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Maximum crosswind integrated ground level concentration in two stability classes

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सार – इस शोध पत्र में निष्प्रभावी और अस्थिर स्थितियों में क्रॉसपवन समाकलित सांद्रण लेने के लिए दो दिशाओं में अभिवहन विसरण समीकरण (ADE) को हल किया गया है। लाप्लास रूपांतरण तकनीक का उपयोग तथा उर्ध्वाधर ऊँचाई पर आधारित पवन गति और भंवर विसरणशीलता की समीक्षा करते हुए यह हल निकाला गया है। इसके साथ ही भू–स्तर और अधिकतम सांद्रणों का भी आकलन किया गया है। हमने इस मॉडल में पूर्वानुमानित और प्रेक्षित सांद्रण आँकड़ों के मध्य तुलना करने के लिए कोपनहेगन (डेनमार्क) से लिए गए आनुभविक आँकड़ों का उपयोग किया है।

ABSTRACT. The advection diffusion equation (ADE) is solved in two directions to obtain the crosswind integrated concentration in neutral and unstable conditions. The solution is solved using Laplace transformation technique and considering the wind speed and eddy diffusivity depending on the vertical height. Also the ground level and maximum concentrations are estimated. We use in this model empirical data from Copenhagen (Denmark) to compare between predicted and observed concentration data.

Key words – Advection diffusion equation, Neutral and unstable conditions, Laplace transform, Maximum and predicted normalized crosswind integrated concentrations.

1. Introduction

The analytical solution of the atmospheric diffusion equation contains different shapes depending on Gaussian and non- Gaussian solutions. An analytical solution with power law for wind and eddy diffusivity with the realistic assumption was studied by (Demuth, 1978). The solution has been implemented in the KAPPA-G model (Zannetti, 1986; Tirabassi *et al*., 1986). (Lin and Hildemann, 1997) extended the solution of (Demuth, 1978) under boundary conditions suitable for dry deposition at the ground. The mathematics of atmospheric dispersion modeling is studied by (John, 2011). In the analytical solutions of the diffusion-advection equation, assuming constant along the whole planetary boundary layer (PBL) or following a power law was studied by (Van Ulden and Hotslag, 1978; Pasquill and Smith, 1983; Seinfeld, 1986; Tirabassi *et al*., 1986; Sharan *et al*., 1996).

Estimation of crosswind integrated Gaussian and non-Gaussian concentration through different dispersion schemes is studied by (Essa and Fouad, 2011). Analytical solution of diffusion equation in two dimensions using two forms of eddy diffusivities is studied by Essa *et al*. (2011).

In this paper the advection diffusion equation (ADE) is solved in two directions to obtain crosswind integrated ground level concentration in neutral and unstable conditions. Laplace transformation technique is used considering the variation of wind speed and eddy diffusivity with height. Also the maximum ground level concentration is estimated. Observed data from Copenhagen (Denmark) and predicted concentration data using statistical technique is compared.

2. Analytical solution

Time dependent advection – diffusion equation is written as (Arya, 1995).

$$\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial x}=\frac{\partial}{\partial x}\left(k_x\frac{\partial C}{\partial x}\right)+\frac{\partial}{\partial y}\left(k_y\frac{\partial C}{\partial y}\right)+\frac{\partial}{\partial z}\left(k_z\frac{\partial C}{\partial z}\right) \quad (1)$$

where,

x is along wind coordinate measured in wind direction from the source (m).

y is the crosswind coordinate direction (m).

z is vertical coordinate measured from the ground (m).

C (x, y, z) is the mean concentration of diffusing substance at a point (x, y, z) (µg/m^3).

k_x, k_y and k_z are the eddy diffusivity coefficients along x, y and z respectively (m^2/s).

For steady state, taking dc/dt = 0 and neglecting diffusion in the x-axis direction because it is small with respect to the advection in the same direction, then Eqn. 1 becomes as follows :

$$u \frac{\partial C}{\partial x} = \frac{\partial}{\partial y}\left(k_y \frac{\partial C}{\partial y}\right) + \frac{\partial}{\partial z}\left(k_z \frac{\partial C}{\partial z}\right) \tag{2}$$

Integrating Eqn. (2) with respect to y, we obtain the normalized crosswind integrated concentration $C_y(x, z)$ of contaminant at a point (x, z) of the atmospheric advection-diffusion equation in the form (Essa *et al.*, 2006):

$$u(z)\frac{\partial C_y}{\partial x} = k_z \frac{\partial^2 C_y}{\partial^2 z} + \left(\frac{\partial C_y}{\partial z}\right)\left(\frac{\partial k_z}{\partial z}\right) \tag{3}$$

The above equation is solved in neutral and unstable stabilities using two different schemes for the eddy diffusivity as follows:

A. *In neutral case*

Taking $k(z) = \mathrm{k_v}\, w_* z$ (4)

where $\mathrm{k_v}$ is the von-Karman constant and w_* is the scale vertical velocity.

Substituting from equation (4) in equation (3), we obtain that:

$$\frac{\partial C_y}{\partial x} = \frac{k_v w_* z}{u(z)}\frac{\partial^2 C_y}{\partial^2 z} + \frac{k_v w_*}{u(z)}\left(\frac{\partial C_y}{\partial z}\right) \tag{5}$$

Applying the Laplace transform on Eqn. 5 with respect to x, we obtain that:

$$\tilde{c}_y(s,z) = L_p\left[c_y(x,z); x \rightarrow s\right]$$

$$L_p\left(\frac{\partial C}{\partial x}\right) = s\tilde{C}_y(s,z) - C_y(0,z) \tag{6}$$

where L_p is the operator of the Laplace transform Substituting from Eqn. 6 in Eqn. 5, we get that:

$$\frac{\partial^2 \tilde{C}_y(s,z)}{\partial z^2} + \frac{1}{z}\frac{\partial \tilde{C}_y(s,z)}{\partial z} - \frac{u\,s}{\mathrm{k_v} w_* z}\tilde{C}_y(s,z) = -\frac{u}{\mathrm{k_v} w_* z}\,C_y(0,z) \tag{7}$$

Equation (3) is subjected to the following boundary condition

(a) The flux at ground and top of the mixing layer can be given by:

$$k_z \frac{\partial C_y}{\partial z} = 0 \text{ at } z = 0, \mathrm{h} \tag{i}$$

(b) The mass continuity is written in the form:

$u(z)\, C_y(x, z) = Q\, \delta\, (z\text{-}h_s)$ at $x = 0$ (*ii*)

where δ is Dirac delta function, Q is the source strength and h_s is the stack height.

(c) The concentration of the pollutant tends to zero at large distance of the source, *i.e.*,

$C_y(x, z) = 0$ at x, $z \rightarrow \infty$ (*iii*)

Substituted from (ii) in equation (7) we obtain that:

$$\frac{\partial^2 \tilde{C}_y(s,z)}{\partial z^2} + \frac{1}{z}\frac{\partial \tilde{C}_y(s,z)}{\partial z} - \frac{u\,s}{\mathrm{k_v} w_* z}\tilde{C}_y(s,z) = -\frac{Q}{\mathrm{k_v} w_* z}\,\delta(z - h_s) \tag{8}$$

Integrated Eqn. (8) with respect to z, we obtain that:

$$\frac{\partial \tilde{C}_y(s,z)}{\partial z} - \frac{u\,s\ln(z)}{\mathrm{k_v} w_*}\tilde{C}_y(s,z) = -\frac{Q}{\mathrm{k_v} w_* h_s} \tag{9}$$

Equation (9) is non-homogeneous differential equation then, this equation has got two solutions, one is homogeneous and other is special solution. In order to solve the homogeneous equation, we putt $\frac{-Q}{k_v w_* h_s} = 0$ in equation 9, then the solution becomes:

$$\frac{\tilde{C}_y(s,z)}{Q} = c_1 e^{\left(\frac{s\,u\,In\,z}{k_v w_*}\right) z} \tag{10}$$

After taking Laplace transform in equation (10) and substituting from (*ii*), we obtain that:

$$c_1 = \frac{1}{u\,s}\delta(z - h_s) \tag{11}$$

Substituting from equation (11) in equation (10) we obtain that :

$$\frac{\tilde{C}_y(s,z)}{Q} = \frac{\delta(z-h_s)}{us} e^{\left(\frac{s\,u\,In\,z}{k_v w_*}\right) z}$$

$$\Rightarrow \frac{\tilde{C}_y(s,z)}{Q} = \frac{1}{us} e^{\left(\frac{s\,u\,In\,h_s}{k_v w_*}\right) h_s} \tag{12}$$

The special solution of equation (9), is in the form as follows:

$$\left(\frac{\partial}{\partial z} - \frac{u\,s\ln(z)}{k_v w_*}\right)\tilde{C}_y(s,z) = -\frac{Q}{k_v w_* h_s}$$

$$\Rightarrow \frac{\tilde{C}_y(s,z)}{Q} = -\frac{1}{k_v w_* h_s\left(D - \frac{u\,s\ln(z)}{k_v w_*}\right)} \tag{13}$$

where, D= $\frac{\partial}{\partial z}$

Then solution of equation (13) is in the form as follows:

$$\frac{\tilde{C}_y(s,z)}{Q} = \frac{1}{u\,s\,h_s\ln h}\left[e^{-s\left(\frac{u(h\ln h - z\ln z)}{k_v w_*}\right)}\right] \tag{14}$$

The general solution of equation (13) is given by summing two solutions equations (14) and (12) in the form:

$$\frac{\tilde{C}_y(s,z)}{Q} = \frac{1}{us} e^{\left(\frac{s\,u\ln h_s}{k_v w_*}\right) h_s} + \frac{1}{u\,s\,h_s\ln h}\left\{e^{-s\left[\frac{u(h\ln h - z\ln z)}{k_v w_*}\right]}\right\} \tag{15}$$

Taking Laplace inverse on Eqn. (15), we get that :

$$\frac{C_y(x,z)}{Q} = \frac{1}{u\left[x - \frac{u h_s\ln(h_s)}{k_v w_*}\right]} + \frac{1}{u\,h_s[\ln(h)]\left\{x - \frac{u\{h[\ln(h)] - z[\ln(z)]\}}{k_v w_*}\right\}} \tag{16}$$

Since,

$$L^{-1}(AB) = L^{-1}(A)L^{-1}(B) \quad , L^{-1}\left(\frac{1}{s}\right) = 1,$$

$$L^{-1}[\exp-(a\,s)] = \frac{1}{x+a} \text{ and } L^{-1}[\exp(a\,s)] = \frac{1}{x-a}$$

L^{-1} is the operator of the inverse Laplace transform by (Shamus, 1980). To calculate the integrated ground level concentration, we put z = 0 in Eqn. (16), we get that:

$$\frac{C_y(x,0)}{Q} = \frac{1}{u\left[x - \frac{u\,h_s\ln(h_s)}{k_v w_*}\right]} + \frac{1}{u\;h_s[\ln(h)]\left\{x - \frac{u\{h[\ln(h)]\}}{k_v w_*}\right\}} \tag{17}$$

Differentiating Eqn. (17) with respect to *x* and equating the result to zero, we get the maximum downwind distance in the form :

$$x_{\max} = \frac{u\left[\sqrt{h_s \ln(h)}\right](h-h_s)\ln(h)}{\mathrm{k_v} w_* \left[1+\sqrt{h_s \ln(h)}\right]} \qquad (18)$$

Substituting from Eqn. (18) in Eqn. (17), we get the maximum crosswind integrated ground level concentration as follows:-

$$\frac{C_{\max}(x,0)}{Q}$$

$$= \frac{1}{u\left\{\dfrac{u(h_s-h)[\ln(h)]\left[h_s\{\ln(h)\}\right]^{1/2}}{\mathrm{k_v} w_*\left[1+\{h_s[\ln(h)]\}^{1/2}\right]} - \dfrac{u\, h_s \ln(h_s)}{\mathrm{k_v}\, w_*}\right\}}$$

$$+ \frac{1}{u\ h_s[\ln(h)]\left[\dfrac{u(h_s-h)\{\ln(h)\}\{h_s[\ln(h)]\}^{1/2}}{\mathrm{k_v} w_*\left\{1+\left(h_s[\ln(h)]\right)^{1/2}\right\}} - \dfrac{u[h\{\ln(h)\}]}{\mathrm{k_v}\, w_*}\right]} \qquad (19)$$

B. *In unstable case*

In unstable case we take the value of the vertical eddy diffusivity in the form:

$$k(z) = \mathrm{k_v}\, w_* z \quad (1-z/h) \qquad (20)$$

Substituting from equation (20) in equation (3), we get that:

$$\frac{\partial C_y}{\partial x} = \frac{\mathrm{k_v} w_* z\left(1-\frac{z}{h}\right)}{u(z)}\frac{\partial^2 C_y}{\partial^2 z} + \frac{\mathrm{k_v} w_*\left(1-\frac{2z}{h}\right)}{u(z)}\left(\frac{\partial C_y}{\partial z}\right) \qquad (21)$$

Applying the Laplace transform on Eqn. (21) with respect to *x* and considering that:

$$\hat{c}_y(s,z) = L_p\left[c_y(x,z); x \to s\right]$$

$$L_p\left(\frac{\partial C}{\partial x}\right) = s\tilde{C}_y(s,z) - C_y(0,z) \qquad (22)$$

where L_p is the operator of the Laplace transform

Substituting from Eqn. (22) in Eqn. (21), we obtain that:

$$\frac{\partial^2 \tilde{C}_y(s,z)}{\partial z^2} + \frac{\left(1-\frac{2z}{h}\right)}{\left(z-\frac{z^2}{h}\right)}\frac{\partial \tilde{C}_y(s,z)}{\partial z} - \frac{us}{\mathrm{k_v} w_*\left(z-\frac{z^2}{h}\right)}\tilde{C}_y(s,z)$$

$$= -\frac{u}{\mathrm{k_v} w_*\left(z-\frac{z^2}{h}\right)}\, C_y(0,z) \qquad (23)$$

Substituting from (*ii*) in Eqn. 23 we get:

$$\frac{\partial^2 \tilde{C}_y(s,z)}{\partial z^2} + \frac{\left(1-\frac{2z}{h}\right)}{\left(z-\frac{z^2}{h}\right)}\frac{\partial \tilde{C}_y(s,z)}{\partial z}$$

$$-\frac{us}{\mathrm{k_v} w_*\left(z-\frac{z^2}{h}\right)}\tilde{C}_y(s,z) = -\frac{Q\delta(z-h_s)}{\mathrm{k_v} w_*\left(z-\frac{z^2}{h}\right)} \qquad (24)$$

After integrating equation (24) with respect to *z*, we obtain that:

$$\frac{\partial \tilde{C}_y(s,z)}{\partial z} + \frac{u\,s\ln\left|\frac{z-h}{z}\right|}{\mathrm{k_v} w_*}\tilde{C}_y(s,z)$$

$$= -\frac{Q}{\mathrm{k_v} w_* h_s\left(1-\frac{h_s}{h}\right)} \qquad (25)$$

Eqn. (25) is non-homogeneous differential equation then, above equation has got two solutions, one is homogeneous and other is special solution, in order to solve the homogeneous, we put, $\frac{-Q}{\mathrm{k_v} w_* h_s\left(1-\frac{h_s}{h}\right)} = 0$ in Eqn. (25), the solution becomes:

$$\frac{\tilde{C}_y(s,z)}{Q} = c_z e^{-\left(\frac{s\,u\,In\left|\frac{z-h}{z}\right|}{\mathrm{k_v} w_*}\right) z} \tag{26}$$

After taking Laplace transform in equation (26) and substitute from (*ii*), we obtain that:

$$c_z = \frac{1}{u\,s}\delta\left(z-h_s\right) \tag{27}$$

Substituting from equation (27) in equation (26) we get that:

$$\frac{\tilde{C}_y(s,z)}{Q} = \frac{1}{u\,s} e^{-\left(\frac{s\,u\,\ln\left|\frac{h_s-h}{h_s}\right|}{\mathrm{k_v} w_*}\right) z} \tag{28}$$

The special solution of equation (25) becomes:

$$\frac{\tilde{C}_y(s,z)}{Q} = \frac{1}{\mathrm{k_v} w_* h_s\left(\frac{h_s}{h}-1\right)} e^{-\left(\frac{s\,u\,\ln\left|\frac{z-h}{z}\right|}{\mathrm{k_v} w_*}\right) z} \tag{29}$$

Then, the general solution of eqn. (25) is as follows:

$$\frac{\tilde{C}_y(s,z)}{Q} = \frac{1}{u\,s} e^{-\left(\frac{s\,u\ln\left|\frac{h_s-h}{h_s}\right|}{\mathrm{k_v} w_*}\right) z} + \frac{1}{\mathrm{k_v} w_* h_s\left(\frac{h_s}{h}-1\right)} e^{-\left(\frac{s\,u\ln\left|\frac{z-h}{z}\right|}{\mathrm{k_v} w_*}\right) z} \tag{30}$$

Taking Laplace inverse of equation (30), we get that:

$$\frac{C_y(x,z)}{Q} = \frac{1}{u\left(x - \frac{u\,\ln\left|\frac{h_s-h}{h_s}\right|}{\mathrm{k_v}\,w_*}\right)} + \frac{1}{\mathrm{k_v}\,w_* h_s\left(\frac{h_s}{h}-1\right)\left(x + \frac{u\,\ln\left|\frac{z-h}{z}\right|}{\mathrm{k_v}\,w_*}\right)} \tag{31}$$

Since:

$$L^{-1}(AB) = L^{-1}(A)L^{-1}\ (B)\ \ , L^{-1}\left(\frac{1}{s}\right) = 1,$$

$$L^{-1}\left[\exp-\left(a\,s\right)\right] = \frac{1}{x+a}\ \text{and}\ L^{-1}\left[\exp\left(a\,s\right)\right] = \frac{1}{x-a}$$

L^{-1} is the operator of the Laplace inverse transform by (Shamus, 1980).

To get the crosswind integrated ground level concentration, we put $z = 0$ in Eqn. 31, we get that:

$$\frac{C_y(x,0)}{Q} = \frac{1}{u\left(x - \frac{u\,\ln\left|\frac{h_s-h}{h_s}\right|}{\mathrm{k_v}\,w_*}\right)} + \frac{1}{\mathrm{k_v}\,w_* h_s\left(\frac{h_s}{h}-1\right)x} \tag{32}$$

Differentiating Eqn. (32) with respect to *x* and equating it to zero, we get on the maximum downwind distance as follows:

$$x_{\max} = \frac{u\ln\left|\frac{h_s-h}{h_s}\right|}{\mathrm{k_v} w_*\left[\left\{\sqrt{\mathrm{k_v} w_* h_s\left(1-\frac{h_s}{h}\right)}\right\} - \sqrt{u}\right]} \tag{33}$$

TABLE 1

Comparison between observed, predicted and maximum normalized crosswind integrated ground level concentrations under different stabilities, mixing height, wind speed and maximum downwind distance

Run no.	Stability	h(m)	U (m/s)	distance (x) (m)	w$_*$	$C_{yglc.}$ (x, 0)*10^{-4} (s/m^2)			$x_{max.}$	x/h
						Observed	Predicated	Maximum		
1	Very unstable(A)	1980	3.34	1900	1.8	6.48	5.01	2.15	1.19	0.96
1	Very unstable (A)	1980	3.34	3700	1.8	2.31	2.62	1.30	0.71	1.87
2	Slightly unstable (C)	1920	7.79	2100	1.8	5.38	4.36	2.51	3.22	1.09
2	Slightly unstable (C)	1920	7.79	4200	1.8	2.95	2.26	1.41	1.79	2.19
3	Moderately unstable (B)	1120	3.82	1900	1.3	8.2	5.01	5.89	3.32	1.70
3	Moderately unstable (B)	1120	3.82	3700	1.3	6.22	2.61	3.67	1.97	3.30
3	Moderately unstable (B)	1120	3.82	5400	1.3	4.3	1.80	2.30	1.21	4.82
5	Slightly unstable (C)	820	4.93	2100	0.7	6.72	4.50	10.74	9.92	2.56
5	Slightly unstable (C)	820	4.93	4200	0.7	5.84	2.27	7.12	5.42	5.12
5	Slightly unstable (C)	820	4.93	6100	0.7	4.97	1.57	4.42	2.97	7.44
6	Slightly unstable (C)	1300	11.45	2000	2	3.96	4.35	2.60	4.27	1.54
6	Slightly unstable (C)	1300	11.45	4200	2	2.22	2.21	1.42	2.30	3.23
6	Slightly unstable (C)	1300	11.45	5900	2	1.83	1.60	0.85	1.37	4.54
7	Moderately unstable (B)	1850	6.52	2000	2.2	6.7	4.57	1.58	1.65	1.08
7	Moderately unstable (B)	1850	6.52	4100	2.2	3.25	2.32	0.88	0.91	2.22
7	Moderately unstable (B)	1850	6.52	5300	2.2	2.23	1.81	0.62	0.64	2.86
8	Neutral (D)	810	6.68	1900	2.2	4.16	4.89	0.02	1.89	2.35
8	Neutral (D)	810	6.68	3600	2.2	2.02	2.68	0.05	1.17	4.44
8	Neutral (D)	810	6.68	5300	2.2	1.52	1.85	0.07	0.73	6.54
9	Slightly unstable (C)	2090	8.11	2100	1.9	4.58	4.34	2.15	2.93	1.00
9	Slightly unstable (C)	2090	8.11	4200	1.9	3.11	2.26	1.21	1.63	2.01
9	Slightly unstable (C)	2090	8.11	6000	1.9	2.59	1.60	0.70	0.94	2.87

Substituting from Eqn. (33) in Eqn. (32), we get the maximum crosswind integrated ground level concentration as follows :

$$\frac{C_y(x,0)}{Q}=\frac{1}{u\left(\dfrac{u\ln\left|\dfrac{h_s-h}{h_s}\right|}{\mathrm{k_v}w_*\left[\left\{\sqrt{\mathrm{k_v}w_*h_s\left(1-\dfrac{h_s}{h}\right)}\right\}-\sqrt{u}\right]}-\dfrac{u\ln\left|\dfrac{h_s-h}{h_s}\right|}{\mathrm{k_v}\,w_*}\right)}+\frac{1}{\mathrm{k_v}\,w_*h_s\left(\dfrac{h_s}{h}-1\right)\left(\dfrac{u\ln\left|\dfrac{h_s-h}{h_s}\right|}{\mathrm{k_v}w_*\left[\left\{\sqrt{\mathrm{k_v}w_*h_s\left(1-\dfrac{h_s}{h}\right)}\right\}-\sqrt{u}\right]}\right)} \tag{34}$$

The data used was observed from the atmospheric diffusion experiments conducted at the northern part of Copenhagen, Denmark, under neutral and unstable conditions (Gryning and Lyck, 1984; Gryning *et al.*, 1987). Table 1 shows that the comparison between observed, predicted and maximum integrated crosswind ground level concentrations under different stabilities, mixing height, wind speed and maximum downwind distance.

Fig. 1. Shows comparison between the observed, predicated and maximum normalized crosswind integrated ground level concentrations under different stabilities.

Fig. 2. Shows comparison between the observed, predicated and maximum normalized crosswind integrated

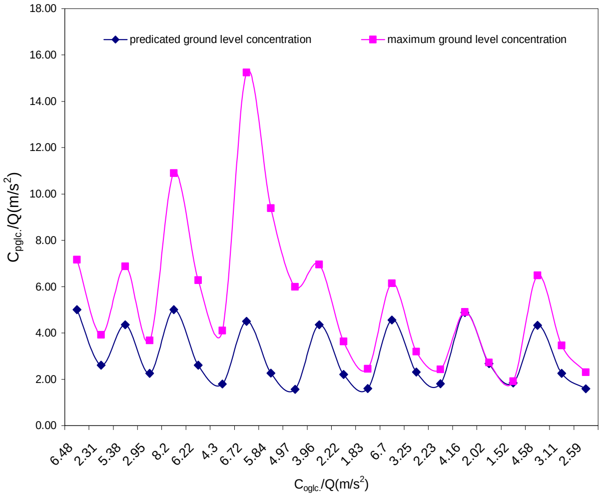

Fig. 1. Comparison between the observed with predicated and maximum normalized crosswind integrated ground level concentration

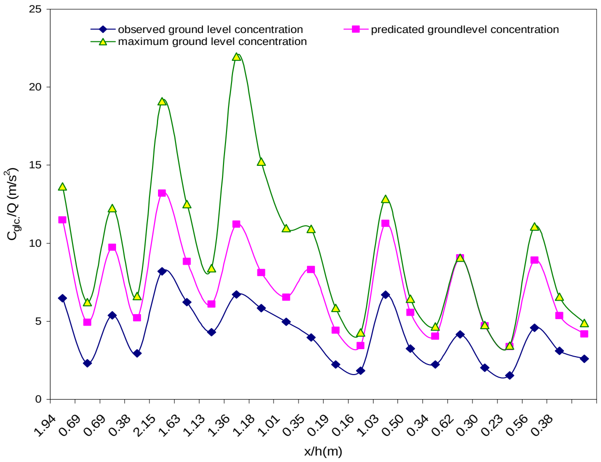

Fig. 2. Comparison between the observed, predicated and maximum normalized crosswind integrated ground level concentrations via downwind distance per height

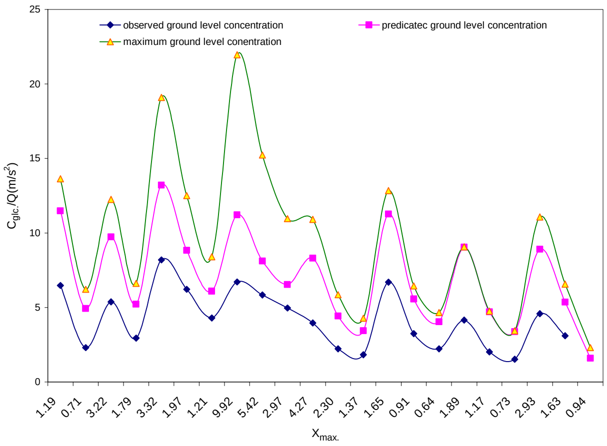

Fig. 3. Comparison between maximum downwind distance and the observed, predicated and maximum normalized crosswind integrated ground level concentrations

ground level concentrations under different stability via downwind distance over height.

Fig. 3. Shows comparison between maximum downwind distances and the observed, predicated and maximum normalized crosswind integrated ground level concentrations under different stabilities.

3. Statistical method

Now, the statistical method is presented and comparison between predicted and observed results will be offered by (Hanna, 1989). The following standard statistical performance measures that characterize the agreement between prediction ($C_p = C_{\text{pred}}/Q$) and observations ($C_o = C_{\text{obs}}/Q$):

$$\text{Fractional Bias}\left(\text{FB}\right)=\frac{\left(\overline{C_o}-\overline{C_p}\right)}{\left[0.5\left(\overline{C_o}-\overline{C_p}\right)\right]}$$

$$\text{Normalized Mean Square Error}\left(\text{NMSE}\right)=\frac{\overline{\left(C_p-C_o\right)^2}}{\left(\overline{C_p}\,\overline{C_o}\right)}$$

$$\text{Correlation Coefficient}\left(\text{COR}\right)$$

$$=\frac{1}{N_m}\sum_{i=1}^{N_m}\left(C_{pi}-\overline{C_p}\right)\times\frac{\left(C_{oi}-\overline{C_o}\right)}{\left(\sigma_p\sigma_o\right)}$$

$$\text{Factor of Two}\left(\text{FAC2}\right)=0.5\leq\frac{C_p}{C_o}\leq 2.0$$

Where σ_p and σ_o are the standard deviations of C_p and C_o respectively. Here the over bars indicate the average over all measurements. A perfect model would have the following idealized performance: NMSE = FB = 0 and COR = 1.0.

From the statistical method, we find that the two models are within a factor of two with observed data. According to NMSE and FB, the predicted normalized crosswind integrated ground level concentration is better than maximum normalized crosswind integrated ground level concentration. The correlation of predicated model equals (0.67) and maximum model equals (0.70) (Table 2).

4. Conclusions

This method solving Laplace transforms technique and eddy diffusivity depends on the vertical height in

TABLE 2

Comparison between two models according to standard statistical performance measure

Models	NMSE	FB	COR	FAC2
Predicted concentration	0.26	0.32	0.67	0.80
Maximum concentration	0.62	0.52	0.70	0.51

neutral and unstable conditions. Also the predicted and maximum normalized crosswind integrated ground level concentrations are estimated. We find that the predicted and maximum normalized crosswind integrated concentrations are within a factor of two with observed concentration data. One finds that there is agreement between maximum and predicted normalized crosswind integrated concentrations with the observed normalized crosswind integrated concentrations.

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