Fault in a Half Space of Linear Viscoelastic Material Representing the Lithosphere-Asthenosphere System", *Frontiers in Science*, **4**, 2, 21-28.
- Debnath., P. and Sen, S., 2015, "A Vertical Creeping Strike Slip Fault in a Viscoelastic Half Space under the Action of Tectonic Forces Varying with Time", *IOSR Journal of Mathematics (IOSR-JM)*, **11**, 3, 105-114.
- Ghosh, U., Mukhopadhyay, A. and Sen, S., 1992a, "On two aseismically creeping and interacting vertical strike - slip faults - one buried and the other extended up to the surface in a twolayer model of the lithosphere", *Bull. Ind. Soc. Earth. Tech.*, Paper No. 313, **29**, 1, 1-15.
- Ghosh, U., Mukhopadhyay, A. and Sen, S., 1992b, "On two interacting creeping vertical surface breaking strike - slip faults in a twolayer model of the lithosphere", *Physics of the Earth and Planetary Interiors*, **70**, 119-129.
- Ghosh, U. and Sen, S., 2011, "Stress accumulation near locked buried faults in the lithosphere-asthenosphere system", *International Journal of Computing*, **1**, 4, 786-795.
- Karato, S., 2010, "Rheology of the Earth's mantle", *A historical review Gondowana Research* , **18**, 1, 17-45.
- Kayal, J. R., Zhao, D., Mishra, O. P., De, R. and Singh, O. P., 2002, "The 2001 Bhuj earthquake: Tomographic evidence for fluids at the hypocenter and its implications for rupture nucleation", *Geophysical Research Letters*, **29**, 24, 5.1-5.4.
- Maruyama, T., 1966, "On two dimensional dislocations in an infinite and semi-infinite medium", *Bull. Earthq. Res. Inst.*, Tokyo University, .**44**, Part 3, 811-871.
- Mishra, O. P. and Zhao, D., 2003, "Crack density, saturation rate and porosity at the 2001 Bhuj, India, earthquake hypocenter: A fluid-driven earthquake?", *Earth and Planetary Science Letters*, **212**, 3, 393-405.
- Mishra, O. P., Zhao, D. and Wang, Z., 2008, "The genesis of the 2001 Bhuj, India, earthquake (MW 7.6): A Puzzle for Peninsular India?", Special issue in *Indian Minerals*, **61**, 62, 149-170.
- Mukhopadhyay, A. and Mukherji, P., 1978a, "On stress accumulation in a viscoelastic lithosphere", Proceedings of the Sixth International Symposium on Earthquake Engineering, Roorkee, **1**, 71-76.
- Mukhopadhyay, A., Sen, S. and Maji, P., 1978b, "On the interaction between two locked strike-slip faults", Proceedings of the Sixth International Symposium on Earthquake Engineering, Roorkee, **1**, 77-82.
- Mukhopadhyay, A. and Mukherji, P., 1979b, "On stress accumulation and fault slip in the lithosphere", Indian J. *Meteol. Hydrol. Geophys. (Mausam)*, **30**, 353-358.
- Mukhopadhyay, A., Maji, M. and Sen, S., 1979c, "On stress accumulation in the lithosphere and interaction between two strike-slip faults", *Indian J. Meteol. Hydrol. Geophys. (Mausam)*, **30**, 359-363.
- Mukhopadhyay, A., Sen, S. and Pal, B. P., 1980a, "On stress accumulation in a viscoelastic lithosphere containing a continuously slipping fault", *Bull. Soc. Earthquake Technol.*, **17**, 1, 1-10.
- Mukhopadhyay, A., Pal, B. P. and Sen, S., 1980b, "On stress accumulation near a continuously slipping fault in a two layer model of the lithosphere", *Bull. Soc. Earthquake Technol.*, **17**, 4, 29-38.
- Mukhopadhyay, A. and Mukherji, P., 1984, "On two interacting creeping vertical surface breaking strike-slip faults in the lithosphere", *Bull. ISET*, **21**, 4, 163-191.
- Mukhopadhyay, A. and Mukherji, P., 1986, "On two aseismically creeping and interacting buried vertical strike-slip faults in the lithosphere", *Bull. ISET*, **23**, 3, 91-117.
- Rundle, J. B. and Jackson, D. D., 1977, "A three dimensional viscoelastic model of a strike-slip fault", *Geophys. J. R. Astr. Soc*., **49**, 575-591.
- Rybicki, K., 1971, "The elastic residual field of a very long strike-slip fault in the presence of a discontinuity", *Bull. Seis. Soc. Am*., **61**, 79-92.
- Rybicki, K., 1973, "Static deformation of a multilayered half-space by a very long strike-slip fault", *Pure and Applied Geophysics*, **110**, 1955-1966.
- Sen, S., Sarkar, S. and Mukhopadhyay, A., 1993, "A creeping and surface breaking long strike-slip fault inclined to the vertical in a viscoelastic half space", *Indian J. Meteol. Hydrol. Geophys. (Mausam)*, **44**, 4365-4372.
- Sen, S., Karmakar, A. and Mondal, B., 2012, "A nonplanar surface breaking strike-slip fault in a viscoelastic half-space model of the lithosphere", *IOSR Journal of Mathematics*, **2**, 5, 32-46.
- Singh, A. P., Mishra, O. P., Rastogi, B. K. and Kumar, S., 2013, "Crustal heterogeneities beneath the 2011 Talala, Saurashtra earthquake, Gujrat, India source zone: Seismological evidence for neo-tectonics", *Journal of Asian Earth Sciences*, **62**, 672-684.
- Singh, A. P. and Mishra, O. P., 2015, "Seismological evidence for monsoon induced micro to moderate earthquake sequence beneath the 2011 Talala, Saurashtra earthquake, Gujrat, India", *Tectonophysics*, **661**, 38-48.