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Role of Hamiltonian energy in thunderstorms

G. K. SAWAISARJE, SOMENATH DUTTA and S. JAGTAP*

India Meteorological Department, Pune – 411 005, India

** Dr. Kalmadi Shamarao Junior College (KSJC), Pune – 411 038, India*

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e mail : gksawaisarje@gmail.com

सार - इस शोध पत्र में हमने एक परिकल्पना प्रस्तावित की है कि गर्ज भरे तूफान की हैमिलटनियन ऊर्जा जो पर्यावरण में संवहनी उपलब्ध विभव ऊर्जा उन्मुक्त होती है और मुक्त संवहन के स्तर के पार्सल से संवहनी ऊर्जा आती है वह इस ऊर्जा में सहायक होती है। हमने इस परिकल्पना को सिद्ध करने का प्रयास किया है। हमने हैमिलटनियन संरचना का गर्ज भरे बादल पर प्रयोग किया जो मौसम वैज्ञानिक वेधशाला स्टेशन के ऊपर उर्ध्वाधर रूप से घटित हुआ था। इसके अलावा पालम और डमडम दोनों स्टेशनों के लिए कुल 62 गर्ज भरे तूफानों का चयन किया है। संवहनी उपलब्ध विभव ऊर्जा की अपेक्षा महत्वपूर्ण वृहत संवहनी ऊर्जा के मामलों के लिए हैमिलटियन ऊर्जा की परिकलन की गई और जांच की गई। हमने यह दिखाने का प्रयास किया है कि हैमिलटियन ऊर्जा वह ऊर्जा है जो मुक्त संवहन के स्तर को पार्सल के स्तर तक संवहनी अवरोधी ऊर्जा को उठाती है और वर्षा वाले गर्ज भरे तूफान में मुख्य भूमिका निभाती है।

इन परिणामों से यह पता चलता है कि हैमिलटेनियन ऊर्जा धरातल पर अधिकतम होती है और संवहनी अवरोधी ऊर्जा और संवहनी उपलब्ध विभव ऊर्जा दोनों को ही प्रभावित करती है। निम्न क्षोभमंडल में यह संवहनी अवरोधी ऊर्जा को निष्क्रिय करती है और वायु संहति को धरातल से उन्मुक्त संवहन के स्तर तक ले जाने के लिए आवश्यक साधन जुटाती है। जब ऊपरी क्षेभमंडल में यह संवहनी उपलब्ध ऊर्जा को प्रभावित करती है तब विभव ऊर्जा का भाग गतिक ऊर्जा में परिवर्तित होता है तथा उष्ण और आर्द्र वायु संहति (अस्थिर) एक्सलरेशन दाब ऊर्जा से बढ़ जाता है।

साथ ही छह और विशेष मामलों में स्थिरता सूचक गर्ज भरे तूफान की संभावना का संकेत देते हैं। इसके अलावा सिनॉप्टिक स्थितियां भी इसके लिए अनुकूल होती हैं।

ABSTRACT. In the present study, we propose a hypothesis that "Hamiltonian energy of thunder storm is contributing towards the energy that overcomes convective inhibition energy to lift the parcel to the level of free convection and releases convective available potential energy in the environment". We attempt to substantiate the hypothesis. We have applied Hamiltonian structure to a thundercloud which has occurred vertically above the meteorological observatory station. Further, a total of 62 cases of thunderstorms are selected for both stations Palam and Dumdum. Hamiltonian energy is computed and investigated the cases having significant large convective inhibition energy as compared to that of convective available potential energy. We attempt to show that Hamiltonian is the energy that overcomes convective inhibition energy to lift the parcel to the level of free convection and plays a major role in thunderstorms for giving rain.

Results reveal that Hamiltonian energy is seen to be maximum at the surface and contributes to both convective inhibition energy and convective available potential energy. At the lower troposphere, it overcomes the convective inhibition energy and provides necessary trigger for air mass to move from surface to the level of free convection. While in the upper troposphere, it is contributing to the convective available potential energy such that the part of potential energy converted into kinetic energy & warm and moist air mass (unstable) acceleration is enhanced by pressure energy.

Further, in all the six special cases stability indices had indicated possibility of thunderstorm. In addition, synoptic conditions were also favorable for the same.

Key words – CAPE, Cine, Thunderstorms, Hamiltonian energy.

1. Introduction

It is well known that convective instability in the layers of the atmosphere is a contributory factor in the formation of violent storms like thunderstorms. It is deduced from T-ϕ gram (Te-phi gram) and quantified by different stability indices. Several studies have investigated the convective available potential energy (CAPE) and convective inhibition energy (CIN) for isolated mesoscale convective activity (Moncrieff and

Miller, 1976; Williams and Renno, 1993; Srivastava and Sinha Ray, 1999; De and Dutta, 2005; Sen, 2005). Lifted index is used as a predictor of latent instability (Galway, 1956).

CAPE is the proxy for the amount of kinetic energy that an air parcel can gain from temperature differences between the parcel and environmental air. CIN acts to suppress the release of CAPE and is a proxy for the amount of energy needed to lift a parcel to its Level of Free Convection (LFC). In physics, Lagrangian (L) for conservative system is given by T-V, where, T = Kinetic energies of particles in the system and V = potential energies of particles in the system. We know that if L does not contain time explicitly, the Hamiltonian (H) expressed as $\sum \frac{dq}{dt} p - L$ is equal to 2T – L = T + V. Here, T + V are the total energy of the system. $\frac{dq}{dt}$ is the generalized velocity and q is generalized coordinates and *p* is the generalized momenta. Here T is the homogeneous quadratic function of generalized velocities. Hence, Hamiltonian is a constant of motion [*i.e.*, $\frac{dL}{dt} = 0$ *and* $\frac{dH}{dt} = -\frac{\partial L}{\partial t}$

Therefore, $\frac{dH}{dt} = 0$. It implies that H is constant. As a result, T + V = constant.

In the present study, we propose a hypothesis that "Hamiltonian energy of thunder storm is contributing towards the energy that overcomes CIN to lift parcel to the LFC and releases CAPE in the environment".

Bokhove and Lynch (2007), in the study of application of Hamiltonian for dynamics of the air parcel have used Hamiltonian particle mesh or particle element method and pointed out that equations of motion used in the atmospheric climate simulations are Hamiltonian in the absence of forcing and dissipation. Their study was motivated by preliminary results in low-dimensional models suggesting that this preservation of the Hamiltonian structure on the discrete level is important even in the presence of forcing & dissipation (Hairer *et al*., 2006).

In the present study, we have applied Hamiltonian structure to a thundercloud which has occurred over the meteorological observatory station. For this thundercloud, L is considered independent of time, hence conservative forces are significant. Here L depends on *q* and $\frac{dq}{dt}$. H depends on *q* and *p* and is energy only and constraints do not depend on time. H in cylindrical coordinate system (*r*, *θ*, *z*) has spatial influence up to *r*. So H is the sum of kinetic (dynamic) energies and potential (static) energies with inclusion of friction in the atmospheric layer. Additional energies terms like pressure energy, viscous energy, coriolis energy and molecular energy have been taken into consideration as this system is an open system. In the open system matter is passed in and out of segments of system boundaries and other segments of system boundaries may pass only heat or work and not matter and in thermodynamic equilibrium all flow must vanish.

Further, total 62 cases of thunderstorms are selected for both stations Palam and Dumdum. H is computed and is investigated for the cases having significant large CIN as compared to CAPE. We attempt to show that H is the energy that overcomes CIN to lift the parcel to the LFC and plays a major role in thunderstorms for giving rain.

2. Data and methodology

Present study utilizes the available daily Radiosonde/Rawin data for Stations Palam and Dumdum from India Meteorological Department data archive centre (NDC) at Pune. Information of occurrence of thunderstorm available in Days summary surface observation data was obtained from NDC, India Meteorological Department and RS/RW data from University of Wyoming upper air soundings [http://weather.uwyo.edu/upperair/sounding.html] was also utilized for this study.

The profiles of soundings up to 200 hPa levels with Thunderstorm events are considered for computation of stability indices like k Index (KI), Total-Totals Index (TTI), Sweat Index (SWI), Vorticity Generation Parameter (VGP), Bulk Richardson Number (BRN), Potential Instability Index (PII), Convective Instability Index (CII), CAPE, CIN and lifted index (LI). H was also computed for all the selected cases. The mathematical expression for H used is

$$H = \Big\{\frac{1}{2}[(usin\theta + vcos\theta)^2 + (u^2sin^2\theta + v^2cos^2\theta -2uvsin\theta cos\theta + \frac{w^2}{g^2}] + R_d(1 + 0.608q_v)T +2u\Omega cos\phi z + u\frac{\partial(\delta u)}{\partial z} + kT\Big\} + gz$$

for unit mass of air parcel in thunderstorm environment. k is Boltzman constant, R_d is specific gas constant for dry air, q_v is the mixing ratio, T is the temperature, u, v are zonal and meridional components of wind, ϕ is the latitude of the station, Ω is angular velocity of earth and g is gravity.

TABLE 1

Stability indices computed from 1200 UTC RS/RW Observations

Station	Date of occurrence of Thunderstorm	KI	TTI	SWI	VGP	BRN	PII	CII	DCI	CAPE J/kg	CIN J/kg	LI	TS dur. minutes	Rain DUR minutes
Palam	21 Mar 2000	18.7	46.3	-59.4	.1	1.11	-1.8	12.3	43.4	680.5	280.8	-35.2	220	338
	14 Mar 2000	28.8	51.0	85.6	.0	0.502	-1.8	12.6	50.4	144.5	304.9	-29.2	125	085
	26 May 2001	33.4	49.1	23.4	.1	531.0	-1.8	7.4	61.5	265.7	515.0	-31.8	100	150
	21 May 2011	6.6	-12.2	273.4	.7	859	-0.4	-51.8	34.3	1790.9	-1578.0	9.1	415	Sand/ dustorm 25 min. Squall 2 min.
DumDum	8 May 2000	31.9	40.6	164.4	.1	4.66	-1.8	29.9	50.4	673.8	133.2	-20.4	025	135
	16 May 2000	28.7	40.6	193.4	.0	3.24	-1.8	20.3	60.4	161.8	3.6	-23.9	230	085

TABLE 2

Divergence computation

Station	Date of occurrence of Thunderstorm	Divergence	ω = dP/dt hPa/sec	Vertical velocity W m/sec	(1/2)w^2 Vertical Kinetic energy m^2/sec^2
Palam	21 Mar 2000	-596.046×10^{-4} sec^{-1}	-2.98	0.304	0.0462
	14 Mar 2000	-11.9209×10^{-4} sec^{-1}	-0.0595	0.00607	0.000018
	26 May 2001	Data not available for computation			
	21 May 2011	-0.05939×10^{-4} sec^{-1}	-0.000296	0.009472	4.4859×10^{-5}
DumDum	8 May 2000	-1.90079×10^{-4} sec^{-1}	-0.0095	0.000969	4.6948×10^{-7}
	16 May 2000	-1.038×10^{-4} sec^{-1}	-0.005	0.00051	1.300×10^{-7}

We have selected such cases having CIN greater value as compared to CAPE and comparable to threshold values of CAPE and CIN (Sen, 2005). Hence, H contribution to CIN is shown with six special case studies of Thunderstorm events.

3. Results and discussion

Six special cases of thunderstorm (during March and May) are considered. Table 1 shows the computed stability indices. Table 2 gives the computed divergence. Table 3 gives the computed different energy parameters for the two layers of atmosphere *viz.*, surface to 850 hPa and 800 hPa to 200 hPa. Figs. 1-6 depicts the Hamiltonian energy contributing to CIN for lower troposphere (surface to 850 hPa level) and Hamiltonian energy for mid and upper troposphere (800 to 200 hPa levels). Duraisamy *et al*. (2011) have given the critical values of stability indices for LI, KI, TTI and SWI as < 0 °C, >24 °C, >44.5 °C and >100 °C respectively at 0000 UTC over Delhi. Davies (1998) has attempted to predict storm type using BRN, *i.e.*, by balance of instability and shear. Khole *et al*. (2007) in their study have found that the probability of occurrence of thunderstorm is higher when total-totals index value is higher. They have taken radiosonde data of Kolkata (Dumdum) during pre-monsoon months. The special cases are:

(a) Thunderstorm event having CAPE 680.5 J/kg and CIN 280.8 J/kg dated 21 March, 2000

(b) Thunderstorm event having CAPE 144.5 J/kg and CIN 304.9 J/kg dated 14 March, 2000

(c) Thunderstorm event having CAPE 265.7 J/kg and CIN 515.0 J/Kg dated 26 May, 2001

(d) Thunderstorm event having CAPE 673.8 J/kg and CIN 133.2 J/kg dated 8 May, 2000

(e) Thunderstorm event having CAPE 161.8 J/kg and CIN 3.6 J/kg dated 16 May, 2000

(f) Thunderstorm event having CAPE 1790.9 J/kg and CIN -1578.0 J/kg dated 21 May, 2011

TABLE 3

Values of different energy parameters

S. No.	Date	Layer	Different energy parameters						
			Hamiltonian Energy (MJ)	Potential Energy (MJ)	Kinetic Energy (MJ)	Pressure Energy (MJ)	Coriolis Energy (MJ)	Viscous Energy (MJ)	Molecular Energy (MJ)
1.	14 Mar 2000	Surface to 850 hPa	3.690	0.0383	0.138×10^{-3}	3.651	0.2×10^{-5}	0.2×10^{-5}	0.0
		800 hPa to 200 hPa	3.067	0.0792	8.860×10^{-3}	2.266	5.8×10^{-5}	0.0	0.0
2.	21 Mar 2000	Surface to 850 hPa	2.938	0.036	0.3888×10^{-3}	2.901	0.1×10^{-5}	0.1×10^{-5}	0.0
		800 hPa to 200 hPa	3.519	0.784	5.938×10^{-3}	2.729	5.2×10^{-5}	0.1×10^{-5}	0.0
3.	8 May 2000	Surface to 850 hPa	7.586	0.0363	0.037×10^{-3}	7.550	0.2×10^{-5}	0.0	0.0
		800 hPa to 200 hPa	4.654	0.8045	2.093×10^{-3}	3.847	4.9×10^{-5}	0.0	0.0
4.	16 May 2000	Surface to 850 hPa	6.082	0.03396	0.008×10^{-3}	6.0483	0.07×10^{-5}	0.0	0.0
		800 hPa to 200 hPa	4.836	0.80935	0.219×10^{-3}	4.0267	8.9×10^{-5}	0.01×10^{-5}	0.0
5.	21 May 2011	Surface to 850 hPa	10.084	0.0804	1.847×10^{-3}	10.0019	2.5×10^{-5}	0.07×10^{-5}	0.0
		800 hPa to 200 hPa	19.979	1.625	37.3×10^{-3}	18.3016	83.25×10^{-5}	6.51×10^{-5}	0.0
6.	26 May 2001	Surface to 850 hPa	3.105	0.0339	0.504×10^{-3}	3.070	0.3×10^{-5}	0.3×10^{-5}	0.0
		800 hPa to 200 hPa	4.269	0.8068	2.590×10^{-3}	3.460	7.4×10^{-5}	0.06×10^{-5}	0.0

Case (a) : *Thunderstorm of 21 March, 2000*

On this day over station Palam, CAPE was 680.5 J/kg (weak instability) and CIN 280.8 J/kg (not conducive to development of Thunderstorm) indicated stability of stratification is too high to overcome. A high negative value of LI (-35.2) further suggests severe thunderstorms likely to occur. KI was 18.7 (0% to 20% thunderstorm probability). TTI (46.3) suggested possibility of occurrence of Thunderstorm. However SWI with a value -59.4 was too low to suggest thunderstorm threshold. VGP was having value of 0.1. It indicates rare occurrence of super cell tornadoes. BRN having value of 1.11 suggested thunderstorm occurrence is unlikely. PII was -1.8 < 2.67. It means that thunderstorm may occur. Deep Convective Index (DCI) was having value of 43.4 > 30 °C which indicated possibility of thunderstorm.

In all, out of 11 stability indices, 5 stability indices suggested occurrence of thunderstorm. On this day, thunderstorm lasted for 3.6 hours with rain 5.6 hours.

Divergence was computed for 950 hPa level (Lower Troposphere) and was seen to be having a value of -596.046×10^{-4} sec^{-1}. This suggested that on this day a high convergence and rising motion of air parcel was there which also supported for synoptic large scale features. However, vertical velocity (w) was 0.304 m/sec and vertical kinetic energy was 0.0462 m^2/sec^2 equivalent to 0.0462 J/Kg which is less than CIN. Thus, contribution of air parcel which has vertical kinetic energy (0.0462 m^2/sec^2) to trigger atmosphere for occurrence of thunderstorm. So, Hamiltonian energy ~2.938 MJ, provided the necessary trigger to atmosphere on this day for occurrence of thunderstorm.

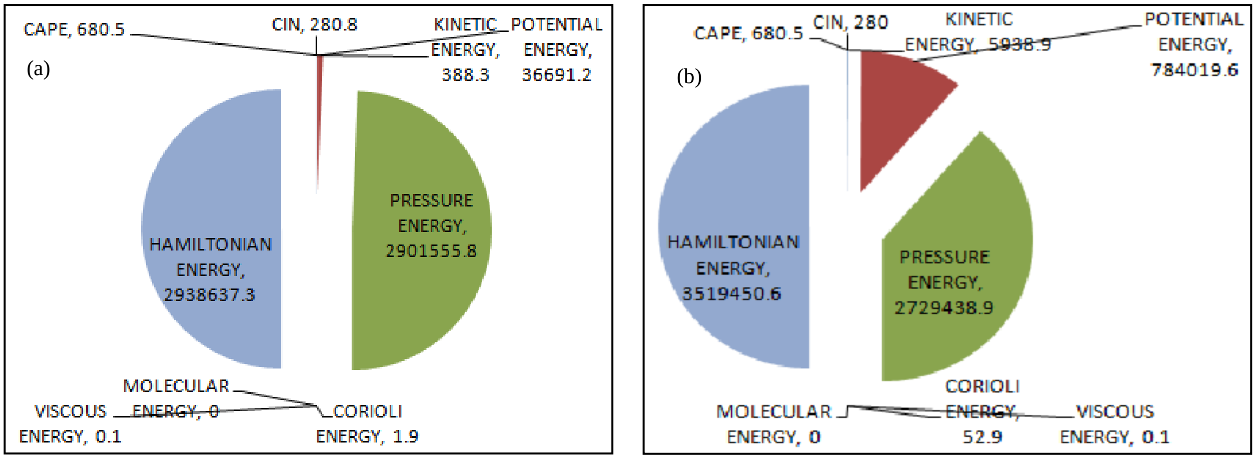


Fig. 1. (a&b). (a) Hamiltonian energy contributing to overcome CIN in lower troposphere (surface to 850 hPa) on March 2000 (b) Hamiltonian energy (800 hPa to 200 hPa) on 21 March, 2000. Note : All energies are in joules (Station Palam)

On this day, vertically integrated Hamiltonian energy for lower troposphere was 2938637.3 J (~2.938 MJ) [Fig. 1 (a)]. This contributed to overcome CIN. CIN energy is needed to lift an air parcel vertically and pseudo adiabatically from its original level to Level of Free Convection and source of CIN is air parcel heating and moistening. Kinetic energy (388.3 J) provided the required movement to lift the parcel to its level of free convection. Some part of potential energy is getting converted to kinetic energy which is providing the necessary trigger according to law of conservation of energy. Pressure energy provides a potential for generating Hamiltonian energy and is a measure of energy contained in each unit of the air parcel due to thermal kinetic motions of air parcels. The air near ground being warm and moist rises and cools leading to condensation and release of heat, and increases its kinetic energy. As a result more air being pulled up leading to moisture condensation and release of heat in to the atmosphere. Viscous energy (Friction) is taken into consideration while formulating Hamiltonian energy for these thunderstorm events.

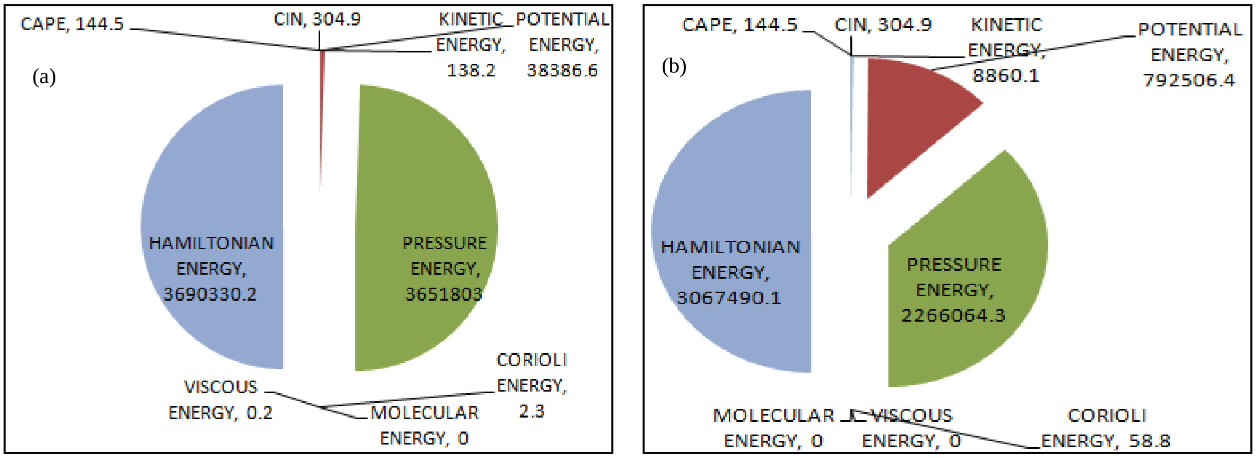
However, vertically integrated Hamiltonian energy in mid and upper troposphere was 3519450.5 J (~3.519 MJ) [Fig. 1(b)]. Potential energy is 0.784 MJ while pressure energy (~2.729 MJ) *i.e.*, pressure differential between the displaced air mass and environmental air at higher altitude to which this air mass is displaced.

Case (b) : *Thunderstorm of 14 March, 2000*

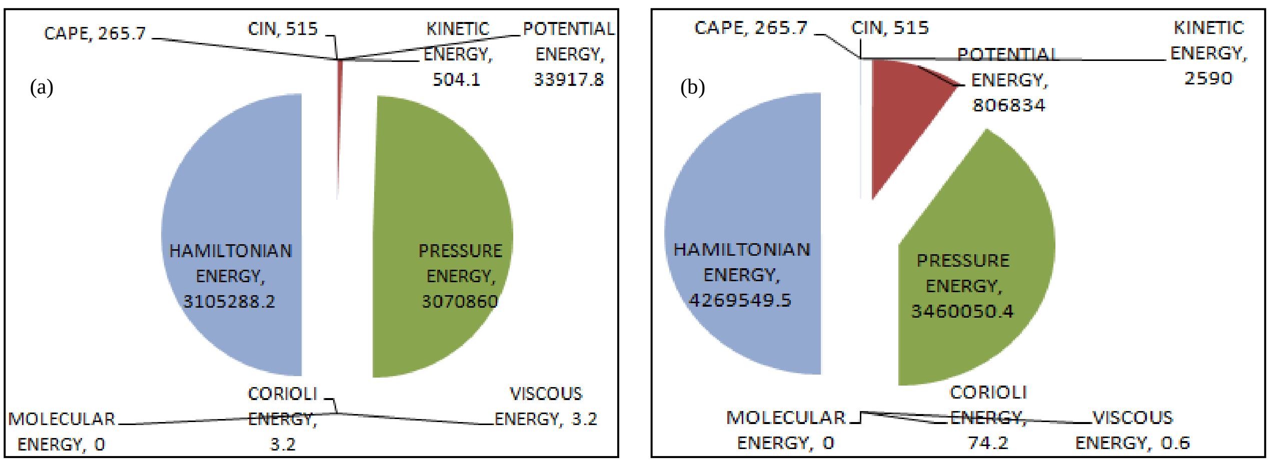
On this day over Palam, CAPE was 144.5 J/kg (weak instability) with high negative value of LI (-29.2) which suggests extreme instability and possibility of severe thunderstorm. The high value of CIN (304.9 J/kg) suggested that stability of stratification was too high to develop and no thunderstorm development occurred. KI was 28.8 indicating 40% to 60% probability of thunderstorm. TTI (51.0) denoted possibility of severe thunderstorm. VGP was .0 suggesting rare occurrence of super cell tornadoes. BRN (0.502) suggested thunderstorm is unlikely. PII was -1.8 showing possibility of thunderstorm. CII was 12.6 > 5 °C threshold indicating thunderstorm to occur. However, DCI was 50.4 > 30 °C which is the threshold for the possibility of thunderstorm. From total 11 stability indices, 5 indices suggest occurrence of thunderstorm. Thunderstorm duration was for 2 hours and rainfall duration was 1 hour.

Divergence at 950 hPa level suggested convergence at that level (value -11.9209 × 10^{-4} sec^{-1}) atmosphere was conducive for occurrence of thunderstorm activity on synoptic scale. However, vertical velocity (w) was 0.00607 m/sec and vertical kinetic energy was 0.000018 m^2/sec^2 equivalent to 0.000018 J/Kg which is less than CIN. Thus, contribution by this rising motion of air parcel (vertical kinetic energy was 0.000018 m^2/sec^2) to trigger atmosphere was very low. Hamiltonian energy on this day was ~ 3.69 MJ [Fig. 2(a)].

High value of Hamiltonian energy (~ 3.69 MJ) was contributing towards to overcome CIN. Kinetic energy in lower troposphere (138.2 J) provided the necessary movement to lift air parcel to the level of free convection. Pressure energy (~ 3.6 MJ) along with Corioli's energy (~2.3 J) and viscous energy (~0.2 J) have contributed thunderstorm. High value of pressure energy indicated that moist warm air near the surface is seen in lower troposphere (up to 850 hPa level).



Figs. 2(a&b). As in Fig.1 (a&b), but for the date 14 March, 2000 (Station Palam)



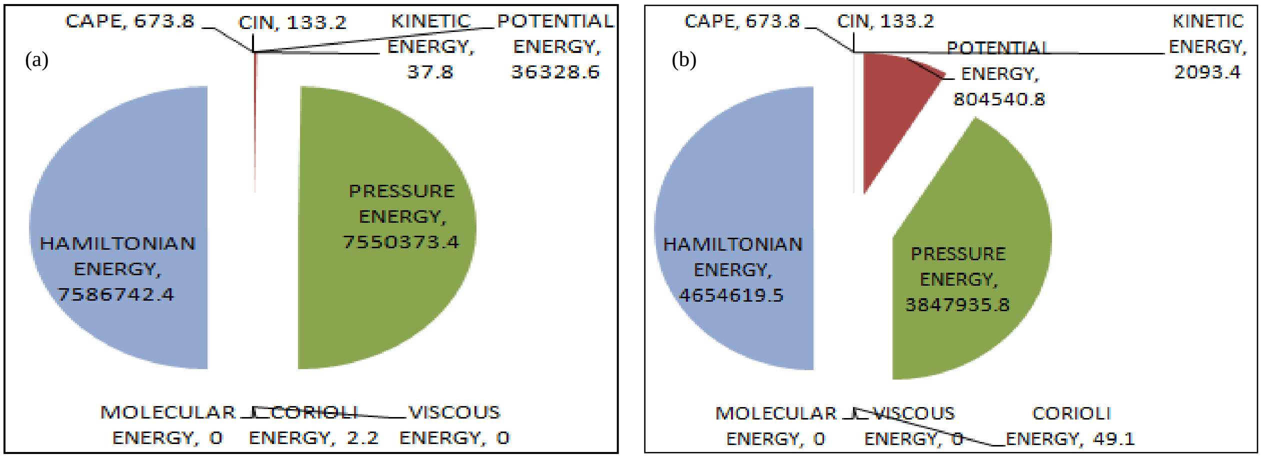
Figs. 3(a&b). As in fig. 1 (a&b), but for the date 26 May, 2001 (Station Palam)

However, observed Hamiltonian energy in the mid and upper troposphere is ~3.067 MJ. Observed Potential energy is ~0.792 MJ, which is getting converted into kinetic energy and acceleration of air parcel (warm and moist)in upper atmosphere is enhanced by pressure energy which is ~2.266 MJ (Fig. 2b).

Case (c): *Thunderstorm of 26 May, 2001*

On this day, over station Palam, CAPE was 265.70 J/kg (weak instability) with a high CIN (515.0 J/kg) suggesting almost no possibility of thunderstorm as stability of stratification was too high to overcome it. KI was 33.4 indicating 60% to 80% thunderstorm probabilities. TTI value of 49.1 indicated thunderstorm possibility. However, SWI was 23.4; it was too low for occurrence of thunderstorm. On this day, VGP was 0.1 indicating super cell tornadoes rare. BRN was 531.0, suggesting single cell and multi cells possibility. PII was -1.8, indicating occurrence of thunderstorm. CII was 7.4, indicating occurrence of thunderstorms likely. DCI was 61.5 indicated possibility of thunderstorm. LI was -31.8, suggesting severe possibility of thunderstorm.

From Fig. 3(a), it is seen that High value of Hamiltonian energy (~ 3.105 MJ) was contributing to overcome CIN. Kinetic energy in lower troposphere (504.1 J) provided the necessary movement to lift air parcel to level of free convection. Pressure energy (3.07 MJ) along with Corioli’s energy (3.2 J) and viscous energy (3.2 J) have contributed to thunderstorm. High value of pressure energy indicated moist warm air near the surface in lower troposphere (up to 850 hPa level).



Figs. 4(a&b). As in fig. 1 (a&b), but for the date 8 May, 2000 (Station Dum Dum)

However, observed Hamiltonian energy [Fig. 3(b)] in the mid and upper troposphere is ~4.269 MJ, which has a value comparable to pressure energy (~3.46 MJ). Observed Potential energy was ~0.806 MJ and also observed air parcel is accelerated by pressure energy in mid and upper troposphere.

Case (d) : *Thunderstorm of 8 May, 2000*

On this day, over station Dumdum, CAPE was 673.8 J/kg (weak instability) with a high CIN (133.2 J/kg) suggesting possibility of multi cell thunderstorms. KI was 31.9 indicating 60% to 80% thunderstorm probabilities. TTI value of 40.6 indicated thunderstorm possibility. However, SWI was 164.4, suggesting slight severe occurrence of thunderstorm. On this day, VGP was 0.1 indicating super cell tornadoes rare. BRN was 4.66, suggesting rare occurrence of tornadoes. PII was -1.8, indicating occurrence of thunderstorm. CII was 29.9, it means that the thunderstorms occurrence is likely. DCI was 50.4 indicated possibility of thunderstorm. LI was -20.4, suggesting severe possibility of thunderstorm.

Divergence on this day at 950 hPa level was -1.90079×10^{-4} sec^{-1}. Thunderstorm lasted for 25 minutes and rain duration was 2 hours 15 minutes. Convergence in the lower troposphere supported occurrence of thunderstorm on synoptic scale. However, vertical velocity (w) was 0.000969 m/sec and vertical kinetic energy was 4.6948×10^{-7} m^2/sec^2 equivalent to 4.6948×10^{-7} J/Kg which is less than CIN. Thus, suggesting vertical kinetic energy of air parcel was very low. Further Hamiltonian energy on this day was ~ 7.58 MJ.

From Fig. 4(a), it is seen that High value of Hamiltonian energy (~ 7.58 MJ) was contributing to overcome CIN. Kinetic energy in lower troposphere (37.8 J) provided the necessary movement to lift air parcel to the level of free convection. Pressure energy (0.363 MJ) along with Corioli's energy (2.2 J) have contributed to thunderstorm. High value of pressure energy (*i.e.*, 0.363 MJ) indicated moist warm air near the surface in lower troposphere (up to 850 hPa level).

However, observed Hamiltonian energy in the mid and upper troposphere is ~4.654 MJ, Potential energy is ~0.804 MJ and pressure energy is ~3.847 MJ for this case of thunderstorm event [Fig. 4(b)].

Case (e) : *Thunderstorm of 16 May, 2000*

On this day, over station Dumdum, CAPE was 161.8 J/kg (weak instability) with a high CIN (3.6 J/kg) suggesting possibility of development of minor cumuli. KI was 28.7 indicating 40% to 60% thunderstorm probabilities. TTI value of 40.6 indicated thunderstorm possibility. However, SWI was 193.4, suggesting slight severe occurrence of thunderstorm. On this day, VGP was 0.0 indicating super cell tornadoes rare. BRN was 3.24, suggesting rare occurrence of tornadoes. PII was -1.8, indicating occurrence of thunderstorm. CII was 20.3; it indicates that thunderstorms are likely to occur. DCI was 60.4, indicated possibility of thunderstorm. LI was -23.9, suggesting severe possibility of thunderstorm.

Only one stability indices indicated non occurrence of thunderstorm. Divergence at 950 hPa level was -1.03810^{-4} sec^{-1} meaning convergence in lower troposphere was supporting large scale synoptic condition. However, vertical velocity (w) was 0.00051 m/sec and vertical kinetic energy was 1.300×10^{-7} m^2/sec^2 equivalent to 1.300×10^{-7} J/Kg which is less than CIN.

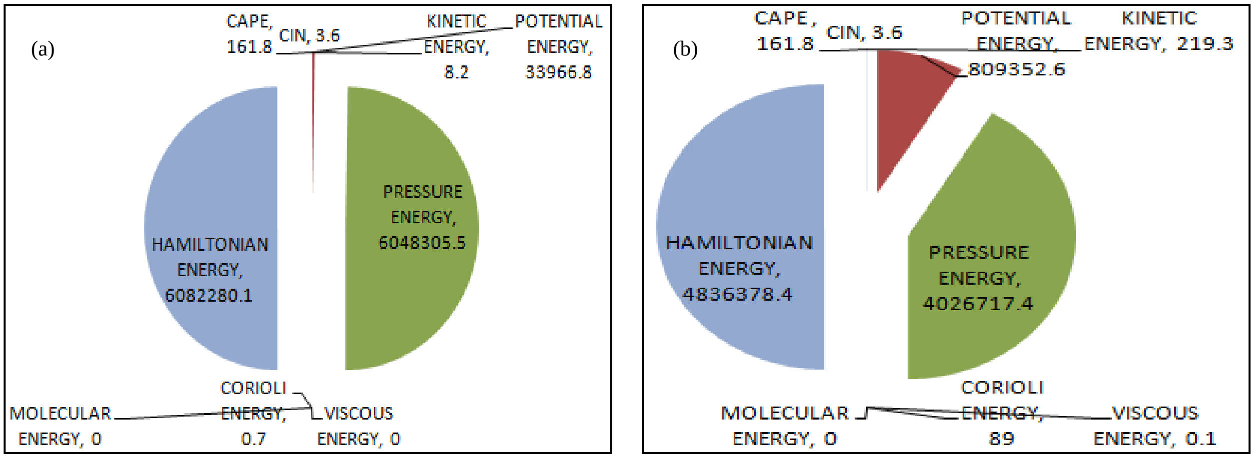


Fig. 5(a&b). As in Fig. 1(a&b), but for the date 16 May, 2000 (Station Dum Dum)

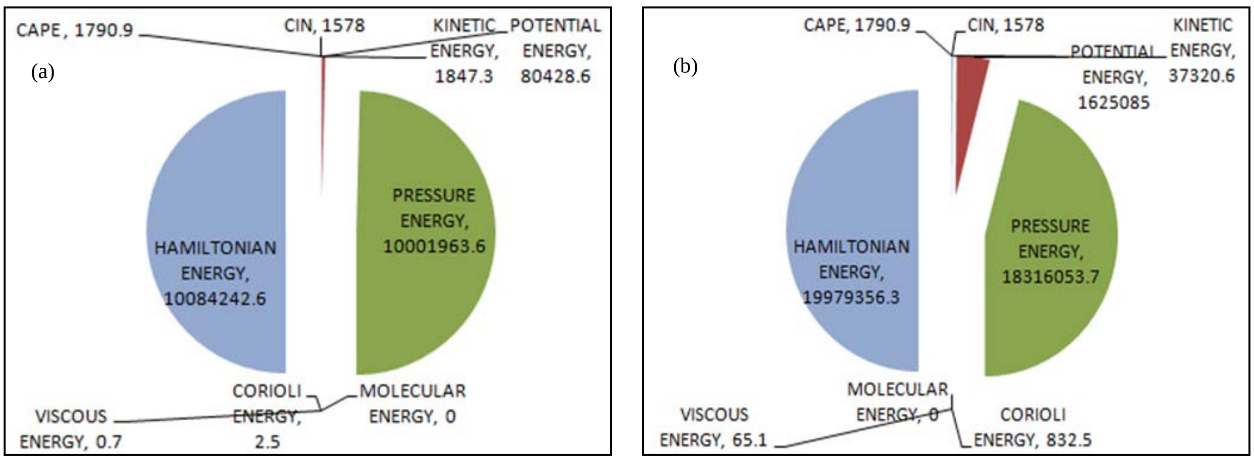


Fig. 6(a&b). As in Fig. 1(a&b), but for the date 21 May, 2011 (Station : Palam)

Thus, for this case vertical kinetic energy was very low and air parcel did not trigger the atmosphere. Further Hamiltonian energy on this day (~ 6.08 MJ) provided trigger to atmosphere for the occurrence of thunderstorm. Thunderstorm lasted for 230 minutes with rain lasting for 85 minutes.

From Fig. 5(a), it is seen that High value of Hamiltonian energy in lower atmosphere (~ 6.08 MJ) was contributing to overcome CIN. Kinetic energy in lower troposphere (~ 8.2 J) provided the necessary movement to lift air parcel to the level of free convection. Also some part of potential energy was converted into kinetic energy, providing the necessary trigger to the atmosphere which is added to Hamiltonian energy. Pressure energy (~ 6.04 MJ) along with Corioli's energy (~ 0.7 J) has contributed to thunderstorm. High value of pressure energy indicated moist warm air near the surface in lower troposphere (up to 850 hPa level).

However, as in Fig. 5(b) observed Hamiltonian energy in the mid and upper troposphere is ~4.836 MJ which is comparable to pressure energy(~4.026 MJ). Potential energy is ~0.809 MJ on this day in mid and upper troposphere indicates enhanced acceleration of air parcel.

Case (f) : *Thunderstorm of 21 May, 2011*

On this day, over station Palam, CAPE was 1790.9 J/kg (moderate instability) with a high CIN (1578.0 J/kg) suggesting multi cell thunderstorm possibility. KI was 6.6 indicating 0% to 20% thunderstorm probability. TTI value of -12.2 indicated thunderstorm coverage rare. However, SWI was 273.4 is more than the threshold of 250, which suggested potential for strong convection. On this day, VGP was 0.7 > 0.6 indicating super cell tornado is likely to occur. BRN was 859 suggesting single cell and multi cells possibility. PII

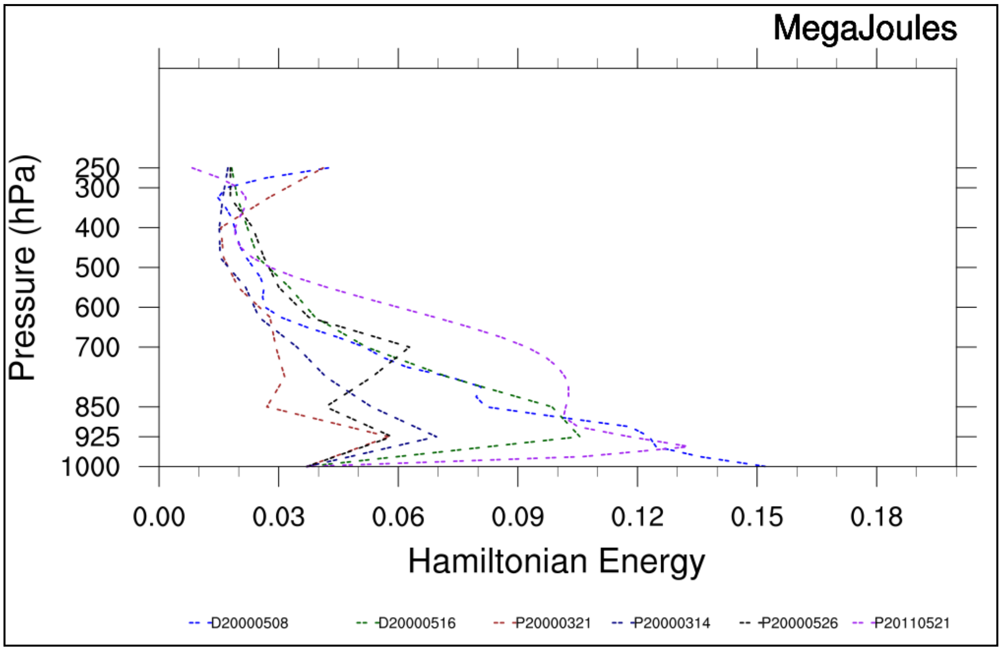


Fig. 7. Vertical Profile of Hamiltonian energy (mJ) at station Dum Dum (denoted by D followed by dates) and Palam (denoted by P followed by dates)

was -0.4 indicating non occurrence of thunderstorm. CII was -51.8 it means that the thunderstorms is likely to occur. DCI was 34.3 indicated possibility of thunderstorm. LI was 9.1 suggesting stable atmosphere and no possibility of thunderstorm.

There was convergence at 968 hPa level (-0.05939×10^{-4} sec^{-1}). Duration of thunderstorm was 6.9 hours with 25 minutes of sand/dust storm and 2 minutes squall. In all, 3 stability indices did not predict thunderstorm out of 11 stability indices. However, vertical velocity (w) was 0.009472 m/sec and vertical kinetic energy was 4.4859×10^{-5} m^2/sec^2 equivalent to 4.4859×10^{-5} J/Kg which is less than CIN. Thus, indicating that contribution of air parcel was very low. Further Hamiltonian energy on this day was high, *i.e.*, ~ 0.084 MJ.

From Fig. 6(a), it is seen that High value of Hamiltonian energy (~ 10.084 MJ) was contributing to overcome CIN. Kinetic energy in lower troposphere (1847.3 J) provided the necessary movement to lift the air parcel to the level of free convection. Pressure energy (10.0 MJ) along with Corioli's energy (2.5 J) and viscous energy (0.7 J) have contributed to thunderstorm. High value of pressure energy indicated moist warm air near the surface in lower troposphere (up to 850 hPa level).

However, as seen from Fig. 6(b), observed Hamiltonian energy in the mid and upper troposphere is ~19.979 MJ, Potential energy is ~1.625 MJ and pressure energy is ~18.31 MJ.

From all the six special case studies, it can be inferred that Hamiltonian energy is maximum at the surface and contributes to both CIN and CAPE. At the lower troposphere it overcomes the CIN and provides necessary drift for air mass to move from surface to the level of free convection. While in mid and upper troposphere, it is contributing to CAPE such that the part of potential energy converted into kinetic energy. Warm and moist air mass (unstable) acceleration is enhanced by pressure energy. CIN is $\int g\left(\frac{T_v, parcel - T_v, envir}{T_v \, envir}\right)dz$ and integral from z_{bottom} to z_{top} while CAPE is $\int g\left(\frac{T_v, parcel - T_v, envir}{T_v \, envir}\right)dz$ and integral from z_f to z_n , where, z_f is level of free convection and z_n is height of equilibrium level (neutral buoyancy). Bracketed terms in above expressions can be interpreted as potential of warm moist air mass which is unstable with respect to environment.

Fig. 7 depicts the vertical profile of Hamiltonian energy (MJ) for the six case studies of thunderstorm events. It is seen that the Hamiltonian energy (MJ) is maximum at the surface on all the six special case studies undertaken and Hamiltonian energy decreases from lower troposphere as we go upward in the atmosphere.

4. Conclusions

It is seen that Hamiltonian energy is maximum at the surface and contributes to both CIN and CAPE. At the lower troposphere, it overcomes the CIN and provides necessary trigger for the air mass to move from surface to the level of free convection. While in mid and upper troposphere, it is contributing to CAPE such that the part of potential energy is converted into kinetic energy, and warm and moist air mass (unstable) acceleration is enhanced by pressure energy.

Further, in all the six special cases under consideration stability indices have indicated the possibility of thunderstorm. In addition synoptic conditions were also favorable for the thunderstorm to occur.

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