

Stochastic modeling of the occurrence of rainfall over some districts of Assam during 1987-1992

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सार – इस शोध पत्र का मुख्य उद्देश्य असम के कुछ जिलों में शुष्क और आर्द्र दौर के मॉड्यूलन पैटर्न के संबंध में एक वैकल्पिक पक्ष प्राप्त करना है। भारत में ग्रीष्म मानसून ऋतु (जून से सितम्बर) के दौरान शुष्क और आर्द्र दौर की अवधि का पूर्वानुमान करने के लिए मार्कोव चेन मॉडल का उपयोग किया गया है। इस सूचना के उपयोग से कृषिशालिग्रयों और कृषि वैज्ञानिकों को फसल योजना बनाने में मदद मिल सकती है। इस शोध पत्र में पाँच जिलों यथा डिब्रूगढ़, कामरूप, सोनितपुर, धेमाजी और उत्तरी लखीमपुर का अध्ययन किया गया है। मार्कोव चेन मॉडल इन सभी जिलों के लिए उपयुक्त पाया गया है और पाँचों जिलों के परिणाम को एक साथ मिला दिया गया है। इस सम्मिलित परिणाम से पता चला है कि वर्ष 1987-1992 के दौरान दिन में आर्द्रता रहने की संभावना थी जब कि उसके ठीक पहले के दिन भिन्न-भिन्न वर्षों में शुष्क रहे जिसकी मात्रा 0.44 से 0.54 के बीच रही। ठीक इसी प्रकार वे दिन आर्द्र रहे जिनके पहले के दिन आर्द्र रहे जिसकी मात्रा अलग अलग वर्षों में 0.74 से 0.86 के बीच रही। यह भी ज्ञात हुआ है कि भारतीय ग्रीष्म ऋतु मानसून ऋतु में प्रत्येक 4 से 7 आर्द्र दिवसों के बाद एक शुष्क दिवस का दौर आने की संभावना देखी गई है जबकि लगातार दो शुष्क दिनों के बाद एक आर्द्रता वाला दिन आने की संभावना होती है। समतुल्यता की अवस्था तक पहुँचने में इस प्रक्रिया में करीब 4 से 7 दिनों तक का समय लगता है।

ABSTRACT. The Primary aim in this paper is to find an alternative approach that consists of modeling the pattern of dry and wet spell over some districts of Assam. The Markov Chain Model is used to predict the length of dry and wet spells during the Indian summer monsoon season (June to September). This information may help the agronomists and agricultural scientists in crop planning. Five districts viz., Dibrugarh, Kamrup, Sonitpur, Dhemaji and North Lakhimpur are considered here for this study. Markov Chain Model is fitted for each of the district and the results of the five districts are pooled. This pooled result reveals that during the period 1987-1992, the probability for the day being wet when the immediately preceding day is dry for different years varies from 0.44 to 0.54 while the probability of the day being wet when the immediately preceding day is wet for different years varies from 0.74 to 0.86. It is also found that in the Indian summer monsoon season after about every consecutive 4 - 7 wet days a dry day is expected to occur whereas alter about consecutive 2 dry days, a wet day is expected to occur. The number of days required for the process to reach the state of equilibrium varies from 4 - 7 days.

Key words – Modeling, Markov chain, Indian summer monsoon season, Occupation probability, Dry and wet days.

1. Introduction

Agriculture occupies a vital position in the Indian economy. Being an agricultural economy, crop yields play a very important role in the national income of India and farming is mainly dependent upon the distribution of rainfall. Based on the 100 years data of India Meteorological Department the Indian Summer Monsoon Season (ISMS) of June to September is the principal rainy season for India. It is seen that crop yield over a region depends mainly upon the temporal distribution of rainfall instead of the total amount of rainfall. In other words, it

depends directly upon the occurrence of dry and wet spells during a particular season.

In the present study, the occurrence of rainfall is investigated by developing a Markov model which provides us various important information about the expected lengths of dry and wet spells during ISMS.

In this direction some studies had been carried out by Gabriel and Neumann (1962 & 1977), Gangopadhya and Sarkar (1965), Gates and Tong (1976), Victor and Sastry (1979), Chowdhury (1981), Robertson (1982), Rao and

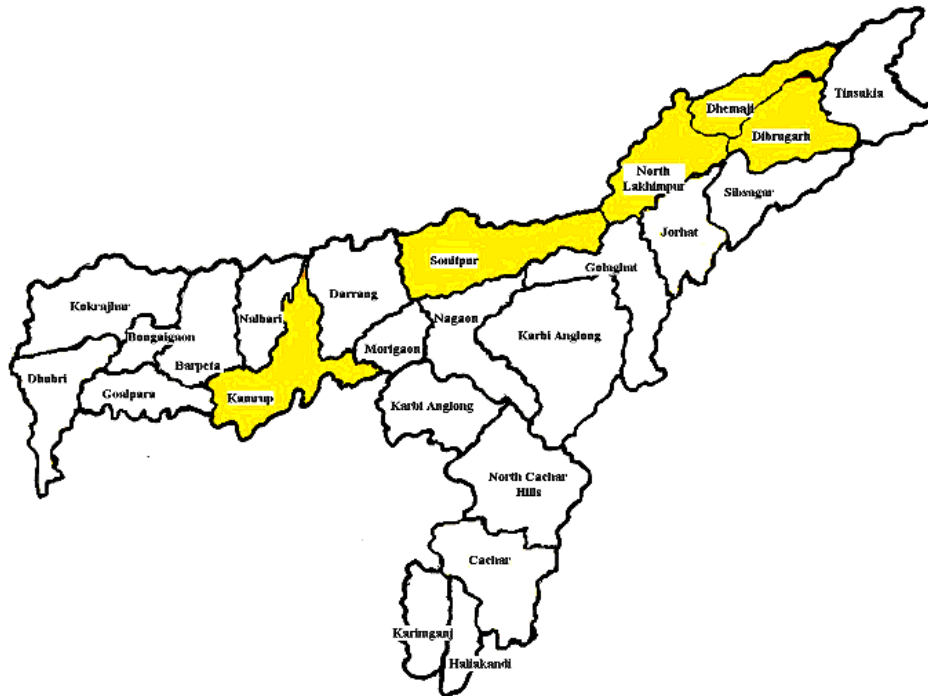


Fig. 1. Districts (shaded) of Assam which are considered for the present study

Biazi (1983), Khambete and Biswas (1984), Pandharinath (1991), Gore and Thapliyal (2000). Caskey (1963) developed the Markov chain model to find the probability of precipitation in intervals of various lengths. Chin (1977) showed that the proper Markov order of the daily precipitation process depends primarily on the season of the year and for the summer months, a majority of the stations displayed 1st order conditional dependence in the daily precipitation occurrence process. Cowpervait (1994 & 1998) developed model for daily rainfall using generalized point process and Poisson-cluster process. In this study, we investigate the occurrence of dry and wet spells for Assam using two-state Markov Chain.

2. Data and methodology

Data of daily rainfall amount during ISMS (1st June to 30th September) over Assam for the period 1987-1992 have been considered for the present study. The district averaged rainfall data of five districts, *viz.*, Dibrugarh, Kamrup, Sonitpur, Dhemaji and North-Lakhimpur (Fig. 1) are considered here for the study purpose. The data source is National Data Centre, India Meteorological Department, Pune. Method of interpolation has been carried out to filling the missing data. Markov Chain model is fitted for each of the district (pooled for six years). Again the results of the five districts are pooled to

get the temporal variation in the occurrence of rainfall over Assam during the study period 1987-1992.

The most common form of Markov Chain is a 1st order two-state Markov Chain. The two-state Markov Chain Model is appropriate as the occurrence and non-occurrence of rainfall can be represented as binary random sequence.

Following (Rao and Biazi, 1983), each day during 1st June to 30th September is classified as a wet day if the amount of rainfall on that particular day is equal to or greater than 0.1 mm and a dry day if the amount of rainfall is less than 0.1 mm.

Two conditional probabilities p_{00} and p_{01} represent the probability of a dry day following a dry day and probability of a rainy day following a dry day respectively. Similar statements can be made for p_{10} and p_{11} . Estimating p_{01} and p_{11} will suffice the description of Markov Chain. As the sequence of dry and wet day is an infinite sequence, dry and wet day can be considered as the initial day of the sequence of dry and wet day. This process after sometime gets settled down to a state of equilibrium whose state occupation probabilities are π_1 and π_2 . In such case, π_1 and π_2 become unconditional probabilities, *i.e.*, they are independent of initial

TABLE 1

The estimates of the parameters and the various properties of Markov Chain

Name of the districts	Year	p ₀₁	p ₁₁	Expected length of Dry spells	Expected length of Wet spells	Expected length of weather cycle	Expected number of Dry days	Expected number of Wet days	Equilibrium state probability π ₁	Equilibrium state probability π ₂	S.D of Dry/ Wet days	Number of days to equilibrium
Dibrugarh	1987-1992	0.53	0.80	2	5	7	203	523	0.72	0.28	6	4
Kamrup	1987-1992	0.50	0.74	2	4	6	250	476	0.66	0.34	8	5
Sonitpur	1987-1992	0.54	0.78	2	5	6	210	516	0.71	0.29	7	4
Dhemaji	1987-1992	0.44	0.82	2	5	8	214	512	0.70	0.30	5	7
North-Lakhimpur	1987-1992	0.50	0.86	2	7	9	158	568	0.78	0.22	5	5
Combined	1987-1992	0.53	0.97	2	33	35	40	686	0.95	0.05	2	8

conditions. The climatological probability (which is the unconditional probability) of the occurrence of rainfall is given by

$$\pi_1 = p_{01} / (1 + p_{01} - p_{11})$$

If ‘D’ be the no. of successive dry days followed by a wet day, then expected length of a dry runs is given by

$$E(D) = (p_{01})^{(-1)}$$

Similarly, if ‘W’ be the no. of successive wet days followed by a dry day, then expected length of a wet runs is given by

$$E(W) = (1 - p_{11})^{(-1)}$$

If ‘T’ denotes the length of weather cycle which is defined as a wet spell followed by a dry spell or a dry spell followed by a wet spell, then expected length of a weather cycle is given by

$$E(T) = E(D) + E(W)$$

If y_k (‘k’ being total number of wet and dry days) is the total number of wet days, then it can be easily shown that y_k is asymptotically normally distributed with

$$\text{Mean} = E(y_k)$$

$$\approx k * p_{01} / (1 - p_{11} + p_{01})$$

$$\text{Variance} = V(y_k)$$

$$\approx k * p_{01}(1 - p_{11}) * (1 + p_{11} - p_{01}) / (1 - p_{11} + p_{01})^{*3}$$

TABLE 2

Results of Chi-square test for pooled observations over six years for each district and for combined district

Name of the districts	Value of the computed Chi-square test statistic	Tabulated value of Chi-square test statistic	Results
Dibrugarh	52.38	3.841	Significant
Kamrup	38.84	3.841	Significant
Sonitpur	43.70	3.841	Significant
Dhemaji	102.84	3.841	Significant
North-Lakhimpur	94.74	3.841	Significant
Combined	324.30	3.841	Significant

For each district, Chi-square test has been performed to test the discrepancies between the observed and expected value for pooled observations over six years. For the analysis of pooled observation of all districts, a binary sequence has been constructed for the rainfall observations making assignment with entry ‘0’ if none of the individual district under consideration reports rainfall and with entry ‘1’ if at least one of the district reports rainfall. For the combined district, common estimates have been computed for each year and for observations pooled over six years and for these cases, chi-square test has been performed.

3. Results and discussion

The results of study of occurrence of rainfall over different districts of Assam considered here for the study

during the ISMS for the period 1987-1992 are summarized in the Tables 1 & 2.

From Table 1, the probability for a day being wet when the immediately preceding day is dry (p_{01}) is significantly less than the probability of the day being wet when the immediately preceding day is wet (p_{11}) as p_{01} varies from 0.44 to 0.54 and p_{11} varies from 0.74 to 0.86.

After, about every consecutive 4 - 7 wet days, a dry day is expected to occur whereas after about consecutive 2 dry days a wet day is expected to occur.

Out of 726 days (in 6-years, 1987-1992), the highest number (expected) of wet days is observed over North-Lakhimpur while that of dry days is observed over Kamrup.

The number of days required for the process to reach the state of equilibrium varies from 4-7 days over different districts of Assam under consideration.

Considering the results of the five districts (pooled), it is found that during ISMS, after every consecutive 33 wet days, a dry day is expected to occur whereas after every 2 consecutive dry days a wet day is expected to occur. The number of days required for the process to reach the state of equilibrium is found to be 8 days. So far as the equilibrium state occupation probabilities (π_1) is concerned it can be said that in long run 95 per cent of the time, the process spends in wet state while only 05 per cent of the time, the process spends in dry state.

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