# **Crosswind integrated concentration for various dispersion parameter systems**

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सार – स्थानीय स्तर प्रकीर्णन के लिए गौसीयन पिच्छक मॉडल (Gaussian Plume Model) का व्यापक रूप से प्रयोग किया जाता है। अनुप्रस्थ पवन की कुल सांद्रता ज्ञात करने के लिए गौसीयन सूत्र (फॉमूला) को संगठित किया है। अनुप्रस्थ पवन की कुल सांद्रता की गणना करने के लिए प्रकीर्णन प्राचलों की भिन्न–भिन्न प्रणालियों का उपयोग किया गया है। सतह स्तर में ऊँचाई के अनुसार पवन गति की भिन्नता का वर्णन करने के लिए लागरिथ्मीक विंड प्रोफाइल का उपयोग किया गया है। इसमें छोड़ी जाने वाली प्रभावी ऊँचाई को ध्यान) में रखा गया है। भिन्न भिन्न प्रकीर्णन प्राचल प्रणालियों के लिए पूर्वानुमानित सांद्रताओं और कोपेनहेगन के विसरण प्रयोग से प्राप्त किए गए प्रेक्षित आँकड़ों की तुलना करने के लिए सांख्यिकीय परिमापों का उपयोग किया गया है।

**ABSTRACT.** The Gaussian plume model is the most widely used model for local scale dispersion. The Gaussian formula has been integrated to obtain the crosswind-integrated concentration. Different systems of dispersion parameters are used to calculate the crosswind integrated concentration. A logarithmic wind profile is used to describe the variation of wind speed with height in the surface layer. The effective release height was taken into consideration. Statistical measures are utilized in the comparison between the predicted concentrations for different dispersion parameter systems and the observed concentrations data obtained from Copenhagen diffusion experiment.

**Key words** – Gaussian plume model, Crosswind-integrated concentration, Logarithmic wind profile, Plume rise, Sigma schemes, Statistical analysis.

## **1. Introduction**

The Gaussian plume model (GPM) derived first by Sutton (1953), Csanady (1973), Smith (1973) and Turner (1970) provides the primary method for calculating concentrations of non-reactive air pollutants. The GPM has found widespread application in design of stacks and environmental impact analysis.

The formulations of the most commonly used models in air quality analysis assume wind speed to be constant. However, in reality it increases with the vertical height (Stull, 1988). A power law profile is generally used to describe the variation of wind speed with height in the surface layer (Smith, 1957; Pasquill and Smith, 1983; Khaled *et al*., 2005).

In present work, we used a logarithmic wind profile to describe the variation of wind speed with height in the surface layer (Smith, 1990).

We performed a statistical analysis concerning the agreement of the measured and predicted concentrations by using different dispersion schemes.

## **2. Mathematical description**

The Gaussian formula for estimating the concentration of pollutant released from a continuous point source at some point above the ground is given by (WMO, 1982, Lines *et al*., 1997):

$$
C(x, y, z, H) = \frac{Q}{2\pi u \sigma_y \sigma_z} e^{-y^2/2\sigma_y^2}
$$

$$
\left[ e^{-(z-H)^2/2\sigma_z^2} + e^{-(z+H)^2/2\sigma_z^2} \right]
$$

(1)

Where,



The crosswind-integrated concentration  $(C_v)$  can be obtained by integrating both sides of Eqn. (1) with respect to y from  $-\infty$  to  $+\infty$ , in the form:

$$
C_{y}(x, z, H) = \frac{Q}{\sqrt{2\pi} u \sigma_{z}}
$$

$$
\left[ e^{-(z-H)^{2}/2\sigma_{z}^{2}} + e^{-(z+H)^{2}/2\sigma_{z}^{2}} \right]
$$
(2)

where,

$$
\int_{-\infty}^{\infty} e^{(-y^2/2\sigma_y^2)} dy = \sqrt{2\pi} \sigma_y
$$
 (3)

## **3. Wind speed profile**

A logarithmic wind profile is used to describe the variation of wind speed with height in the surface layer (Smith, 1990) as:

## 3.1. *In unstable conditions*

$$
u(z) = \frac{u_{*}}{k} \left[ \ln \left( \frac{\mu - 1}{\mu + 1} \cdot \frac{\mu_{o} + 1}{\mu_{o} - 1} \right) + 2 \tan^{-1} \mu - 2 \tan^{-1} \mu_{o} \right]
$$
(4)

## **TABLE 1**

**Relation between Pasquill stability and Monin Obukhove length L (Gifford, 1976)** 



Where,

$$
\mu = \left(1 + 16 \frac{z}{|L|}\right)^{1/4} \text{ and } \mu_0 = \left(1 + 16 \frac{z_0}{|L|}\right)^{1/4} \tag{5}
$$

3.2. *In stable conditions* 

$$
u(z) = \frac{u_{*}}{k} \left[ \ln \left( \frac{z}{z_0} \right) + \frac{5.2}{L} (z - z_0) \right]
$$
 (6)

#### 3.3. *In neutral conditions*

$$
u(z) = \frac{u_{*}}{k} \ln\left(\frac{z}{z_{o}}\right)
$$
 (7)

where,  $u_*$  is the friction velocity and k is the von

Karman's constant, often taken to be  $0.4$ ,  $z<sub>o</sub>$  is the roughness length, expresses the effect of the varying ground surface roughness on the wind profile and *L* is the Monin Obukhov scale length, its value is negative in unstable conditions. In stable conditions *L* is positive and infinite under neutral conditions.

The values of *L* for different stability classes have been determined according to John and Robert, 1983 (Table 1). The table presents a relationship between the Pasquill stability classes and Monin Obukhov scale length.

#### **4. Data used**

1987). The tracer sulfur hexafluoride  $(SF_6)$ , an inert gas The data used was obtained from the atmospheric diffusion experiments conducted at the northern part of Copenhagen, Denmark, under the neutral and unstable conditions (Gryning and Lyck, 1984; Gryning *et al*., tracer, was released from a tower at a height of 115 m without buoyancy and collected near ground level in

Run No.	Distance (m)	P-G stability	$u_{10}$ (m/s)	Obs. conc. $(10^{-4} \text{ s/m}^2)$	L(m)	$u_*$ (m/s)	$u_{115}$ (m/s)
$1\,$	1900	$\boldsymbol{\rm{A}}$	2.1	6.84	$-2.5$	0.60	3.06
$\mathbf{1}$	3700	$\boldsymbol{\mathsf{A}}$	2.1	2.31	$-2.5$	0.60	3.06
$\sqrt{2}$	2100	$\mathbf C$	4.9	5.38	$-13.5$	0.98	7.30
$\sqrt{2}$	4200	$\mathcal{C}$	4.9	2.95	$-13.5$	0.98	7.30
$\mathfrak z$	1900	$\, {\bf B}$	2.4	8.2	$-4.5$	0.60	3.51
$\mathfrak{Z}$	3700	$\, {\bf B}$	2.4	6.22	$-4.5$	0.60	3.51
$\mathfrak{Z}$	5400	$\, {\bf B}$	2.4	4.3	$-4.5$	0.60	3.51
$\overline{4}$	4000	$\mathbf C$	2.5	11.7	$-13.5$	0.50	3.73
$\sqrt{5}$	2100	$\mathcal{C}$	3.1	6.72	$-13.5$	0.62	4.62
$\sqrt{5}$	4200	$\mathbf C$	$3.1$	5.84	$-13.5$	0.62	4.62
$\sqrt{5}$	6100	$\mathbf C$	3.1	4.97	$-13.5$	$0.62\,$	4.62
$\sqrt{6}$	2000	$\mathbf C$	$7.2\,$	3.96	$-13.5$	1.45	10.73
$\sqrt{6}$	4200	$\mathbf C$	$7.2\,$	2.22	$-13.5$	1.45	10.73
$\sqrt{6}$	5900	$\mathbf C$	$7.2\,$	1.83	$-13.5$	1.45	10.73
$\overline{7}$	2000	$\, {\bf B}$	4.1	6.7	$-4.5$	1.02	6.00
$\boldsymbol{7}$	4100	$\, {\bf B}$	4.1	3.25	$-4.5$	1.02	6.00
$\boldsymbol{7}$	5300	$\mathbf B$	4.1	2.23	$-4.5$	1.02	6.00
$\,8\,$	1900	$\mathbf D$	$4.2\,$	4.16	$\infty$	0.60	7.85
$\,$ 8 $\,$	3600	D	4.2	2.02	$\infty$	0.60	7.85
$\,8\,$	5300	$\mathbf D$	4.2	1.52	$\infty$	0.60	7.85
9	2100	$\mathcal{C}$	5.1	4.58	$-13.5$	1.02	7.60
$\overline{9}$	4200	$\mathbf C$	5.1	3.11	$-13.5$	1.02	7.60
$\boldsymbol{9}$	6000	$\mathbf C$	$5.1\,$	2.59	$-13.5$	1.02	7.60

Data used from Copenhagen experiments and the estimated values of  $L$ ,  $U_*$  and  $u_{115}$  at different distances

crosswind arcs 2 to 6 km from the source. The tracer 4.1. *The effective stack height*  sampling time was 1 hour. The roughness length was 0.6 m (Sharan and Modani, 2006).

The value of  $u_*$  corresponding to different stability classes have been estimated applying Eqns. (4-7) in addition to the wind speed measurements at a height  $(z = 10 \text{ m})$ . Consequently, the values of the wind speed at the release height (115 m) was estimated.

 The data used at different distances [wind speed at 10 m height  $(u_{10})$ , the atmospheric stability classes and the observed concentrations during the experiment] and the estimated values of  $L$ ,  $u_*$  and wind speed at the release height  $u_{115}$  are presented in Table 2.

The effective stack height is generally presented in the form (IAEA Safety Guide, 1983):

$$
H = h_S + \Delta h \tag{8}
$$

where,  $h_s$  is the physical stack height and  $\Delta h$  is the plume rise given by:

$$
\Delta h = 3 \frac{w_o}{u(h_s)} D_i \tag{9}
$$

where,

 $w_0$  is the exit velocity (m/s). It is taken to be (4 m/s),

#### **Coefficients of the Pasquill-Gifford system for all stability classes (Vogt, 1977)**





#### **Values of the dispersion parameters corresponding to Pasquill stability**



- $D_i$  is the internal stack diameter (m). It is taken to be  $(1m)$ ,
- $u(h_s)$  is the wind speed (m/s) at the stack height.

## 4.2. *Diffusion parameters*

Since the Gaussian plume model has been expressed in terms of diffusion parameters, σ*y* and σ*z*, the subjective aspect of using this model is the selection of appropriate horizontal and vertical diffusion parameters. The following are some of the most important systems of diffusion parameters; we used to calculate the concentration of pollutant.

## 4.2.1. *Pasquill-Gifford system*

Pasquill (1961) suggested values of σ*y* and σ*z* as functions of distance for use with his suggested stability categories. Gifford **(**1961) suggested modified values of σ*y* and σ*z* for use with the original Pasquill stability categories. The combination of Pasquill and Gifford parameters is called P-G scheme. In this scheme  $\sigma_v$  and  $\sigma_z$ are obtained from graphs as a function of downwind distance, *x*, for each stability class. These curves can be approximated by the following equations (John and Robert, 1983):

$$
\sigma_y(x) = (a_1 \ln x + a_2)x \tag{10}
$$

$$
\sigma_z(x) = \frac{1}{2.15} \exp(b_1 + b_2 \ln x + b_3 \ln^2 x)
$$
 (11)

where the constants  $a_1$ ,  $a_2$ ,  $b_1$ ,  $b_2$ , and  $b_3$  depend on the atmospheric stability and their values are presented in Table 3.

#### 4.2.2. *Standard scheme*

In this scheme, the crosswind dispersion parameter σ*<sup>y</sup>* (*x*) and the vertical dispersion parameter σ*<sup>z</sup>* (*x*) for various stability classes can be analytically expressed

**Formulas recommended by Briggs (1973) for**  $\sigma_y(x)$  **and**  $\sigma_z(x)$ ;  $10^2 \leq x \leq 10^4$  m

Atmospheric stability	$\sigma_v(x)$ (m)	$\sigma_z(x)$ (m)		
A and B	$0.32x(1+0.0004x)^{-1/2}$	$0.24x(1+0.001x)^{-1/2}$		
C	$0.22x(1 + 0.0004x)^{-1/2}$	0.20x		
C		$0.16x (1 + 0.0004x)^{-1/2}$ $0.14x (1 + 0.0003x)^{-1/2}$		
E and F		$0.11x (1 + 0.0004x)^{-1/2}$ $0.08x (1 + 0.00015x)^{-1/2}$		

based on Pasquill-Gifford (P-G) curves as follows (Green *et al*., 1980):

$$
\sigma_y = \frac{\mathbf{r} \, x}{\left(1 + \frac{x}{a}\right)^p} \tag{12}
$$

$$
\sigma_z = \frac{\text{s}x}{\left(1 + x/a\right)^q} \tag{13}
$$

where r, s, a, p and q are constants which depend on the atmospheric stability. Their values are given in Table 4 (Green *et al*., 1980).

#### 4.2.3. *Briggs system*

Briggs (1973) developed the set of analytical formulas for  $\sigma_y$  and  $\sigma_z$  in urban conditions given in Table 5. These are valid only for downwind distances *x*, between 0.1 and 10 km and are intended for use in estimating ground level concentrations.

#### 4.2.4. *Irwin method*

The standard deviations of plume concentration distribution in the horizontal and vertical directions, σ*<sup>y</sup>* and σ*z* respectively have been proposed by Irwin (1983) as follows:

$$
\sigma_y(x) = \sigma_V t f_y \tag{14}
$$

and

$$
\sigma_Z(x) = \sigma_W t f_Z \tag{15}
$$

where,  $t = x/u_{115}$  is the travel time of the pollutant (sec) and  $f_y$  and  $f_z$  are non-dimensional function of travel time and given by Irwin (1983) as :

$$
f_y = \frac{1}{1 + 0.9\sqrt{\frac{t}{1000}}},\tag{16}
$$

#### **TABLE 6**

**Values of the standard deviation of the horizontal and vertical wind directions for different atmospheric stability** 

Stability	в		Ð	E.	ы
$\sigma_{\phi}$	20		10		2.5
$\sigma_{\theta}$	8	6.5	5.5	2.5	

$$
f_Z = 1, \text{ for unstable condition and} \tag{17}
$$

$$
f_Z = \frac{1}{1 + 0.9\sqrt{\frac{t}{50}}}
$$
, for stable condition. (18)

The standard deviations  $\sigma_v$  and  $\sigma_w$  of the wind speed in the lateral and vertical directions for small angles are given as:

$$
\sigma_{\mathcal{V}}(x) = \sigma_{\theta} \ u_{115} \tag{19}
$$

$$
\sigma_W(x) = \sigma_{\phi} \ u_{115} \tag{20}
$$

wind direction in the horizontal and vertical respectively. where  $\sigma_{\theta}$  and  $\sigma_{\phi}$  are the standard deviations of the Therefore, Eqns. (14) and (15) can be rewritten as (Khaled *et al*., 2003):

$$
\sigma_y(x) = \frac{\sigma_{\theta} x}{1 + 0.9 \sqrt{\frac{x}{1000 u_{115}}}}
$$

for stable and unstable conditions (21)

and

$$
\sigma_z(x) = \sigma_{\phi} x
$$
 for unstable condition, (22)

$$
\sigma_z(x) = \frac{\sigma_{\phi} x}{1 + 0.9 \sqrt{\frac{x}{50 u_{115}}}}
$$
(23)

#### for stable condition

The specifications of  $\sigma_{\theta}$  and  $\sigma_{\phi}$  can be found in Gifford (1976) and Hanna *et al*. (1982). Based on Pasquill stability classes from A to F, they are given in Table 6.

#### **Coefficients of different systems of diffusion parameters for all stability classes (Vogt, 1977)**



#### **TABLE 8**

## **Observed diffusion parameters, release height and applicable downwind distances**



## 4.2.5. *Power law method* and

# 4.2.5.1. *Klug system* ( ) *qz*

 $\overline{a}$ 

Klug (1969) specified a system of diffusion parameters that is applicable for short-term ground-level release over terrain with a low surface roughness (John and Robert, 1983). Klug does not exceed source distances of 2 or 3 km. In this range the diffusion parameters can be described by power law functions as:

$$
\sigma_y(x) = p_y x^{qy} \tag{24}
$$

$$
\sigma_z(x) = p_z x^{qz} \tag{25}
$$

where  $x$  is the source distance and the coefficients  $p$ and *q* are specified in Table 7.

## 4.2.5.2. *Jülich system*

The tracer experiments carried out in the vicinity of the Jülich Nuclear Research Center at emission heights of  $\int_0^{p_y} x^{y}$  (24) 50 and 100 m and during emission periods of 1 hr (John



## **Observed and calculated normalized crosswind integrated concentrations Cy/Q (10-4 sm-2) at ground surface in diffusion experiment in northern part of Copenhagen**

and Robert, 1983). The experiments carried out up to source distance of 11 km. The diffusion parameters are described by power law functions, the coefficients of Jülich system at emission height 100 m are listed in Table 7.

# 4.2.5.3. *Brookhaven system*

The tracer experiments at Brookhaven were carried out under conditions typical for the release of pollutants from industrial plants (the tracer was released at a height of 108 m with emission periods of 1 hr and its dispersion was measured over terrain of medium roughness). The diffusion parameters are described by power law functions, the coefficients of Brookhaven system (John and Robert, 1983) are listed in Table 7.

The system of diffusion parameters, release height of pollutant and the applicable distance of diffusion parameters are shown in Table 8.

The normalized crosswind-integrated concentrations  $C_y/Q$  (10<sup>-4</sup> sm<sup>-2</sup>) of SF<sub>6</sub> were calculated by using the different sigma schemes and the results are presented in Table 9. The comparisons of the observed and predicted concentrations are represented graphically as in Figs. (1 & 2).



Fig. 1. Comparison of the observed and predicted normalized crosswind integrated concentrations of SF<sub>6</sub> *via* download distance



**Fig. 2.** Comparison between normalized integrated concentration and observed concentration

## **5. Statistical analysis of the observed and predicted concentration**

Willmott (1981) discussed the application of various statistical parameters, which are utilized in the comparison of predicted and observed data sets. These statistical measures are defined using the following notation. Let *Cp* denote the predicted concentration and *Co* be the corresponding observed concentration. The most commonly used statistical measures for model evaluation were chosen for the present analysis (Fariba and Hanadi, 2004):

(*i*) Normalized mean square error (NMSE): It is an estimator of the overall deviations between predicted and observed concentrations. Smaller values of NMSE indicate a better model performance. It is defined as:

$$
NMSE = \frac{(C_o - C_p)^2}{\overline{C}_o \overline{C}_p}
$$
 (26)

The model is considered acceptable when NMSE  $\leq 0.5$ 

(*ii*) Fractional bias (FB): It provides information on the tendency of the model to overestimate or underestimate the observed concentrations. The values of FB lie between  $-2$  and  $+2$  and it has a value of zero for an ideal model. It is expressed as:

$$
FB = \frac{(\overline{C}_o - \overline{C}_p)}{0.5(\overline{C}_o + \overline{C}_p)}
$$
(27)

The model is assumed acceptable when  $-0.5 \leq FB \leq 0.5$ 

(*iii*) Correlation coefficient (R): It describes the degree of association between predicted and observed concentrations and is given by:

$$
R = \frac{(C_o - \overline{C}_o)(C_p - \overline{C}_p)}{\sigma_o \sigma_p}
$$
 (28)

where  $\sigma_o$  and  $\sigma_p$  are the standard deviations of  $C_o$ and  $C_p$  respectively. The over bars denote the mean values.

The square of correlation coefficient is called coefficient of determination  $R^2$ . Its value lies between 0 and 1 and for good model performance it should be close to unity.

**TABLE 10** 

**Statistical measures evaluating the model performance** 

R	NMSE	<b>FB</b>	FAC2
0.48	1.37	0.83	0.48
0.18	66.00	1.93	0.02
0.68	0.18	0.04	1.08
0.61	0.24	0.05	1.09
0.70	0.19	$-0.07$	1.20
0.67	0.30	0.29	0.85
0.56	0.32	0.25	0.93

<sup>(</sup>*iv*) Fraction within a factor of two (FAC2) is defined as:

 $FAC2$  = fraction of the data for which

$$
0.5 \le (C_p/C_o) \le 2\tag{29}
$$

The model is accepted if  $FAC2 > 0.8$ 

These statistical measures were computed for each of the schemes predictions of normalized concentration  $(C_v/Q)$  values and comparing them with the corresponding observed values. The results are presented in Table 10.

# **6. Conclusions**

Gaussian plume model is the most widely used model for local scale dispersion**.** The Gaussian formula has been integrated to obtain the crosswind integrated concentration. Since the Gaussian plume model has been expressed in terms of diffusion parameters  $\sigma_v$  and  $\sigma_z$  the subjective aspect of using this model is the selection of appropriate horizontal and vertical diffusion parameters. Some of the most important systems of dispersion parameters were used to calculate the concentration of pollutant to determine the accurate system.

The data used was obtained from the atmospheric diffusion experiments conducted at the northern part of Copenhagen, Denmark, under the neutral and unstable conditions (Gryning and Lyck, 1984; Gryning *et al*., 1987). The tracer sulfur hexafluoride  $(SF_6)$ , an inert gas tracer, was released from a tower at a height of 115 m without buoyancy and collected near ground level in crosswind arcs 2 to 6 km from the source. The roughness length was 0.6 m.

The concentration of pollutant has been estimated by using each system of diffusion parameters at a release height (115 m) of Copenhagen experiment.

Table 9 shows that the concentrations computed by the schemes; Standard, P-G, Klug, Jülich (100 m), and Brookhaven (108 m) are in good agreement with those observed, while the computed concentrations using the schemes Briggs and Irwin do not agree with the observed values.

The statistical measures (FB, NMSE, R, FAC2) were computed for each of the schemes predictions of normalized concentration  $(C_v/Q)$  values and comparing them with the corresponding observed values.

From the statistical measures used for model performance Eqns. (26 to 29) and the estimated values of these statistics Table 10 we found that the models; Standard, P-G, Klug, Jülich (100 m), and Brookhaven are acceptable models while the models; Briggs and Irwin are not acceptable. The model Irwin gives the poorest performance. Klug model gives better performance. Standard, Jülich and P-G models give very good performance. Brookhaven gives best performance.

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