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# Relation between pressure defect and maximum wind in the field of a Tropical Cyclone – Theoretical derivation of proportionality constant based on an idealised surface pressure model

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**सार** – उष्णकटिबंधीय चक्रवात में सतह दाब त्रुटि $\Delta P$ तथा सतह पर अधिकतम पवन गति $V_m$ उष्णकटिबंधीय चक्रवात की तीव्रता के दो मुख्य घटक होते हैं औरी ये सुविख्यात सूत्र $V_m = \mathrm{K}\sqrt{\Delta P}$ को दर्शाते हैं, जहाँ K समानुपतिक नियतांक है। अनेक शोध पत्रों में चक्रवात क्षेत्र के अंदर अधिकतम पवन गति $V_m$ और दाब त्रुटि $\Delta P$ के प्रेक्षेणों के आधार पर K के आनुभविक मान प्राप्त किए हैं। इस प्रकार से प्राप्त किए गए K का मान 10.5–16.0 की सीमा के अंतर्गत पाया गया है जहाँ $V_m$ और $\Delta P$ का मान क्रमश : नॉट्स और हेक्टापास्कल में मापा गया है। इस शोध पत्र में K का मान आकलित करने की समस्या को बिल्कुल ही भिन्न तरीके से प्राप्त करने का प्रयास किया गया है। सांख्यिकीय वितरण सिद्धांत में पियर्सन के वितरण की विचारधारा को उपयोग करते हुए एक सर्वमान्य दाब मॉडल तैयार किया गया है। यह तार्किक एवं स्वीकार करने योग्य पूर्वानुमानों पर आधारित है। जैसे कि चक्रवात केन्द्र के समीप चक्रगतिक संतुलन की वैधता, अधिकतम दाब प्रवणता की त्रिज्या, कुल संचयी सतह दाब निक्षेप का अभिसरण, केन्द्र से कुछ दूरी तक सापेक्षिक भ्रमिलता का धनात्मक बने रहना, चक्रवातीय क्षेत्र में निरपेक्ष भ्रमिलता का हमेशा धनात्मक बने रहना – इस प्रकार की भिन्नताओं की सीमाएँ सामान्य दाब मॉडल को दर्शाती हैं और पवनगति $V_m$ अथवा सतह दाब त्रुटि $\Delta P$ के वास्तविक प्रेक्षण के बिना ही K का मान प्राप्त कर लिया जाता है। घर्षण बल, पर्यावरणीय बहाव ओर उष्णकटिबंधीय चक्रवात की गति से उत्पन्न बल को सम्मेलित करने के बाद K का अंतिम मान 11.0 प्राप्त किया गया है। सैद्धांतिक तरीके से प्राक्कलित किए गए इस के मान की तुलना प्रायोगिक तरीके से प्राप्त किए मान के साथ करने पर यह पाया गया है कि न्यूनतम मान इससे थोड़ा ही कम है। ऊपरी तौर पर अन्य शोध पत्रों दिए गए $\Delta P$ के लिए $V_m$ मानों को अधिक आकलित करके प्रायोगिक K मानों को ज्ञात किया गया है और इस विषय पर इस शोध पत्र में विवेचना की गई है।

**ABSTRACT.** The surface pressure defect $\Delta P$ and the surface maximum wind speed $V_m$ of a tropical cyclone which are two important measures for the intensity of a tropical cyclone are related by the well known relation $V_m = \mathrm{K}\sqrt{\Delta P}$ where K is the proportionality constant. Based on composites of observations of $V_m$ and $\Delta P$ within the cyclone field, the empirical values of K have been derived in a large number of studies. The value of *K* thus derived has been found to vary in the range 10.5-16.0 when $V_m$ and $\Delta P$ are measured in knots and hPa respectively. In this study the problem of estimating the value of K has been approached and treated from an entirely different angle. A general idealised pressure model derived using the concept of Pearson's Distributions in Statistical Distribution Theory has been initially assumed. Based on a few logical and acceptable assumptions such as - validity of cyclostrophic balance near the centre of the cyclone, existence of radius of maximum pressure gradient, convergence of integral defining cumulative surface pressure drop, relative vorticity to remain positive up to some distance from the centre, absolute vorticity to remain always positive within the cyclone field – the ranges of variables defining the general pressure model, and hence for K, have been derived without in any way relying upon actual observations of $V_m$ or $\Delta P$. After incorporating frictional forces, environmental flow and force due to translation speed of a tropical cyclone, the final value of K has been derived as 11.0. This theoretically estimated K value compared very well with the empirically derived values but was slightly on the lower side. Apparently most of the empirical K values derived in other studies generated overestimated $V_m$ values for given $\Delta P$ and this aspect has been discussed.

**Key words** – Tropical cyclone, Pressure defect, Maximum wind, Proportionality constant, Pressure gradient, Cyclostrophic wind, Gradient wind, Cumulative surface pressure drop, Vorticity, Friction, Translation speed, Pearson's distribution.

## 1. Introduction

The tropical cyclone (TC) is an intense atmospheric warm core vortex characterised by several recognisable features such as, decreasing mean sea level pressure (MSLP) from the periphery to the centre, presence of a belt of strong surface winds close to the centre, heavy rain area in the wall cloud region and a relatively calm eye region around the centre in respect of very intense storms. The TC originates as an incipient low pressure area over the warm oceans, intensifies gradually and then moves. It generally reaches its peak intensity after 4-5 days of sea travel and weakens when it crosses the coast and enters land or when it enters into colder sea areas. Some TCs do weaken over the warm sea areas also due to other factors. There are several basins where the TCs form and move. Atlantic, where the TCs are called hurricanes, Pacific, where TCs are called typhoons and the Indian Ocean are the major basins. Anthes (1982), Asnani (1993), Elsberry *et al*. (1987), Riehl (1954), WMO (1996), Raghavan (2003) and several similar treatises provide detailed and excellent description of the various aspects of TCs.

The MSLP at its centre is an important measure of the intensity of a TC. The difference between the pressure at the outer isobar and the central pressure is the pressure defect (PD) $\Delta P$, which is taken as an index of the intensity of the TC. Another related measure defining the intensity is the maximum wind speed (MWS) $V_m$ sustained in the TC regime. The India Meteorological Department (IMD), which monitors the TC activity over the North Indian Ocean, defines various categories of low pressure systems based on MWS only. If the MWS lies in the range of 17-27 knots, the system is defined as a depression, 28-33 knots as deep depression, 34-47 knots as cyclonic storm, 48-63 knots as severe cyclonic storm etc. (IMD, 2003).

Determination and derivation of the correct relationship between PD and MWS of TCs has been a long standing but fascinating problem. Approximate relations since developed, have found extensive use in operational forecasting and post analysis of TCs. A conceptual relation of the form $V_m = \mathrm{K}\sqrt{\Delta P}$ between the MWS and PD of a TC can be derived from the application of dynamical equations governing the balance of forces in a TC regime. In several studies this proportionality factor K has been estimated based on composites of large number of independent observations of $V_m$ and $\Delta P$ collected in the TC field when the TC centred over the oceans. A straight line fit between $V_m$ and $\sqrt{\Delta P}$ finally yields the K value. Such derivations of K have been made for several individual basins such as North Indian, Pacific and the Atlantic oceans. The derivation of K in such studies can therefore be considered as by and large empirical though the conceptual relation $V_m = \mathrm{K}\sqrt{\Delta P}$ can be derived theoretically. A review of few such studies is presented in the subsequent sections.

Whether the value of the proportionality constant K could be derived based entirely on theoretical considerations, is a problem of intense scientific interest. In this paper we show that such a derivation is indeed possible on the basis of reasonable assumptions related to the profiles of basic / derived measures such as MSLP, absolute / relative vorticity, cumulated surface pressure difference (CSPD) etc. of a TC.

The objective of this paper is to build up a theory step by step eventually leading to such a derivation. The subsequent sections describe the detailed theoretical evolution of this derivation.

## 2. General form of surface pressure and wind profiles in a cyclone field and conceptual relation between $V_m$ and $\Delta P$

2.1. We assume that the isobars in the TC field at the surface level are concentric circles. A general form of pressure profile could then be given by

$$\frac{P(r)-P(0)}{P(N)-P(0)} = \psi\left(\frac{r}{R}\right) \tag{1}$$

where $P(r)$ is the MSLP at radius $r$ from the centre , $P(0)$ is the central pressure, and $P(N)$ is the MSLP at an infinite distance from the centre which in practice corresponds to the first anticyclonically curved isobar and is generally taken as the pressure of the outer isobar (Fig. 1) [Anthes (1982), Bretschneider (1982)]. In Eqn. (1) $R$ is the radius of maximum wind (RMW) and $\psi$ is an appropriate mathematical function to be defined satisfying the conditions that $\psi(0) = 0$, $\psi(\infty) = 1$ and that $\psi$ is an increasing function. The function $\psi$ is also called normalised pressure and it is obvious that $0 < \psi(r/R) < 1$. Evidently $P(N) - P(0) = \Delta P$, the PD. We now define a function $H$ by the relation

$$H(r) = P(N) - P(r) \tag{2}$$

The function $H(r)$ is the surface pressure difference or drop or anomaly (SPD) at $r$ and varies from $\Delta P$ to 0 when $r$ varies from 0 to $\infty$. It would sometimes be convenient to deal with $H(r)$ rather than $P(r)$. The equation (1) can now be written as

$$H(r) = \left[1-\psi\left(\frac{r}{R}\right)\right]\Delta P \tag{3}$$

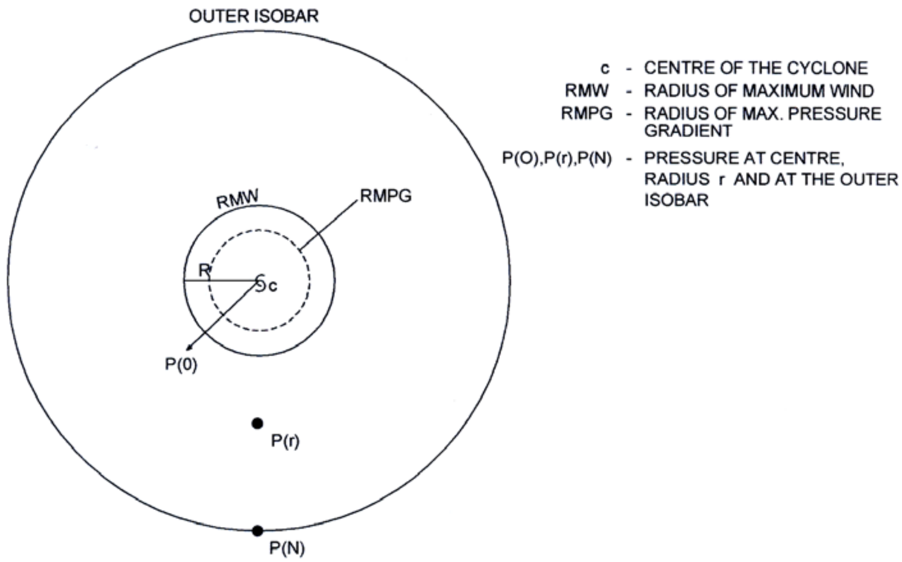



**Fig. 1.** Horizontal distribution of pressure and wind at the surface level in a tropical cyclone field [Anthes (1982), Bretschneider (1982), Basu and Ghosh (1987)]

Fig. 1 presents a simple thematic picture of the horizontal structure of a TC depicting horizontal distribution of pressure and wind at the surface level, location of RMW, radius of maximum pressure gradient (RMPG) and the outer isobar.

2.2. We assume that the TC is stationary and the flow at the surface level in the TC regime is governed by the gradient wind balance between pressure gradient force acting towards the centre and the Coriolis and centrifugal forces, acting away from the centre. The other forces influencing the flow such as frictional force and force due to movement of the TC are ignored, to begin with. In natural coordinates, the gradient wind speed $V$ is given by the equations (Hess, 1959; Holton, 1979)

$$\frac{\mathrm{d}V}{\mathrm{d}t} = -\frac{1}{\rho}\frac{\partial P}{\partial s}, \quad \frac{V^2}{R_s} + fV = -\frac{1}{\rho}\frac{\partial P}{\partial n} \tag{4}$$

where $V$ is the wind speed, $R_s$ is the radius of curvature, $f = 2\Omega\sin\varphi$ is the Coriolis parameter with $\Omega$ the angular velocity of earth, $\varphi$ is the latitude, $\rho$ is the density of surface air, $P$ is the MSLP at radius $r$, $n$ is measured normal to the tangent, $s$ is the distance measured along the motion and $t$ is the time. In the field of a TC with concentric isobars, $\partial P/\partial s = 0$ and the speed is constant following the motion. Further $R_s = r$ and as $r$ is measured from the centre to the periphery then Eqn. (4) becomes

$$\frac{V^2}{r} + fV = \left(\frac{1}{\rho}\right)\left(\frac{\mathrm{d}P}{\mathrm{d}r}\right) = -\left(\frac{1}{\rho}\right)\left(\frac{\mathrm{d}H}{\mathrm{d}r}\right) \tag{5}$$

In the neighbourhood of the TC centre the centrifugal force is considerably larger than the Coriolis force. Thus we have, $V^2/r >> fV$, known as the cyclostrophic assumption and so we obtain the cyclostrophic wind equation

$$\frac{V^2}{r} = \left(\frac{1}{\rho}\right)\left(\frac{\mathrm{d}P}{\mathrm{d}r}\right) \tag{6}$$

Now, $V$ and so $V^2$ reaches maximum, *i.e.*, when

$$P'(r) + rP''(r) = 0 \tag{7}$$

which is obtained by differentiating $V^2$ in Eqn. (6). Differentiating Eqn. (1) twice, we get

$$P'(r) = \frac{\Delta p}{R}\psi'\left(\frac{r}{R}\right), \quad P''(r) = \frac{\Delta p}{R^2}\psi''\left(\frac{r}{R}\right)$$

and so Eqn. (7) becomes

$$\psi'\left(\frac{r}{R}\right) + \frac{r}{R}\psi''\left(\frac{r}{R}\right) = 0 \tag{8}$$

Again from Eqns. (1) and (6) we obtain

$$V^2 = \left(\frac{1}{\rho}\right)\left(\frac{r}{R}\right)\psi'\left(\frac{r}{R}\right)\Delta P$$

**TABLE 1**

**Empirically derived values of K**

| S. No | Name / Reference | K | Remarks |
|---|---|---|---|
| 1. | Takahashi (1939 & 1952) | 13.4, 11.5 | |
| 2. | McKnown (1952) | [20 – (φ/5)] | φ is the latitude angle |
| 3. | Fletcher (1955) | 10.7, 16 | Based on 63 Atlantic hurricanes |
| 4. | Kraft (1961) | 14 | |
| 5. | Meyers (1957) | 11 | |
| 6. | Natarajan and Ramamurthy (1975) | 13.6 | Based on 41 hurricanes of Atlantic |
| 7. | Mishra and Gupta (1976) | 14.2 | Based on 29 cyclones of North Indian ocean |
| 8. | Atkinson & Holliday (1977) | $6.7\ \Delta P^{0.6444}$ | |

$V_m = \mathrm{K}\sqrt{\Delta P}$ , $V_m$ & $\Delta P$, maximum wind speed and pressure defect of a cyclone measured in knots and hPa respectively.

As $V_m$ is realised at $r = R$ we obtain the conceptual relation

$$V_m = \mathrm{K}\sqrt{\Delta P}, \quad \mathrm{K} = \sqrt{\frac{\psi'(1)}{\rho}} \tag{9}$$

between $V_m$ and $\Delta P$. This relation is popularly known as Fletcher's formula. It is clear from Eqn. (9) that $V_m$ is independent of $R$ or $f$. However, $R$ has to be smaller such that the cyclostrophic assumption remains valid. Perhaps a value of $R$ up to 60 km could be acceptable. In the relation Eqn. (9), generally $V_m$ is measured in knots and $\Delta P$ in hPa.

## 3. Brief review of past studies on estimation of K

In Sec.1, we mentioned that in a few studies the value of K has been estimated based on composites of observations of $V_m$ and $\Delta P$. Some of these are presented below.

### 3.1. *On empirical estimation of K*

Takahashi (1939) was perhaps the first to derive an estimated value of K. For typhoons of Pacific, he obtained a K value of 13.4, for the units of $V_m$ and $\Delta P$ as given in Sec.2.2. Subsequently, Takahashi (1952) rederived the value of K as 11.5. McKnown (1952), based on 230 observations taken from the typhoons over the Pacific ocean derived the equation

$$V_m = \left(20 - \frac{\phi}{5}\right)\sqrt{\Delta P} \tag{10}$$

Thus in Eqn. (10) K is not taken as a constant for the whole basin, but it decreases with latitude (φ).

Fletcher (1955) derived a value of 16.0 for K based on data of one Atlantic Hurricane of 1949 but obtained a lower value of 10.7 based on wind data of 63 hurricanes in Atlantic. Utilising wind and pressure data of hurricanes of Atlantic, Natarajan and Ramamurthy (1975) obtained a K value of 13.6. Atkinson and Holliday (1977) derived the relation

$$V_m = 6.7\Delta P^{0.644} \tag{11}$$

based on tropical cyclone data of Western North Pacific, collected over a long period of 28 years. It may be noted that the relation Eqn. (11) differs from the conceptual relation of Eqn. (9) in that here the exponent of $\Delta P$ itself has been estimated from the data. Meyers (1957) and Kraft (1961) have obtained K values of 14 and 11 respectively for the hurricanes of Atlantic.

For North Indian Ocean, Mishra and Gupta (1976) derived the relation

$$V_m = 14.2\sqrt{\Delta P} \tag{12}$$

based on wind and pressure data of 29 TCs. A linear correlation coefficient (CC) of 0.8 between $V_m$ and $\sqrt{\Delta P}$ was obtained by them based on 35 pairs of observations. The empirical relation Eqn. (12) is widely used operationally in IMD (IMD, 2003).

Table 1 lists the different values / ranges of K derived empirically and also the corresponding references.

**TABLE 2**

**Values of K based on a few Pressure models referred in the literature**

| S. No. | Name / Reference | $\Psi(x)$, $x = (r/R)$ | K |
|---|---|---|---|
| 1. | Takahashi (1939) | $\frac{1}{1+x}$ | 13.4 |
| 2. | Graham and Nunn (1959) | $\exp\left(-\frac{1}{x}\right)$ | 11.3 – 11.7 |
| 3. | Fujita model, Bretschneider (1982) | $1-\frac{1}{\sqrt{1+2x^2}}$ | 11.6 - 12.0 |
| 4. | Bret-X model, Bretschneider (1982) | $\frac{1}{1+x^2}$ | 13.2 – 13.6 |
| 5. | Holland (1980) | $\exp\left(-\frac{A}{r^B}\right)$ | 11.1 – 17.5 For B varying between 1 and 2.5 |
| 6. | Basu & Ghosh (1987) | $1-\exp(-x^n)$ | 11.0 – 15.6 |
| 7. | Asnani (1993) | $\frac{1}{(1+x)^2}$ | |

$V_m = K\sqrt{\Delta P}$ , $V_m$ & $\Delta P$, as in Table 1

3.2. *Analytical models of radial profiles of MSLP in the regime of a tropical cyclone*

In the literature a few analytical models of MSLP have been used to derive the pressure profiles within a cyclone regime. Such models defined through suitable mathematical forms of $\psi(x)$ (Sec.2) are widely used in forecast schemes for TCs based on numerical weather prediction techniques and also in the modelling of storm surge. Some of them found in the literature are listed and discussed below:

Takahashi (1939) used the model defined by

$$\psi(x) = \frac{1}{1+x} .$$

Graham and Nunn (1959) made use of the model

$$\psi(x) = 1 - \exp\left(-\frac{1}{x}\right), \; x = \frac{r}{R} \tag{13}$$

which yielded a K value of 11.3-11.7. Bretschneider (1982) referred to the general model

$$\psi(x) = \frac{1}{(1+ax^2)^b} \tag{14}$$

Two specific cases of the model Eqn. (14) have been defined and used. When $a$=2, $b$=½, Eqn. (14) becomes

$$\psi(x) = \frac{1}{(1+2x^2)^{1/2}} \tag{15}$$

which has been called as the Fujita model. When $a$=$b$=1, Eqn. (14) is called the Bret Model–X and has the form

$$\psi(x) = \frac{1}{1+x^2} \tag{16}$$

The Fujita and Bret-X models yielded K values of 11.6-12.0 and 13.2-13.6 respectively. Asnani (1993) has referred to the model

$$\psi(x) = \frac{1}{(1+x)^2}$$

Holland (1980), based on the assumption that surface pressure profiles of hurricanes resemble a family of rectangular hyperbolas, derived a form

$$\psi(x) = 1 - \exp\left(-\frac{A}{r^B}\right) \quad (17)$$

where $R = A^{1/B}$ and K as defined in Eqn. (9) is given by

$$K = \sqrt{\frac{B}{\rho e}}\,. \quad (18)$$

where ρ is the density of the surface air and e is the exponential constant. By incorporating the concept of conservation of angular momentum, Holland indicated that $B$ should lie between 1 and 2.5 which provided a range of 11.1-17.5 for K as defined in Eqn. (18) above. Evidently (17) is a generalisation of Eqn. (13).

Basu and Ghosh (1987) used a derivative of the exponential model

$$\psi(x) = \exp\left(-x^n\right) \quad (19)$$

Analysing the pressure distributions of 44 TCs that occurred over the North Indian Ocean, they derived that $n$ in Eqn. (19) should lie between 1 and 2. Table 2 lists the above models along with the values of K. Brown *et al.* (2006), Harry (2006) and Knaff & Zehr (2007) have provided a detailed review and discussed several aspects of the tropical cyclone wind - pressure relationship.

How the mathematical form of the function $\psi(x)$ has been arrived at in each case is an interesting question. In models given in Eqns. (14), (17) and (19), the values of the variables can be varied and the values that yield most realistic profiles of MSLP can be chosen. In some cases parameter values that yield K values which are closer to its empirical value appear to have been taken. Barring Holland (1980), other studies did not clearly provide justification for the choice of the function $\psi(x)$ or the selection of values for the variables of $\psi(x)$. Whether it is possible to derive the value of K without relying upon actual observations of $V_m$ and $\Delta P$ and without resorting to an overtly empirical approach is an interesting and challenging problem, which we are attempting in this study.

With this brief background we now proceed to derive the value of K theoretically, based on assumptions that are well known or can be accepted as logical.

## 4. A generalised idealised surface pressure model for a tropical cyclone

### 4.1. *General form for P(r)*

We express the functions $P(r)$ and $H(r)$ as defined in Eqns. (1) and (2) by the following form

$$H(r) = P(N) - P(r) = \frac{\Delta P}{\left(1 + ax^n\right)^b}\,, \quad x = \frac{r}{R} \quad (20)$$

where $a$, $b$ and $n$ are positive variable constants whose values are to be determined and $R$ is the RMW. The principle behind the choice of this specific function and its generality would be discussed in detail in a subsequent section.

### 4.2. *Expressions for P′(r) and P″(r)*

Differentiating Eqn. (20) twice we obtain

$$H'(r) = -P'(r) = -\left(\frac{nab\,\Delta P}{R}\right)\frac{x^{n-1}}{\left(1 + ax^n\right)^{b+1}} \quad (21)$$

$$H''(r) = -P''(r) = \frac{H'(r)}{R}\left[\frac{(n-1) - (a + nab)x^n}{x(1 + ax^n)}\right] \quad (22)$$

If the maximum wind is to be realised at $r = R$, *i.e.*, $x = 1$, we have from Eqn. (7)

$$H'(R) + R\,H''(R) = 0 \quad (23)$$

Substituting $x = 1$ in Eqn. (22) and effecting Eqn. (23), we obtain $n\,(1\text{-}ab) = 0$ and as $n$ cannot be 0, we further obtain

$$ab = 1 \quad (24)$$

Thus Eqn. (21) and (22) simplify to

$$H'(r) = -P'(r) = -\frac{\left(\frac{n}{R}\right)x^{n-1}}{(1 + ax^n)^{b+1}}\Delta P \quad (25)$$

and

$$H''(r) = -P''(r) = -\left(\frac{n}{R^2}\right)x^{n-2}\left[\frac{(n-1) - (a + n)\,x^n}{(1 + ax^n)^{b+2}}\right]\Delta P \quad (26)$$

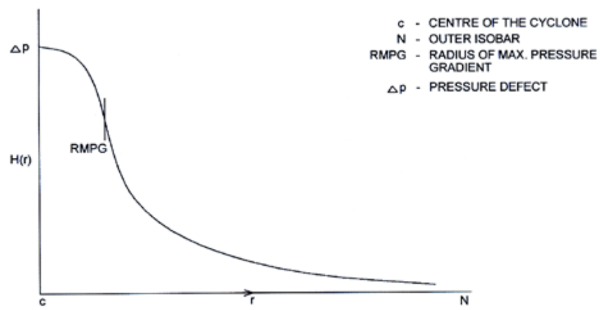



**Fig. 2.** Graph of H(r)

4.3. *Expressions for gradient and cyclostrophic winds and maximum wind derived from the pressure profile*

Let $V_g(r)$ and $V_c(r)$ denote respectively the gradient and cyclostrophic winds at the surface at the radius $r$. From Eqns. (5) and (25) we obtain the expression for $V_g(r)$ as

$$V_g(r) = -\frac{fr}{2} + \frac{1}{2}\sqrt{\left(f^2r^2 - \left(\frac{4r}{\rho}\right)H'(r)\right)}$$

$$V_g(r) = \left(\frac{Rx}{2}\right)\left(-f + \sqrt{f^2 + \left(\frac{4n\Delta P}{\rho R^2}\frac{x^{n-2}}{(1+ax^n)^{b+1}}\right)}\right) \tag{27}$$

The cyclostrophic wind speed $V_c(r)$ as derived from Eqns. (6) & (25) is given by

$$V_c(r) = \sqrt{\frac{n\Delta P}{\rho}}\left(\frac{x^{n/2}}{(1+ax^n)^{(b+1)/2}}\right), \quad x = \frac{r}{R} \tag{28}$$

The above expression can also be obtained by substituting $f = 0$ in Eqn. (27).

From Eqns. (3), (9) and (23) or by substituting $x = 1$ in Eqn. (28) we obtain an expression for $V_m$ as $V_m = \mathrm{K}\sqrt{\Delta P}$ , where K is given by

$$\mathrm{K} = \frac{\sqrt{n}}{\sqrt{\rho}(1+a)^{(b+1)/2}}$$

The value of ρ has been assumed to be $1.13 \times 10^{-3}$ gm/cm$^3$ corresponding to a MSLP value of 990 hPa and air temperature of 30° C, which could be considered to represent the ranges of MSLP and air temperature over the tropical oceans in a TC field. If $V_m$ is measured in knots and $\Delta P$ in hPa we get

$$\mathrm{K} = 18.8\sqrt{n}\left(\frac{b}{b+1}\right)^{(b+1)/2} \tag{29}$$

Thus we have derived an expression for K in terms of two independent variables $n$ and $b$.

4.4. The idealised pressure model defined in Eqn. (20) has to first satisfy the characteristics of the surface pressure profile of a TC. It also should be able to generate profiles of derived parameters or measures such as pressure gradient, wind speed, relative and absolute vorticities, momentum, divergence etc. which are consistent with the profiles derived and assimilated from actual long term data on TCs over different basins. Values/ranges of $b$ and $n$ which satisfy such requisites alone would qualify for inclusion in Eqn. (20). If such values/ranges of $b$ and $n$ are identified which generate, through (20), acceptable profiles of TC measures, they could be invoked in (29) to derive values/ranges for K as well.

From Eqn. (20) we have $H(0)=\Delta P$, $H(\infty)=0$ with the function $H(r)$ strictly decreasing from $r$=0 to ∞, thus satisfying the basic requisites of the surface pressure distribution within a TC field. Based on some broad based concepts which are logically acceptable as well as empirically known to exist we now derive constraints on $b$ and $n$ eventually leading to constraints on K.

4.5. *Constraint on n based on the concept of non zero radius of maximum pressure gradient*

In a TC field the pressure gradient is normally 0 at the centre, increases away from the centre, reaches the maximum within the wall cloud region and then decreases (Anthes, 1982; Basu and Ghosh, 1987) (Fig. 1). It is evident from the expressions of $P'(r)$ and $P''(r)$ given in Eqns. (25) and (26) that $P'(r)$ reaches maximum at $x$, given by

$$x^n = \frac{n-1}{n+a}, \quad x = \frac{r}{R}$$

*i.e.*,

$$\mathrm{RMPG} = \left(\frac{n-1}{n+a}\right)^{1/n} R \tag{30}$$

**TABLE 3**

**Variation of K, $V_m$, $\zeta$ and $z_g$ given φ, ∆*P*, *R*, *n* and *b*, in the field of a Tropical Cyclone**

| φ | ∆*P* | *R* | *n* | *b* | $R_{pg}$ | K | $V_m$ | $z_g$ | $\zeta_L$ | $\zeta_{aL}$ | $\zeta_1$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 12.5 | 50 | 30 | 1.1 | 0.9 | 0.1 | 9.7 | 68.7 | 14.5 | -0.4 | 2.8 | 573.3 |
| 12.5 | 50 | 30 | 1.1 | 1.9 | 0.1 | 10.7 | 75.6 | 6.3 | -1.4 | 1.7 | 609.2 |
| 12.5 | 50 | 30 | 1.1 | 3.1 | 0.1 | 11.1 | 78.6 | 4.3 | -3.1 | 0.1 | 624.7 |
| 12.5 | 50 | 30 | 1.5 | 0.7 | 0.3 | 10.7 | 75.9 | 12.5 | -0.4 | 2.8 | 536.5 |
| 12.5 | 50 | 30 | 1.5 | 1.1 | 0.4 | 11.6 | 82.2 | 6.8 | -1.1 | 2.1 | 567.4 |
| 12.5 | 50 | 30 | 1.5 | 1.7 | 0.4 | 12.3 | 87 | 3.7 | -2.9 | 0.2 | 590.2 |
| 12.5 | 50 | 30 | 2 | 0.5 | 0.5 | 11.7 | 82.5 | 12.5 | -0.4 | 2.7 | 510.8 |
| 12.5 | 50 | 30 | 2 | 0.7 | 0.5 | 12.5 | 88.4 | 7.3 | -0.9 | 2.3 | 537.8 |
| 12.5 | 50 | 30 | 2 | 1.1 | 0.6 | 13.5 | 95.3 | 3.1 | -2.9 | 0.3 | 568.2 |
| 12.5 | 50 | 60 | 1.1 | 0.9 | 0.1 | 9.7 | 68.7 | 9.3 | -0.3 | 2.8 | 284.5 |
| 12.5 | 50 | 60 | 1.1 | 2.5 | 0.1 | 10.9 | 77.4 | 4.1 | -1.7 | 1.5 | 307.2 |
| 12.5 | 50 | 60 | 1.1 | 4.3 | 0.1 | 11.3 | 80.1 | 3.3 | -3.1 | 0.1 | 314.1 |
| 12.5 | 50 | 60 | 1.5 | 0.7 | 0.3 | 10.7 | 75.9 | 8.3 | -0.4 | 2.8 | 266.2 |
| 12.5 | 50 | 60 | 1.5 | 1.1 | 0.4 | 11.6 | 82.2 | 4.9 | -1 | 2.2 | 281.6 |
| 12.5 | 50 | 60 | 1.5 | 1.7 | 0.4 | 12.3 | 87 | 3.3 | -2.3 | 0.9 | 293 |
| 12.5 | 50 | 60 | 2 | 0.5 | 0.5 | 11.7 | 82.5 | 7.8 | -0.4 | 2.7 | 253.3 |
| 12.5 | 50 | 60 | 2 | 0.7 | 0.5 | 12.5 | 88.4 | 5.3 | -0.8 | 2.3 | 266.8 |
| 12.5 | 50 | 60 | 2 | 0.9 | 0.6 | 13.1 | 92.4 | 3.5 | -1.5 | 1.7 | 275.7 |
| 12.5 | 80 | 30 | 1.1 | 0.9 | 0.1 | 9.7 | 86.9 | 16.5 | -0.3 | 2.9 | 726.3 |
| 12.5 | 80 | 30 | 1.1 | 1.9 | 0.1 | 10.7 | 95.6 | 6.8 | -1.5 | 1.6 | 771.7 |
| 12.5 | 80 | 30 | 1.1 | 2.9 | 0.1 | 11.1 | 99 | 4.7 | -3.1 | 0.1 | 789.1 |
| 12.5 | 80 | 30 | 1.5 | 0.7 | 0.3 | 10.7 | 96 | 14.5 | -0.4 | 2.8 | 679.7 |
| 12.5 | 80 | 30 | 1.5 | 1.1 | 0.4 | 11.6 | 104 | 7.3 | -1.1 | 2 | 718.9 |
| 12.5 | 80 | 30 | 1.5 | 1.5 | 0.4 | 12.1 | 108.5 | 4.5 | -2.3 | 0.8 | 740.3 |
| 12.5 | 80 | 30 | 2 | 0.5 | 0.5 | 11.7 | 104.3 | 14.5 | -0.4 | 2.8 | 647.3 |
| 12.5 | 80 | 30 | 2 | 0.7 | 0.5 | 12.5 | 111.9 | 8.3 | -0.9 | 2.3 | 681.4 |
| 12.5 | 80 | 30 | 2 | 1.1 | 0.6 | 13.5 | 120.6 | 3.3 | -3 | 0.1 | 719.8 |
| 12.5 | 80 | 60 | 1.1 | 0.9 | 0.1 | 9.7 | 86.9 | 10.5 | -0.3 | 2.8 | 361 |
| 12.5 | 80 | 60 | 1.1 | 2.3 | 0.1 | 10.9 | 97.2 | 4.7 | -1.7 | 1.5 | 388 |
| 12.5 | 80 | 60 | 1.1 | 3.7 | 0.1 | 11.2 | 100.5 | 3.5 | -3 | 0.1 | 396.3 |
| 12.5 | 80 | 60 | 1.5 | 0.7 | 0.3 | 10.7 | 96 | 9.8 | -0.4 | 2.8 | 337.8 |
| 12.5 | 80 | 60 | 1.5 | 1.3 | 0.4 | 11.9 | 106.5 | 4.5 | -1.5 | 1.7 | 363.4 |
| 12.5 | 80 | 60 | 1.5 | 1.9 | 0.4 | 12.4 | 111.4 | 3.1 | -3.1 | 0 | 374.8 |
| 12.5 | 80 | 60 | 2 | 0.5 | 0.5 | 11.7 | 104.3 | 9.3 | -0.4 | 2.7 | 321.5 |
| 12.5 | 80 | 60 | 2 | 0.7 | 0.5 | 12.5 | 111.9 | 5.8 | -0.9 | 2.3 | 338.6 |
| 12.5 | 80 | 60 | 2 | 0.9 | 0.6 | 13.1 | 116.9 | 3.9 | -1.5 | 1.6 | 349.8 |

**TABLE 3 (*Contd.*)**

| φ | $\Delta P$ | $R$ | $n$ | $b$ | $R_{pg}$ | K | $V_m$ | $z_g$ | $\zeta_L$ | $\zeta_{aL}$ | $\zeta_1$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 25 | 50 | 30 | 1.1 | 0.9 | 0.1 | 9.7 | 68.7 | 9.3 | -0.7 | 5.5 | 569.3 |
| 25 | 50 | 30 | 1.1 | 2.5 | 0.1 | 10.9 | 77.4 | 4.1 | -3.4 | 2.8 | 614.6 |
| 25 | 50 | 30 | 1.1 | 4.3 | 0.1 | 11.3 | 80.1 | 3.3 | -6.1 | 0.1 | 628.3 |
| 25 | 50 | 30 | 1.5 | 0.7 | 0.3 | 10.7 | 75.9 | 8.3 | -0.8 | 5.4 | 532.5 |
| 25 | 50 | 30 | 1.5 | 1.1 | 0.4 | 11.6 | 82.2 | 4.9 | -1.9 | 4.2 | 563.4 |
| 25 | 50 | 30 | 1.5 | 1.7 | 0.4 | 12.3 | 87 | 3.3 | -4.5 | 1.6 | 586.2 |
| 25 | 50 | 30 | 2 | 0.5 | 0.5 | 11.7 | 82.5 | 7.8 | -0.8 | 5.4 | 506.8 |
| 25 | 50 | 30 | 2 | 0.7 | 0.5 | 12.5 | 88.4 | 5.3 | -1.7 | 4.5 | 533.8 |
| 25 | 50 | 30 | 2 | 0.9 | 0.6 | 13.1 | 92.4 | 3.5 | -2.9 | 3.2 | 551.5 |
| 25 | 50 | 60 | 1.1 | 0.9 | 0.1 | 9.7 | 68.7 | 6.3 | -0.6 | 5.6 | 280.6 |
| 25 | 50 | 60 | 1.1 | 2.1 | 0.1 | 10.8 | 76.3 | 3.7 | -2 | 4.2 | 300.4 |
| 25 | 50 | 60 | 1.1 | 3.5 | 0.1 | 11.2 | 79.2 | 3.1 | -3.4 | 2.7 | 307.9 |
| 25 | 50 | 60 | 1.5 | 0.7 | 0.3 | 10.7 | 75.9 | 5.3 | -0.7 | 5.5 | 262.2 |
| 25 | 50 | 60 | 1.5 | 0.9 | 0.3 | 11.3 | 79.6 | 4.5 | -1.2 | 5 | 271.3 |
| 25 | 50 | 60 | 1.5 | 1.3 | 0.4 | 11.9 | 84.2 | 3.3 | -2.2 | 3.9 | 282.4 |
| 25 | 50 | 60 | 2 | 0.5 | 0.5 | 11.7 | 82.5 | 5.3 | -0.8 | 5.4 | 249.4 |
| 25 | 50 | 60 | 2 | 0.5 | 0.5 | 11.7 | 82.5 | 5.3 | -0.8 | 5.4 | 249.4 |
| 25 | 50 | 60 | 2 | 0.7 | 0.5 | 12.5 | 88.4 | 3.7 | -1.6 | 4.6 | 262.8 |
| 25 | 80 | 30 | 1.1 | 0.9 | 0.1 | 9.7 | 86.9 | 10.5 | -0.7 | 5.5 | 722.3 |
| 25 | 80 | 30 | 1.1 | 2.3 | 0.1 | 10.9 | 97.2 | 4.7 | -3.3 | 2.9 | 776.2 |
| 25 | 80 | 30 | 1.1 | 3.7 | 0.1 | 11.2 | 100.5 | 3.5 | -6 | 0.2 | 792.8 |
| 25 | 80 | 30 | 1.5 | 0.7 | 0.3 | 10.7 | 96 | 9.8 | -0.8 | 5.4 | 675.7 |
| 25 | 80 | 30 | 1.5 | 1.3 | 0.4 | 11.9 | 106.5 | 4.5 | -2.9 | 3.3 | 726.9 |
| 25 | 80 | 30 | 1.5 | 1.9 | 0.4 | 12.4 | 111.4 | 3.1 | -6.1 | 0 | 749.8 |
| 25 | 80 | 30 | 2 | 0.5 | 0.5 | 11.7 | 104.3 | 9.3 | -0.8 | 5.4 | 643.2 |
| 25 | 80 | 30 | 2 | 0.7 | 0.5 | 12.5 | 111.9 | 5.8 | -1.7 | 4.5 | 677.4 |
| 25 | 80 | 30 | 2 | 0.9 | 0.6 | 13.1 | 116.9 | 3.9 | -3 | 3.1 | 699.8 |
| 25 | 80 | 60 | 1.1 | 0.9 | 0.1 | 9.7 | 86.9 | 7.3 | -0.6 | 5.6 | 357.1 |
| 25 | 80 | 60 | 1.1 | 2.5 | 0.1 | 10.9 | 97.9 | 3.7 | -2.8 | 3.4 | 385.7 |
| 25 | 80 | 60 | 1.1 | 4.3 | 0.1 | 11.3 | 101.3 | 3.1 | -4.7 | 1.5 | 394.4 |
| 25 | 80 | 60 | 1.5 | 0.7 | 0.3 | 10.7 | 96 | 6.3 | -0.7 | 5.4 | 333.8 |
| 25 | 80 | 60 | 1.5 | 1.1 | 0.4 | 11.6 | 104 | 4.1 | -1.8 | 4.4 | 353.4 |
| 25 | 80 | 60 | 1.5 | 1.5 | 0.4 | 12.1 | 108.5 | 3.1 | -3.1 | 3.1 | 364.1 |
| 25 | 80 | 60 | 2 | 0.5 | 0.5 | 11.7 | 104.3 | 5.8 | -0.8 | 5.4 | 317.6 |
| 25 | 80 | 60 | 2 | 0.7 | 0.5 | 12.5 | 111.9 | 4.1 | -1.6 | 4.6 | 334.6 |
| 25 | 80 | 60 | 2 | 0.9 | 0.6 | 13.1 | 116.9 | 3.1 | -2.7 | 3.4 | 345.8 |

φ- latitude in deg N
$\Delta P$- Pressure defect of the cyclone in hPa ; $R$ - Radius of maximum wind in km
$n$ & $b$ - Dimensionless numbers ; $R_{pg}$ - Radius of maximum pressure gradient

K: Proportionality constant in $V_m = K\sqrt{\Delta P}$ ; $V_m$ - Maximum wind speed in knots
$z_g$ – *RZRV/RMW*, *RZRV* is the radius of zero relative vorticity
$\zeta_1$ - mean relative vorticity within the region $0 \leq r \leq R$
$\zeta_L$ & $\zeta_{aL}$ - Lowest values of relative / absolute vorticity realised

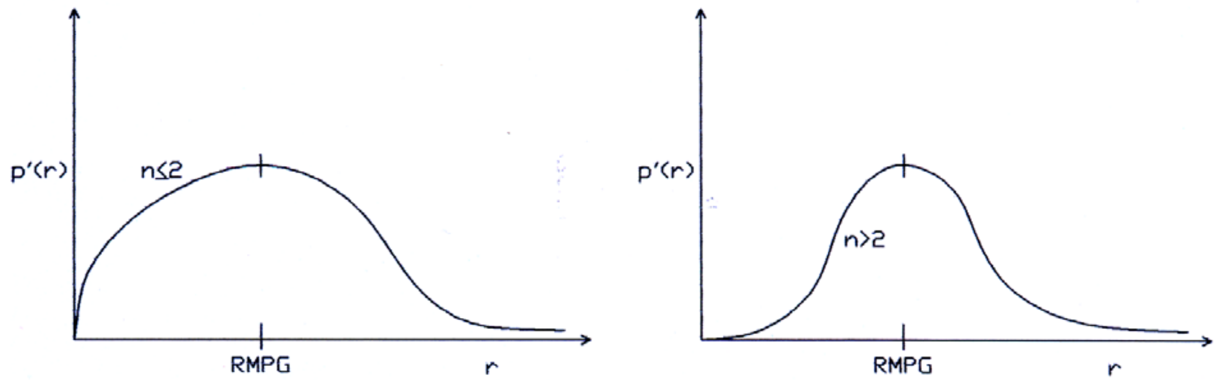



**Fig. 3.** Shape of the graph of $P'(r)$ at r = 0, the centre of the cyclone when $n \leq 2$ and $n > 2$ (RMPG is the radius of maximum pressure gradient)

For the maximum to be reached between the centre and the RMW we must have

$$n > 1 \tag{31}$$

which provides a lower limit for *n*. It is also obvious from Eqn. (31) that 0<*R*MPG<RMW and that when $n > 1$, the pressure gradient is 0 at the centre of the TC.

4.6. *Constraint owing to cyclostrophic assumption and profile of pressure gradient at the centre of the TC*

4.6.1. In the gradient wind equation for wind speed valid in the domain of a TC as given in Eqn. (5), $V^2/r$ which is the first term in the LHS corresponds to the centrifugal acceleration and $fV$ the second term, to the Coriolis acceleration. It has been stated in large number of standard references that the cyclostrophic balance is valid in the central parts of the TC (Hess, 1959; Willoughby, 1996). We have already used this assumption in Sec.2.1 in the derivation of expression for MWS.

The ratio of $V^2/r$ to $fv$ must be very large for the cyclostrophic balance to be realised. Near the RMW, *i.e.*, at $r = R$ or $x = 1$, where the MWS is realised, the higher wind speed ensures that the ratio $V/fr$ remains high. Outside the RMW, as *V* decreases and *r* increases, $V/fr$ decreases and so the cyclostrophic balance would cease to be valid beyond a specific value of *r*.

In the neighbourhood of the TC centre at $r = 0$, both the wind speed *V* and the radius *r* decrease and so $V/fr$ could remain high. Using the expression of *V* based on gradient wind as derived in Eqn. (27) we obtain

$$\frac{V}{fr} = \frac{1}{2}\left(-1 + \sqrt{1 + \frac{4n\Delta P}{\rho R^2 f^2}\frac{x^{n-2}}{(1+ax^n)^{b+1}}}\right) \tag{32}$$

In the neighbourhood of $x = 0$, $x^{n-2}$ and so $V/fr$ would assume very small values if $n>2$ and very large values if $n<2$. If $n = 2$, Eqn. (32) becomes

$$\frac{V}{fr} = \frac{1}{2}\left(-1 + \sqrt{1 + \frac{8\Delta P}{\rho R^2 f^2}}\right) \tag{33}$$

For a low $\Delta P$ value of 8 hPa and a high *R* value of 60 km valid in the domain of a TC, the ratio $V/fr$ at Eqn. (33) works out to 19.3 for $\varphi = 12.5°$ N and 10.0 for $\varphi = 25°$ N thus ensuring validity of cyclostrophic balance for almost the entire spectrum of values of $\varphi$, *R* and $\Delta P$ if $n = 2$.

If $n > 2$ in Eqn. (32), $V/fr \rightarrow 0$ as $x \rightarrow 0$ and so the cyclostrophic balance becomes invalid. If $V/fr$ is close to 0, it means that the gradient wind can be approximated by the geostrophic wind in the neighbourhood of TC centre. The gradient wind always flows parallel to the isobars which in this case are appreciably curved with a high degree of curvature near the TC centre. However the geostrophic wind flows straight and so no curvature is possible. Thus the assumption that $V/fr \rightarrow 0$ near $x = 0$ leads to contradiction of reality and logic. We therefore conclude that values of $n > 2$ do not generate realistic profiles of cyclone parameters near the centre and that *n* should not be greater than 2. If $n < 2$, Eqn. (32) becomes large as $x \rightarrow 0$ and so the cyclostrophic wind which is a curved flow is realised near the TC centre which is realistic. As the value $n$ = 2 also leads to realistic profiles we obtain the following constraint on $n$ which is

$$n \leq 2 \qquad (34)$$

4.6.2. The constraint that $n \leq 2$ can be supported using the concept of pressure gradient also. The expression for the pressure gradient $P'(r)$ and its derivative $P''(r)$ are given in Eqns. (25) and (26). As $n > 1$ it is evident that $P'(0) = 0$. When we consider the graph of $P'(r)$ its shape at $r = 0$ assumes importance in the correct drawing of the graph. If $n>2$, $P''(0) = 0$ and so the horizontal axis would be tangential to the graph at $r = 0$ (Fig. 3). As $P'(0) = 0$ this would imply that from its zero value at the centre, the pressure gradient increases very sluggishly with $r$. Thus a near flat pressure gradient would prevail up to some distance from the centre and this distance would increase with higher values of $n$.

If $n<2$, $P''(0) = \infty$ and so the vertical axis would be tangential to the graph of $P'(r)$ at $r = 0$ (Fig. 3). Thus pressure gradient increases sharply with $r$. If $n = 2$ we get from Eqn. (26)

$$P''(0) = \frac{2\ \Delta P}{R^2}$$

thus generating a non zero positive value of pressure gradient at the centre.

Obviously the case of flat pressure gradient near the centre of the TC should be firmly ruled out of contention yielding $n$ not greater than 2, *i.e.*, $n \leq 2$. Thus the relation $n \leq 2$ derived based on the validity of cyclostrophic balance near the centre of the TC gets supported from the concept of shape of the pressure gradient graph at the TC centre.

4.6.3. Combining Eqn. (31) and (34) we obtain the range for $n$ which is

$$1 < n \leq 2 \qquad (35)$$

4.7. *Constraint on n and b based on the concept of convergence of cumulative surface pressure drop in a TC regime*

4.7.1. We restate the definition of the generalised form of SPD $H(r)$ as defined in Eqn. (20) which is

$$H(r) = \frac{\Delta P}{\left(1 + ax^n\right)^b}, \quad x = \frac{r}{R}$$

The definition of $H(r)$ as above ensures that $H(r)\rightarrow 0$, as $r\rightarrow\infty$, *i.e.*, $H(r)$ which is a decreasing function, eventually converges to 0 asymptotically. Though $H(r)$ is defined for $0<r<\infty$, effectively it is defined over a finite radius only say $R_e$ which can be taken as the radius of the outer isobar (Fig. 1). Normally, the outer closed isobar or the circular cloud mass of a TC can perhaps extend up to a maximum distance of say 800-900 km from the TC centre, but, at some distance from the centre, value of $H(r)$ should be negligible.

It is also evident that for higher values of $r$, the behaviour of $H(r)$ is same as the behaviour of the function $1/x^{nb}$ and so the value of $nb$ plays a crucial role in the convergence of $H(r)$. If $nb$ is large, convergence of $H(r)$ would be faster and $R_e$ would be smaller. If $nb$ is small, $H(r)$ would converge too slowly and $R_e$ would be large. To cite a specific hypothetical case, if we take $a$ = 6, $b$ = 1/6, $n$ = 3/2 and $\Delta P$ = 50 hPa in Eqn. (20), we find that even at $x$ = 50, *i.e.*, $r$ = 50$R$, $H(r)$ = 14 hPa indicating unrealistic weak convergence of $H(r)$. Here $nb$ = 1/4 which is a relatively low value and hence the weak convergence.

It is thus obvious that mere asymptotic convergence of $H(r)$ to 0 as $r\rightarrow\infty$ would not be sufficient for $H(r)$ to reproduce the surface characteristics of a TC and that a lower limit for $nb$ which ensures a minimum rate of convergence has to be set. Now, the problem before hand is on what basis such a limit could be set based on objective as well as logical concepts. We take a recourse to the concept of so called normalised cumulative surface pressure drop (NCSPD) to derive such a lower limit.

In the TC field we define NCSPD denoted by $G$ between two radii $r_1$ and $r_2$ by the integral

$$G(r_1, r_2) = \frac{1}{R}\int_{r_1}^{r_2} H(r)\,\mathrm{d}r$$

Evidently $G$ is a measure providing cumulated SPD taking into consideration the variation of $H$ with $r$. In the above definition of $G$ the division by $R$ is incorporated as a measure of normalisation so that $G$ has the same unit as $H$, *viz*., the unit of pressure.

We have,

$$G(0, R_e) = \frac{1}{R}\int_{0}^{R_e} H(r)\,\mathrm{d}r = h_1 \text{ (say)}$$

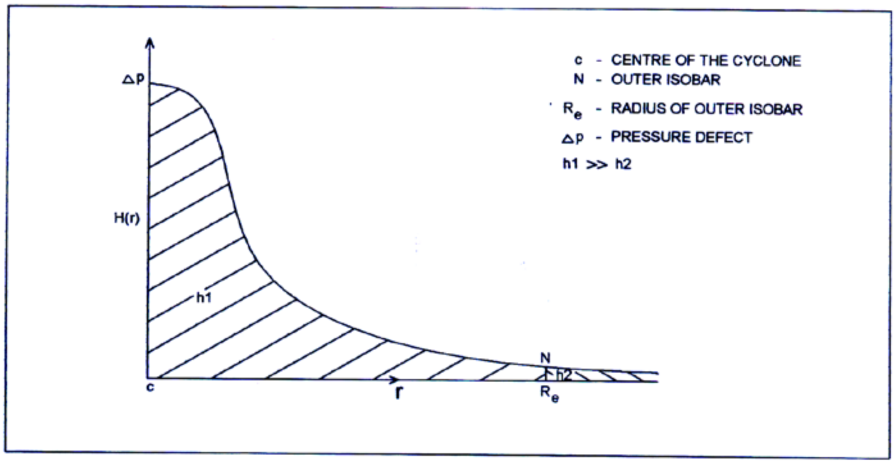



**Fig. 4.** Concept of cumulative pressure drop in a tropical cyclone regime

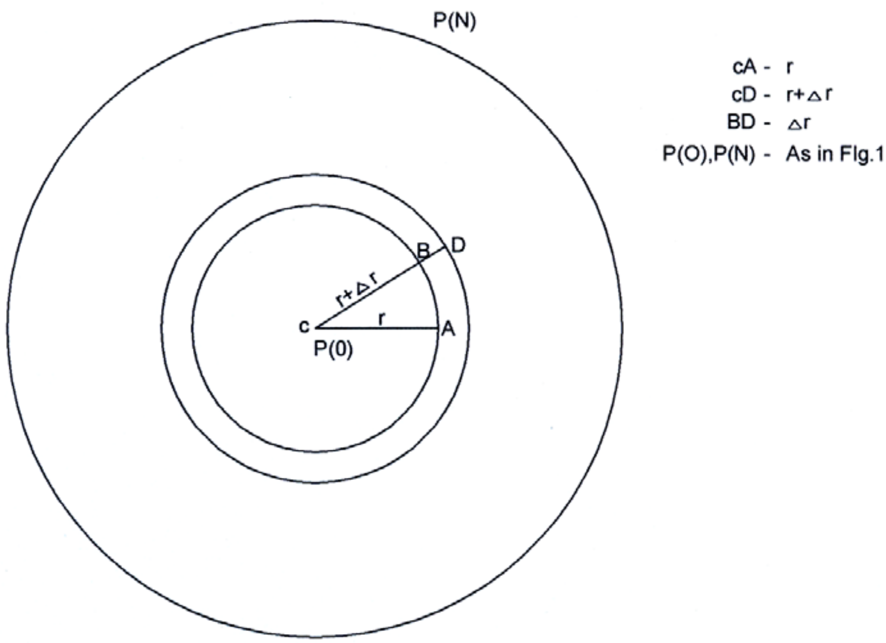



**Fig. 5.** Area of an annulus in the regime of a tropical cyclone

and

$$G(R_e,\infty)=\frac{1}{R}\int_{R_e}^{\infty}H(r)\,\mathrm{d}r=h_2\,(\text{say}) \qquad (36)$$

In the field of a TC sharp increase of MSLP and so sharp decrease of SPD as one moves away from the centre and relatively slack pressure gradient far away from the centre are recognisable features of the MSLP distribution. Invoking such a concept we can definitely conclude

$$h_1 >> h_2 \qquad (37)$$

*i.e.*, $h_1$ is much larger than $h_2$. The inequality Eqn. (37) merely states that the NCSPD realised within the extent of the TC is far more greater than the NCSPD obtained beyond $R_e$, (even if the outer region were allowed to stretch up to infinite distance) an assumption which is realistic as well as objective. This concept is illustrated in Fig. 4.

For the inequality Eqn. (37) to hold, first and foremost $h_2$ of Eqn. (36) should be a finite quantity. Thus the integral

$$\frac{1}{R}\int_{R_e}^{\infty} H(r)\,\mathrm{d}r = \int_{x_e}^{\infty} \frac{\Delta P}{(1+ax^n)^b}\mathrm{d}x, \quad x_e = R_e / R$$

should converge. The above integral converges if and only if the integral $\int_{x_e}^{\infty} \frac{\mathrm{d}x}{x^{nb}}$ converges. It is easily seen that this integral converges when $nb > 1$, *i.e.*, when

$$\frac{1}{n} < b \tag{38}$$

We thus obtain a lower bound for $b$ in terms of $n$.

The basic philosophy behind the above derivation is that mere convergence of $H(r)$ to 0 as $r \to \infty$ would not be adequate and that a minimum rate of convergence has to be present to generate the surface MSLP characteristics of a TC. We have made the very reasonable assumption that the rate of convergence should be such that the NCSPD within the TC field is much larger than the value of NCSPD in the outer region. Such a condition translates into the convergence of the NCSPD integral thereby providing a lower bound for $nb$.

4.7.2. A slightly different style of argument but, in essence, based on the same principle used in Sec. 4.7.1 can be advanced to derive the inequality of Eqn. (38). It follows from the basic definition of MSLP that the SPD $H(r)$ at a distance $r$ from the TC centre could be taken as the loss or depletion of mass at $r$ due to the development of the TC and the consequent decrease of MSLP.

As shown in Fig. 5, let us consider an annulus with radii $r$ and $r + \Delta r$, $\Delta r$ being a small increment. The area of the annulus is $2\pi r\Delta r$ and the mass loss say $m(r)$ within the annulus is therefore given by

$$m(r) = 2\pi r\, H(r)\, \Delta r = \frac{2\pi R^2 x\Delta x}{(1+ax^n)^b}, \quad x = \frac{r}{R}$$

Now let $\quad m_1(x) = \dfrac{x}{(1+ax^n)^b}$ .

Differentiating $m_1(x)$ we get

$$m_1'(x) = \frac{1+(1-nb)x^n}{b\,(1+ax^n)^{b+1}} \tag{39}$$

If $nb \leq 1$, it is obvious from Eqn. (39) that $m_1'(x) > 0$ for all values of $x$, *i.e.*, the function $m_1(x)$ and so $m(r)$ is an increasing function of $r$. Thus the depletion of mass in an annulus increases as one moves away from the centre of the TC, which is unrealistic.

On the other hand, if $nb > 1$, $m_1'(x) = 0$ at a radius say $x_0$ given by $x_0 = 1/(nb-1)$. The function $m_1(x)$ and so $m(r)$ increases within this radius $x_0$ and decreases beyond. This pattern is more realistic and so we conclude that $nb > 1$. Further, it is clear that

$$\lim_{x\to\infty} m_1(x) \begin{cases} = 0 & \text{if } nb > 1 \\ = \infty & \text{if } nb < 1 \end{cases}$$

and obviously only the case of $nb \geq 1$ generates realistic profiles of $m_1(x)$ and so $m(r)$, with the mass depletion reaching zero values as one moves away from the TC centre.

4.8. *Constraint on n and b based on the spatial distribution of relative vorticity*

4.8.1. In the field of a circularly symmetric TC, the relative vorticity $\zeta$ at the surface level is given by (Anthes, 1982)

$$\zeta = \frac{\mathrm{d}V}{\mathrm{d}r} + \frac{V}{r} \tag{40}$$

where $V$ is the wind speed. As is well known the first term in the expression for $\zeta$ as given in Eqn. (40) is the shear vorticity and the second term, curvature vorticity. Both are positive if $r < R$, *i.e.*, $x < 1$. If $x > 1$, the former is negative and the latter remains positive. Thus, $\zeta$ decreases sharply in the outer region of the TC and may become negative also. According to Anthes (1982), the relative vorticity in a TC field is expected to remain positive up to around 500 km from the centre beyond which it may turn negative. If $z$ denotes such a radius, then, $\zeta(z) = 0$, *i.e.*, $z$ is the zero of $\zeta$. As the profile of $\zeta$ reaches maximum very close to the centre and decreases beyond it is evident that $\zeta(x) > 0$ if $x < z$ and $\zeta(x) < 0$ if

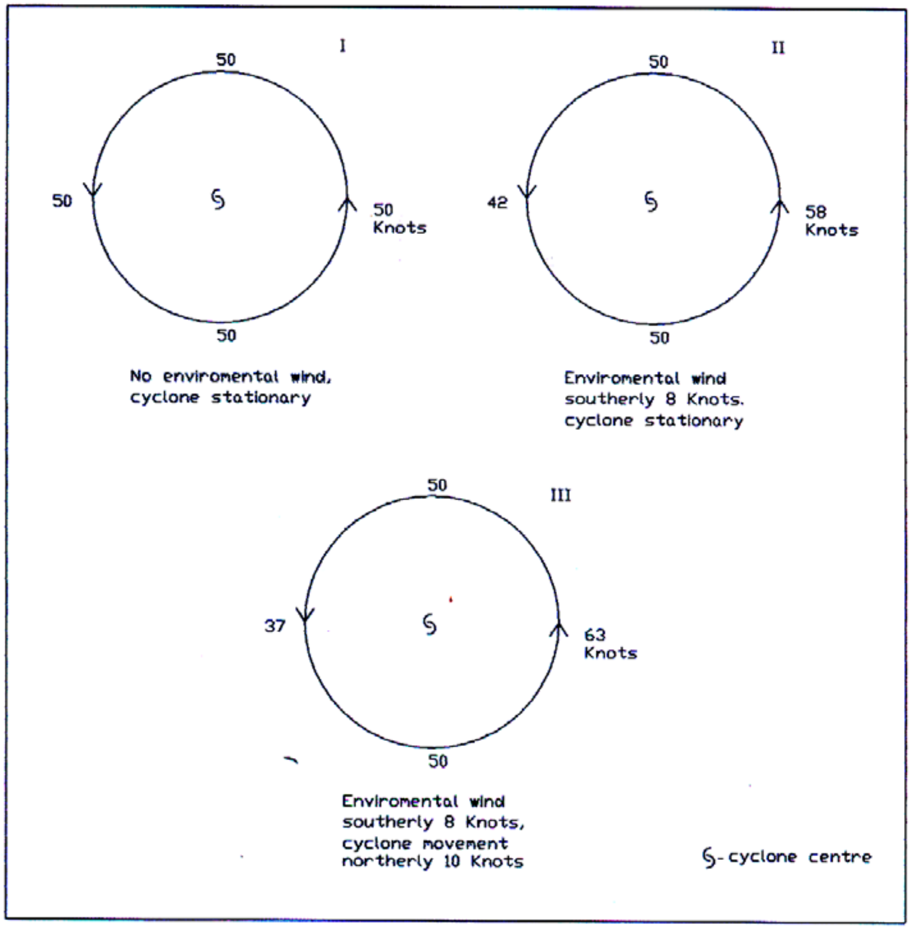



**Fig. 6.** Maximum wind speed in the different sectors of a tropical cyclone – as modified by environmental wind and movement of cyclone

$x > z$. The radius $z$ could be defined as the radius of zero relative vorticity (RZRV).

The absolute vorticity $\zeta_a$ which is the sum of relative and planetary vorticities *i.e.*,

$$\zeta_a = \zeta + f \tag{41}$$

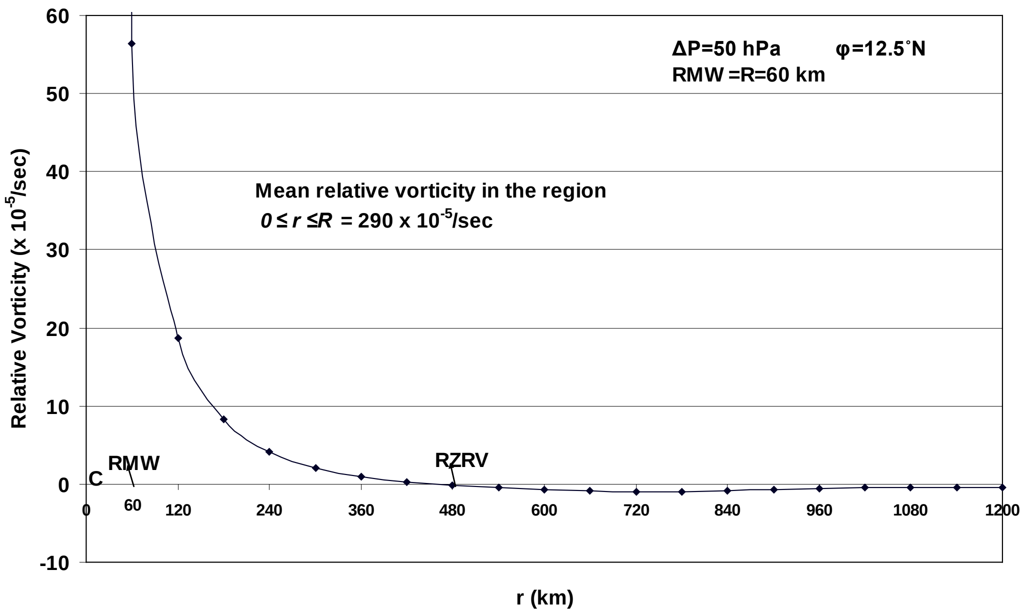
should always remain non negative (Hess, 1959) and obviously very low negative values of $\zeta$ in the TC field are not sustainable.

Differentiating the expression of $V$ based on $H'(r)$ in Eqn. (27) with respect to $r$ the expression for $\zeta$ as defined in Eqn. (40) can be expressed in terms of $H'(r)$ and $H''(r)$ as (Raj, 1991)

$$\zeta(r) = -f + \frac{f^2 - \dfrac{3}{\rho r}H'(r) - \dfrac{H''(r)}{\rho}}{\sqrt{f^2 - \dfrac{4}{\rho r}H'(r)}} \tag{42}$$

Now invoking $H'(r)$ and $H''(r)$ as defined in Eqn. (25) and (26) in Eqn. (40) we obtain

$$\zeta(r) = -f + \frac{f^2 + \left(\dfrac{\Delta P}{\rho R^2}\right) n x^{n-2} \left(\dfrac{2 + n - (n-2a)x^n}{(1+ax^n)^{b+2}}\right)}{\sqrt{f^2 + \left(\dfrac{4n\Delta P}{\rho R^2}\right)\left(\dfrac{x^{n-2}}{(1+ax^n)^{b+1}}\right)}} \tag{43}$$




RMW , RZRV – Radii of maximum wind/ zero relative vorticity, C: centre of the tropical cyclone

**Fig. 7.** Normal profile of variation of relative vorticity in the field of tropical cyclone

where $f$ is the Coriolis parameter, $R$ = RMW, $\rho$ is the density, $x = r/R$, $\Delta P$ is the pressure defect and $ab = 1$.

The cyclostrophic vorticity $\zeta_c$, *i.e.*, vorticity based on cyclostrophic wind is obtained by setting $f = 0$ in Eqn. (43) and is given by

$$\zeta_c(r) = \left(\frac{1}{2R}\right)\sqrt{\frac{n\Delta P}{\rho}}\,\frac{x^{\frac{n-2}{2}}\left[2+n-(n-2a)x^n\right]}{(1+ax^n)^{\frac{b+3}{2}}} \quad (44)$$

4.8.2. It is possible to compute $\zeta(r)$ or $\zeta_c(r)$ from equations (43) or (44) given the values of all the variables/measures defining the two expressions. It must be stated that $\zeta(r)$ or $\zeta_c(r)$ at $r = 0$, *i.e.*, at the TC centre is indefinitely large. However the mean value of $\zeta$ in a small neighbourhood of $x = 0$ could be derived as a finite value, as shown below. In a neighbourhood of $x = 0$ with a radius of $c$, $\zeta_c(r)$ of Eqn. (44) could be approximated by the expression

$$\zeta_0(r) = \frac{2+n}{2R}\sqrt{\frac{n\Delta P}{\rho}}\,x^{\frac{n}{2}-1}, \quad 0 \le r \le c, \quad x = \frac{r}{R} \quad (45)$$

Now the average value of $\zeta_c(r)$ say $\zeta_0$ in the neighbourhood is given by

$$\zeta_0 = \frac{1}{c}\int_0^c \zeta_0(r)\,\mathrm{d}r \quad (46)$$

Evaluating Eqn. (46) we get

$$\zeta_0 = \frac{2+n}{nR}\sqrt{\frac{n\Delta P}{\rho}}\,d^{\frac{n}{2}-1}, \quad d = \frac{c}{R} \quad (47)$$

Thus $\zeta_0$ could be computed for $d \neq 0$ and for smaller values of $d$, say for $d \le 0.2$. For values of $\zeta$ beyond $x = d$, equation (43) could be used. It is thus possible to compute the mean value of the relative vorticity say $\zeta_1$ within RMW, *i.e.*, $0 \le r \le R$ or $0 \le x \le 1$.

4.8.3. A very simplified wind model called Rankine vortex is some times used to model surface wind in a TC field (Anthes, 1982). According to this, the wind speed at radius $r$ within RMW is given by

$$V = kr, \qquad r \le R \quad (48)$$

where $k$ is to be estimated. This is called solid rotation. If $V_m$ is the MWS, from Eqn. (48) we get

$$V = V_m\left(\frac{r}{R}\right) \tag{49}$$

The value of $\zeta$ as defined in Eqn. (40) is constant within RMW and is equivalent to $2V_m / R$. For a typical value of $V_m$ = 40m/s (≈80 knots) and $R$=30 km, ζ works out to nearly $273\times10^{-5}$/s.

4.8.4. Computation of mean value of $\zeta_1$ for $0 \le r \le R$ as per equations (43) and (46) yielded values which were higher than the constant ζ value derived based on expression of wind speed generated by the Rankine vortex. Given $\Delta P$ = 50 hPa, $R$ = 30 km, $\varphi$ = 12.5° N the computed values of $\zeta_1$ were 573, 537 and 515 × $10^{-5}$/s for the values of $n$ = 1.1, 1.5 and 1.9 respectively for lower values of $b$, given $n$. There was sharp variation of relative vorticity from $r$ = 0 to $R$, which decreased strictly from centre to periphery. The mean value of ζ, *i.e.*, $\zeta_1$ should be a better measure of relative vorticity within the RMW rather than the profile of ζ for $0 \le r \le R$.

4.8.5. Fig. 7 presents a typical profile of relative vorticity in the regime of a TC. The profile within RMW is not explicitly shown, only mean value is indicated. As seen the RZRV is located 300-500 km from the cyclone centre beyond which relative vorticity turns negative.

4.8.6. Let us define an expression $A(x)$ as

$$A(x) = 2 + n - (n - 2a)x^n \tag{50}$$

Let $z_c$ be the zero of $\zeta_c(x)$ or $A(x)$. From Eqn. (50), we get

$$z_c = \left(\frac{2+n}{n-2a}\right)^{\frac{1}{n}} \tag{51}$$

From Eqn. (43), it follows that $\zeta(z_c) < 0$. Thus, if $z_g$ is the zero of $\zeta(x)$ then $z_c > z_g$. This shows that $\zeta(x)$ decreases and attains the zero value and turns negative more rapidly than $\zeta_c(x)$.

The RMW around a TC normally has a typical value of 40 km and is taken to vary between 20 and 60 km (Anthes, 1982). If we could make a very reasonable assumption that RZRV should be greater than 3$R$ or up to around 180 km from the centre of the TC, it leads to, the relation $z_g$ > 3 yielding the inequality

$$2 + n - 3^n(n - 2a) > 0 \qquad \text{or by using Eqn. (24)}$$

$$b < \frac{2}{\left(n - \frac{2+n}{3^n}\right)} \tag{52}$$

Combining Eqns. (38) and (52) we get bounds for $b$ in terms of $n$ as

$$\frac{1}{n} < b < \frac{2}{\left(n - \frac{2+n}{3^n}\right)} \tag{53}$$

Now the inequality Eqn. (53) is a necessary but not sufficient condition for $\zeta(x) > 0$ at $x$ = 3. For example, if the values $n$ = 1.5 and $b$ = 2 which satisfy the inequality Eqn. (52) are substituted in Eqn. (43), $\zeta(x)$ need not be greater than 0 for $x$ = 3 as the value of $\zeta(x)$ substantially depends on the values of other variables/measures *viz.*, $\Delta P$, $R$ and $f$ also. As such we must choose $n$ and $b$ such that they satisfy the inequality of Eqn. (53) and at the same time generate realistic profiles of cyclone parameters within the variation of $\Delta P$, $R$ and $f$. The equation $\zeta(r)$=0 of Eqn. (43) does not have a general solution in $r$ or $x$ and cannot be solved in radicals. Therefore we resort to numerical means to identify the suitable values of $n$ and $b$.

## 5. Determination of admissible values of *n* and *b* based on pressure, wind and relative vorticity profiles and estimation of preliminary value of K

The values of $n$ and $b$ within the ranges defined by the inequalities Eqns. (35) and (53) but at the same time generating realistic profiles of cyclone parameters within the cyclone regime are now proposed to be determined. For this we evaluated the profiles of the following parameters *viz*., (*i*) SPD, *i.e.* $H(r)$ (*ii*) gradient wind (*iii*) cyclostrophic wind (*iv*) relative vorticity and (*v*) absolute vorticity. The mean value of relative vorticity $\zeta_1$ within the region $0 \le r \le R$ was also derived as described in Sec. 4.8.2. The profiles were generated for the values of (*i*) $\rho$ = 1.13 × $10^{-3}$ gm/cm$^3$ (*ii*) φ = 12.5 & 25.0° N (*iii*) $\Delta P$ = 50 & 80 hPa (*iv*) $R$ = 30 & 60 km (*v*) $n$ in the interval $1<n\le2$ with an increment of 0.1 and (*vi*) values of $b$ as defined by Eqn. (53) given $n$, with an increment of 0.2, from $x$ = 0 to $x$ = 10 with an increment of 0.2.

A total of 1152 profiles of TC parameters could be generated through the above exercise. The combination of $n$ and $b$ which together generated either (*i*) a relative vorticity profile with RZRV < 3 or an absolute vorticity profile yielding negative value for any value of $x$ - were rejected. The other values of $n$ and $b$ which yielded RZRV>3 and positive absolute vorticity profiles alone were selected. The total number of such cases was 534. The proportionality constant $K$ as defined in Eqn. (9) was evaluated for all the above 534 cases. The mean value of *K* thus obtained was 11.4 with a range of 9.7 to 13.5.

In Table 3 we have listed the values of $\varphi$, $\Delta P$ and *R*, three representative values of *n viz*., 1.1, 1.5 and 2.0 and for each *n*, three representative values of *b*, *viz*., lowest, middle and highest from amongst the *b* values selected and not rejected, thus listing 72 strings. The values of following derived parameters *viz*., (*i*) RMPG as expressed in Eqn. (30) (*ii*) *K* (*iii*) $V_m$, (*iv*) $z_g$ , the RZRV (*v*) $\zeta_L$, the lowest value of relative vorticity (*vi*) $\zeta_{a_L}$, the lowest value of absolute vorticity and (*vii*) $\zeta_1$ the mean relative vorticity within the region $0 \leq r \leq R$ are also listed. All distances are given as ratios to *R* the RMW only.

As RZRV of a TC field is likely to be of nearly 500 km from the cyclone centre, the ratio RZRV/RMW should be nearly 17 for *R* = 30 km and 8.5 for *R* = 60 km. Such a situation is found to be obtained only when *b* assumes the lowest range of values for a given *n*. Holland (1980), has commented that a large separation between RMPG and RMW is unrealistic and has suggested that the ratio RMPG /RMW should be atleast ½. From Table 3, it is seen that such a ratio is obtained only when *n* is closer to 2. However, for such values of *n*, $z_g$ is lower, *i.e*., the relative vorticity turns negative at about 3-4 times RMW from the centre. The size of the TC, which can perhaps be taken as the radius of the outer isobar, can be up to 600-800 km and it is seen that such values are realised for all the values of *n* and for the lowest value of *b*, given *n*.

The important measures of a TC, such as $\Delta P$, $V_m$ and *R* are expected to manifest wide variation. In accordance with the definition by IMD, the MWS of a TC has to be greater than 34 knots but MWS up to 200 knots have been measured in individual cases in respect of intense TCs. The size also varies considerably from cyclone to cyclone, season to season and also from basin to basin. Considering such a wide dispersion it is preferable to allow for adequate variation in the cyclone parameters while selecting pressure profiles. The set of 72 cases listed in Table 3 which is a representation of the 534 cases considered allows for such a variation and as such the mean value of K *viz*., 11.4 could be considered as a representative value.

## 6. Effect of friction, environmental flow and direction of motion on the maximum wind

The range of K as derived in Sec. 4.6 is based on assumptions stated in Sec. 2.2. The following three factors, *viz*. - (*i*) Frictional forces (*ii*) Surface environmental wind flow and (*iii*) Force generated due to the translatory movement of the storm - can modify the maximum wind in the regime of a TC and should be taken into consideration in deriving the final $V_m$ - $\Delta P$ relation. We now consider the effect of these factors on the maximum wind of a TC.

### 6.1. *Frictional forces*

Frictional forces play an important role in the boundary layer by reducing the wind speed and generating cross isobaric wind flow at the surface level in the cyclone regime. Basu and Ghosh (1987) have dealt in detail the effect of frictional forces on the wind field of TCs. For the present study however, we assume a simplified wind model incorporating friction. If *k* is the coefficient relative to friction, the cyclostrophic wind speed can then be modified as

$$\frac{V^2}{r} = \left(\frac{1}{\rho}\right)\left(\frac{\mathrm{d}P}{\mathrm{d}r}\right) - kV^2 \tag{54}$$

If $V_c$ denotes the cyclostrophic wind speed without taking friction into account, *i.e*., $k = 0$ in the above equation and $V_f$ denotes the wind speed as obtained from Eqn. (54) we have

$$V_f = \frac{V_c}{\sqrt{1 + rk}} \tag{55}$$

We can assume a value of $k$ = 0.00560/km as the mean value of *k* for tangential and normal components as suggested in Basu and Ghosh (1987). The expression Eqn. (55) is valid only in the central parts of the TC and not in the outer region owing to the requirement of validity of cyclostrophic assumption. If we assume a 30-60 km range for RMW then the range of $V_f$ is within $0.87V_c$ and $0.93V_c$. Thus friction reduces the maximum wind by 7-13%, depending on the value of RMW.

### 6.2. *Environmental flow*

The superposition of the TC into the environmental wind field in which it is embedded can increase the wind speed in some sectors while decreasing the same in others. For example, an environmental wind speed of 10 knots easterlies would increase (decrease) the maximum wind by 10 knots north (south) of the centre, thus introducing non concentric isotachs.

### 6.3. *Translation speed of TC*

The effect of movement of TC on its wind field has been studied by Myers and Malkin (1961), Basu and

Ghosh (1987). If the Coriolis force is assumed small, we get a simplified expression *viz*.,

$$V = V_g + \frac{V_t \sin\theta}{2}$$

where $V_g$ is the gradient wind speed, $V_t$ is the translatory speed of motion of the TC, θ is the azimuthal angle measured clockwise from the direction of motion and $V$ is the resultant wind speed. Thus a wind speed profile of stronger winds in the right sector of the storm [$\theta = 90°$, $V = V_g + (V_t/2)$] than to the left sector [$\theta = 270°$, $V = V_g - (V_t/2)$] , with reference to the direction of motion is realised.

6.4. *Gradient wind vis-à-vis cyclostrophic assumption*

Though cyclostrophic assumption is frequently used to derive wind speeds in the central parts of a TC due to its simplicity it is the gradient wind which is more accurate than the cyclostrophic wind. If $V_c$ and $V_g$ are the cyclostrophic and gradient wind speeds respectively in the regime of the TC close to the centre, we obtain from Eqns. (4) and (5) the relation

$$\frac{V_c}{V_g} = \sqrt{1 + \frac{fr}{V_g}}$$

Thus, $V_c$ is always larger than $V_g$ in the field of a typical tropical cyclone. It can be shown that when $r$ assumes values closer to $R$ the RMW, this magnitude is up to 1% in lower latitudes and 2% in higher latitudes as gradient wind always turns out higher wind speeds in lower latitudes compared to higher latitudes for a given pressure gradient.

6.5. *Combined effect*

The total contribution of environmental wind and motion vector of the cyclone to the maximum wind of a TC would therefore depend upon the combined effect of the above mentioned factors for a given case. If the environmental wind direction and the direction of motion of the TC are the same, then the MWS of the TC is increased by $V_e + V_t$ where $V_e$ and $V_t$ are environmental wind speed and speed of movement of the TC respectively. The modification of MWS due to environmental wind and translatory movement of the TC is illustrated in Fig. 6 using hypothetical values. A TC which is stationary is assumed to have MWS of 50 knots. A southerly wind of 8 knots increases the MWS to 58 knots in the eastern sector and reduces it to 42 knots in the western sector. Now a northerly movement of the TC with a speed of 10 knots increases the wind speed to 63 knots in the eastern sector and reduces it to 37 knots in the western sector resulting in substantially differential relative MWS in different sectors.

It must be noted that friction reduces the speed uniformly for a given radius, whereas the two vectors considered above increase and decrease the speed in different sectors. If we define a mean maximum wind, *i.e*., mean value of relative maximum wind in different sectors, the incorporation of the two vectors would not change the mean maximum wind at all though the local maximum wind would get modified.

The climatological values of the environmental wind and motion vector are generally available for a given basin. For the basin of North Indian ocean, environmental wind speed ranges from 5 to15 knots and speed of movement of a typical TC is 5-8 knots. Taking into consideration these normal values, we feel, a value of 10 knots could be added to the maximum wind of the TC to incorporate the effects of both the vectors. In North Indian Ocean, only a few TCs attain or exceed the 100 knots (186 kmph) MWS. Thus, by and large, the incorporation of these two vectors could increase the MWS by 10-15%.

Though friction reduces the MWS uniformly in the RMW by 9-14%, the environmental flow and motion vector increase the wind speed only in specific sectors. A reduction of the MWS by 2-3 % to compensate for the combined effects of friction, environmental flow and translation speed and a further reduction of 1-2 % to correct the over estimation by cyclostrophic wind would be appropriate. A total reduction of K value by 3-4% could be effected as final correction to factor in all the other forces influencing maximum wind speed in a cyclone field.

## 7. Final value of *K* and comparison between its theoretical and empirical values

The final correction factor arrived at in the previous section, *i.e*., reduction of K by 3-4% from its preliminary value obtained in Sec.6 yields a value very close to 11.0 for K. Thus 11.0 is the final estimate of the proportionality constant of the empirical relation between PD and MWS of a TC, that we have derived from several theoretical considerations and after incorporating a few correction factors.

How such a theoretically derived value compares with the empirically obtained estimates, some of which are listed in Table 1 will now be discussed. As seen the empirical value of K varies from 10.7 obtained by Fletcher (1955) to 14.2 derived by Mishra and Gupta

**TABLE 4**

**Maximum wind as generated by three different models in a cyclone field, given pressure defect**

| $\Delta P$ | $V_m$ | | |
|---|---|---|---|
| | (*i*) | (*ii*) | (*iii*) |
| 10 | 34.8 | 29.5 | 44.9 |
| 20 | 49.2 | 46.1 | 63.5 |
| 30 | 60.2 | 59.9 | 77.8 |
| 40 | 69.6 | 72.1 | 89.8 |
| 50 | 77.8 | 83.2 | 100.4 |
| 60 | 85.2 | 93.6 | 110.0 |
| 70 | 92.0 | 103.4 | 118.8 |
| 80 | 98.4 | 112.6 | 127.0 |
| 90 | 104.4 | 121.5 | 134.7 |
| 100 | 110.0 | 130.0 | 142.0 |

$V_m$ : Maximum wind speed in knots
$\Delta P$ : Pressure defect in hPa
(*i*) : $11.0\sqrt{\Delta P}$ - derived in this study
(*ii*) : $6.7\ \Delta P^{0.644}$ – Atkinson and Holliday (1977)
(*iii*) : $14.2\sqrt{\Delta P}$ - Mishra and Gupta (1976)

(1976). While interpreting the empirical values, the following aspects need to be taken into consideration.

(*i*) The K values have been based on composites of available observations of MSLP and surface wind from ships' reports, aircraft reconnaissance data and data of coastal/island stations.

(*ii*) Both the PD and MWS values estimated from a set of observations would be slight under estimation of the actual values. However, MWS could be over estimated, as the spot values of wind, which are dominated by convective clouds could get substantially vitiated by gusts.

(*iii*) Given a set of observations in a TC field, at a given time only the observation with highest wind speed is likely to have been taken for the computation of K, which tends to overestimate the actual wind speed.

In the study by Atkinson and Holliday (1977), the authors first took the recorded peak gust values, reduced them to standard anemometer level using power law relationship and then converted to one minute sustained wind speeds using gust factors representative of an air over water environment. Such wind speeds were used as input to derive the $V_m$ - $\Delta P$ relation as given in Eqn. (11). Most of the other studies listed in Table 1 used the spot values which might be over estimates of the sustained wind speed.

In Table 4, we present the maximum wind speeds obtained based on (*i*) K value of 11.0 obtained in this study (*ii*) Atkinson Holliday relation (11) and (*iii*) Mishra and Gupta relation (12). The MWSs are presented for $\Delta P$ values of 10, 20,….., 100 hPa. As seen from Table 3 the $V_m$ values reported as per (*i*) are higher than the values obtained from (*ii*) by 5-10% when $\Delta P$ is up to 30 hPa. When $\Delta P$ is in the range of 40-50 hPa there is hardly any difference between the pairs of values. When $\Delta P$ is higher than 60 hPa, $V_m$ reported by (*i*) are lower than $V_m$ reported by (*ii*) by 5-10%, the difference increasing with $\Delta P$. Thus the $V_m$ values obtained from (*ii*) are lower than (*i*) for low values of $\Delta P$ but higher than (*i*) for higher values of $\Delta P$. This is obviously an offshoot of the higher value of the exponent in the formula for (*ii*) as given in (11). The $V_m$ values obtained from the formula (*iii*) are higher than the values of (*i*) and (*ii*) for all the $\Delta P$ values of Table 3, though the difference is very sharp for lower values of $\Delta P$. For example, for $\Delta P$ = 20 hPa, (*iii*) returns a $V_m$ value of 63.5 knots which is 35% higher than the value reported by (*ii*).

Atkinson and Holliday have commented that most of the empirically derived $V_m$ - $\Delta P$ relations are over estimations due to the vitiation of MWS data by gusts. Similar comments have been made in Mishra & Gupta as well. It thus appears that the slightly lower value of 11.0 for K derived in this study would be closer to reality than the higher values of K obtained in several other studies.

## 8. Certain other aspects of the pressure model

### 8.1. *Bounds for K without invoking the vorticity concept*

We have already derived the range of K as $1 < n \leq 2$ in Eqn. (35) and the constraint that $nb>1$ in Eqn. (38). As shown in Eqn. (29).

$$K = 18.8\sqrt{n}\left(\frac{b}{b+1}\right)^{\frac{b+1}{2}}.$$

The lowest value of K is realised when $n$ = 1 and $b$ > 1, yielding K > 9.4. The upper bound of K is reached for $n$ = 2 and for very large values of $b$ when $[b/(b+1)]^{(b+1)/2} \rightarrow 1/\sqrt{e}$ leading to K < 16.1. Thus we get 9.4 < K < 16.1, a range of K obtained without invoking the concept of minimum RZRV in a TC field. However, as seen, the upper bound at 16.1 is substantially higher than the 13.5 derived in Sec. 5 by invoking the RZRV concept which provides an upper bound for $b$, given $n$.

8.2. *Upper bound for K if RZRV is very large*

In Sec. 4.7 we set that RZRV / RMW >3 which led to the inequality Eqn. (53) and an upper bound for K. Let us now suppose that RZRV / RMW > $d$. We then get

$$b < \frac{2}{n - \frac{2+n}{d^n}} \tag{56}$$

which is a necessary (but not sufficient) condition for $\zeta(x) > 0$ if $x<Rd$, $R$ = RMW. If $d$ is set higher than 3 the RHS of Eqn. (56) would be still lower. If we assume that $d$ is indefinitely large, *i.e.*, positive relative vorticity extends over a very large radius, then Eqn. (56) becomes $b < 2/n$ and Eqn. (53) transforms to

$$\frac{1}{n} < b < \frac{2}{n} \tag{57}$$

When the exercise of determining K as detailed in Sec. 5 was repeated in to for the same values of $\varphi$, $\Delta P$ and $R$ we obtained a mean upper bound for K as 13.1, derived from 302 values, instead of the 13.5 derived for $d$ = 3. This is the lowest upper bound possible to achieve based on the concept of RZRV. The reason for decreasing upper bound for K as RZRV increases could be easily explained. The upper bound $2/n$ for $b$ in Eqn. (57) is lower than that of Eqn. (53) leading to lower value of K. Given $n$, a lower value of $b$ is associated with slack decrease of SPD thereby extending the TC regime to larger distance from the centre.

8.3. *Fitting of the pressure model and derivation of K for a specific TC*

The value of K obtained as 11.0 in Sec.7 is a mean value with range 9.7-13.6. For a specific TC for which estimates of $R$ and $\Delta P$ and also sufficient number of MSLP observations in the field are available, a firm value of K could be estimated. The methodology is briefly described below.

Suppose there are $m$ observations of pressure values *viz.*, $P_1$, $P_2,\ldots,P_m$ at locations $r_1$, $r_2,\ldots,r_m$ with the corresponding $x$ values $x_1$, $x_2,\ldots,x_m$. Now we must have

$$P_i = P_0 + \Delta P\left(1 - \frac{1}{(1+ax_i{}^n)^b}\right) \tag{58}$$

First $n$ must be varied in the range $1 < n \leq 2$, by incrementing by a small value say 0.01. The corresponding limits of $b$ are then computed from Eqn. (53) and as per the discussions in Sec. 5 lower range values of $b$ are to be preferred. For a given $n$ and $b$, the estimated values of $P_i$ say, $\hat{P}_i$, can be computed from Eqn. (58). The linear CC between $P_i$ and $\hat{P}_i$ could then be computed. If need be the profiles of other derived parameters such as relative and absolute vorticities can be derived. A firm pair of $n$ and $b$ values corresponding to the highest CC and so the best fit pressure profile at the same time generating realistic profiles of important cyclone parameters could be selected. A firm and single value of K can then be computed based on the expression Eqn. (29) and the maximum wind derived from the relation $V_m = \mathrm{K}\sqrt{\Delta P}$. Such a value of $V_m$ is expected to be a bit more accurate than the value derived from the mean estimated value of K given in Sec. 7. A more or less similar approach has been suggested by Bretschneider (1982).

8.4. *A few observations based on Table 3*

Table 3 presents data on the profiles of cyclone parameters such as pressure gradient, maximum wind, RZRV etc. The following observations are made in addition to the few remarks made in Sec.7.

(*i*) Given $n$, RZRV decreases with increasing $b$.

(*ii*) For lower values of $n$ close to 1, the RMPG is low at about 0.2R, where $R$ = RMW. If RMPG/RMW is to be higher than ½ as suggested by Holland (1980), the value of $n$ must be greater than 1.5.

(*iii*) Realistic values of RMPG and RZRV are realised if $n \geq 1.5$ and for a given $n$, for lower values of $b$.

(*iv*) Given φ, $\Delta P$, $n$ and $b$, the value of RZRV increases with $R$ though the ratio RZRV/$R$ (given in Table 3) decreases.

(*v*) Given φ, $R$, $n$ and $b$, the value of RZRV increases with $\Delta P$.

(*vi*) Given $\Delta P$, $R$, $n$ and $b$, the value of RZRV decreases with increasing latitude. Thus a TC moving northwards without change in $\Delta P$ and $R$ should progressively have reduced RZRV and consequently reduced extent of the radius of associated circular cloud mass.

(*vii*) The mean relative vorticity $\zeta_1$ within the region $0 \le r \le R$ displays variation as described below:

(a) For the ranges of measures / variables φ, Δ*P*, *R*, *n* and *b* as presented in Table 3, the value of $\zeta_1$ varied between 262 ($\times10^{-5}$/s) (for Δ*P*=50 hPa, *R* = 60 km) to 789 (for Δ*P* = 80 hPa, *R* =30 km).

(b) Given other variables $\zeta_1$ decreases as *n* increases. Given *n*, $\zeta_1$ increases with *b*.

(c) $\zeta_1$ decreases sharply with *R*. For example, when *R* increases from 30 to 60 km, $\zeta_1$ almost halves from 510-600 ($\times10^{-5}$/s) to 250-310.

(d) $\zeta_1$ increases sharply with Δ*P*. When Δ*P* increases from 50 to 80 hPa, $\zeta_1$ increases by nearly 25%.

(e) $\zeta_1$ decreases as φ increases, the decrease is 2-3% as φ increases from 12.5 to 25°N.

8.5. *Basin wise variation of K*

The constant K derived in this paper is defined by the relation in Eqns. (9) and (29). As such it is controlled by only one environmental parameter ρ which is the surface air density and then the variables of the pressure model $\psi$ leading to the range of K obtained in Sec 4.7. As of now the final K value arrived at appears to be same for every basin where TCs form and move though, basins such as North Indian Ocean, Pacific, Atlantic etc are located far apart from one another. Subtle changes in the range and mean value of K could be obtained by incorporating certain basin characteristics. In Eqn. (29), ρ has been computed from the ideal gas law $\rho = P/\mathrm{R}T$ where *T* and *P* are surface air temperature and pressure and R is the gas constant. A range of 26-32° C variation for *T* with no change in pressure can cause only a 1% change in K, lower value of K associated with lower value of *T*. For a *P* variation of 1000-1015 hPa the variation in K is only 0.7%, lower value of K associated with higher value of *P*. Further for TCs at lower latitudes the MWS could be higher than that of higher latitudes for a given Δ*P* as the gradient wind returns a higher wind speed for low value of *f* given the same pressure gradient.

8.6. *Validity over land area*

The PD-MWS empirical relations found in the literature (Table 1) have generally been derived only for TCs located in the seas, utilising observations taken over sea / islands / coastal areas. The validity of such relations when the TC crosses coast and moves into the land has not been discussed in such studies (Table 1). Normally a TC loses its intensity rapidly after land fall and so the assumptions based on which the PD-MWS relation has been derived may not remain valid. However, in the event of an intense TC retaining its cyclone intensity say for one or two days after the land fall, whether the relation remains valid or not could be a relevant question.

The effect of frictional forces over land should be much higher than that over the sea. The friction effect would depend very much on the type of land terrain. For a roughness length of 0.5m and at a reference height of 10m the value of *k* as defined in Sec. 4.7.1 can be derived as approximately 0.0161 / km. In this case as per Eqn. (55) friction reduces the maximum wind speed by as much as 18-30% when the RMW range is 30-60 km, when the TC is over the land. However whether the $V_m$-Δ*P* relationship could be extended to land areas is a moot question. The terrain is capable of producing large changes in the circulation resulting in changes in the horizontal and vertical structure of the cyclone.

## 9. Relative vorticity profile and convergence of the NCSPD integral in respect of other pressure models of TCs

Two concepts that played important roles in deriving the theoretical value of K are (*i*) Convergence of the NCSPD integral defined in Sec. 4.7.1 and (*ii*) Relative vorticity profile to remain positive up to some distance from the TC centre. How the other pressure models which have been listed in Table 2 satisfy these two conditions could be a matter of interest and would be discussed. From Eqn. (42), we note that relative vorticity $\zeta(r)$ becomes negative at *r* if

$$H_1(r) = 3H'(r) + rH''(r) > 0$$

9.1. For the Fujita model

$$H(r) = \frac{\Delta P}{(1+2x^2)^{1/2}}, \qquad x = \frac{r}{R}$$

This is now a specific case of the general model assumed by us in Eqn. (10) with *a* = 2, *b* = ½ and *n* = 2. As *nb* = 1, the integral $\int_0^\infty H(r)\,\mathrm{d}r$ converges. We also obtain that $H_1(r)$ has the same sign as $-(2+4x^2)$, $x = r/R$, and so is always negative. The profile of ζ derived from Eqn. (43) could therefore be realistic.

9.2. The Bret-X model is obtained when $a = b = 1$ and $n = 2$ in Eqn. (10). Behaviour of NCSPD integral and relative vorticity profiles are similar to that of the Fujita model. The value of NCSPD integral between 0 and ∞ is

$$G(0,\infty) = \frac{1}{R}\int_0^\infty \frac{\Delta P \, \mathrm{d}x}{1+(r/R)^2}$$

The value of the above integral and so NCSPD is $\pi/2\ \Delta P$ roughly 1.6 times the PD.

9.3. The exponential model is similar to what is defined in Basu and Ghosh (1987). We have

$$H(r) = \Delta P\, e^{-ax^n}, \quad x = r/R$$

Here, $H'(R) + R\,H'(R) = 0$ yields $a = 1$ and the relation 0 < RMPG < RMW leads to $n > 1$. The condition that ratio $V/fr$ should be large (Sec. 4.6) leads to $n < 2$ and so we get $1 < n \le 2$. The $V_m$ - $\Delta P$ relation can be expressed by $V_m = 11.4\sqrt{n}\sqrt{\Delta P}$ and as $1 < n \le 2$, we obtain bounds for $K$ as $11.4 < K \le 16.1$.

From Eqn. (39) we can derive that $\zeta(x) < 0$ if $x < z_c = \sqrt{\dfrac{2+n}{n}}$. This is a decreasing function of $n$ and even for the lowest value of $n = 1$, we get $z_c = 3$ and so $z_g < 3$ ($z_c$, $z_g$ are as defined in Sec. 4.8.6). As such the model generates profiles of relative vorticity that turn negative too close to the centre. However, the NCSPD integral always converges. This model is a limiting case of Eqn. (10) as

$$\left(1+\frac{x^n}{b}\right)^{-b} \rightarrow e^{-x^n} \text{ when } b \rightarrow \infty.$$

As discussed in Sec.5, higher values of $b$ turn out less realistic relative vorticity profiles than lower values of $b$ in the general model we have assumed. It is evident that the exponential model which is a specific case of Eqn. (20) when $b \rightarrow \infty$ is not able to generate realistic profiles of relative vorticity.

9.4. The model suggested by Holland (1980) can be expressed as

$$H(r) = \Delta P\left[1-\exp\left(-\frac{a}{x^n}\right)\right], \quad x = \frac{r}{R} \qquad (59)$$

For this model, the condition $H'(R) + R\,H''(R) = 0$ yields $a = 1$. The convergence of the NCSPD integral in this case is same as the convergence of the integral $I$ defined by,

$$I = \int_0^\infty \left[1-\exp\left(-\frac{1}{x^n}\right)\right]\mathrm{d}x$$

which is equivalent to the convergence of the integral

$$I_1 = \int_0^\infty \frac{1-e^{-t}}{t^{\frac{1}{n}+1}}\,\mathrm{d}t$$

It can be shown that the above integral converges when $1/n + 1 < 2$, *i.e.*, when $n > 1$. Further $\zeta(r) < 0$ if $z_g < z_c = \left(\dfrac{n}{n-2}\right)^{\frac{1}{n}}$, $n > 2$. When $n \le 2$ $z_c$ does not become zero at all. The condition that $z_c > 3$ leads to the inequality $n < 2.2$. The $V_m$ - $\Delta P$ relation is $V_m = 11.4\sqrt{n}\sqrt{\Delta P}$ same as the relation obtained in Sec 9.3 for the exponential model. Thus we get $11.4 < K < 16.9$ as bounds for K. The relative and absolute vorticity profiles generated by the model are realistic when $1 < n \le 2$, thus narrowing K to $11.4 < K < 16.1$. This yields a mean value of 13.8 and a final corrected estimate of nearly 13.4 for K.

It is thus obvious that some models have not satisfied the criteria requiring convergence of NCSPD integral whereas some models have not been able to generate realistic relative vorticity profiles. The model by Holland (1980) dealt in Sec 9.4 satisfies both but returns a wider interval for K compared to the range of K obtained in Sec. 5.

## 10. Generality of the idealised pressure model assumed

In Sec.4 we defined the pressure model as

$$H(r) = P(N) - P(r) = \frac{\Delta P}{(1+ax^n)^b}, \qquad x = \frac{r}{R}$$

We now discuss the generality of this model. For this we take a recourse to Pearson's frequency curves under Theory of Distributions in Statistics. A typical frequency curve is generally bell shaped and rises from a low frequency to a maximum frequency and then again to a low frequency as the variate increases. Pearson proposed the differential equation

$$F'(x) = \frac{(x-a)\,F(x)}{b_0 + b_1 x + b_2 x^2} \qquad (60)$$

for the bell shaped frequency curve where $a$, $b_0$, $b_1$, $b_2$ constants to be determined / specified. A large number of distributions could be derived from equation (60) (Kendall and Stuart ,1963; Gupta and Kapur, 1994).

The profile of PD in the TC field or the graph of $H(r)$ as defined in Eqn. (2), as $r$ varies from 0 to$\infty$ is depicted in Fig. 2. If the graph of Fig. 2 is extended leftwards for $r$ varying from 0 to $-\infty$, a symmetric curve with the vertical line as line of symmetry results. As such, $H(r)$ can be modelled from the equation (60). As the mode of the distribution is at $x = a$, we get $a = 0$. Thus Eqn. (60) reduces to

$$F'(x) = \frac{-x\,F(x)}{b_0 + b_1 x + b_2 x^2}$$

Retaining all three constants $b_0$, $b_1$, $b_2$ would lead to large number of functional forms of $F(x)$ some of which are too complicated. Taking $b_2 = 0$, $b_0$ & $b_1 \neq 0$ would lead to $F(x)$ assuming a functional form known as Gamma function (Kendall and Stuart, 1963). This function though can be used to build up a TC model is not tried in this study. Taking $b_1$=0, $b_0$, $b_2 > 0$, we get

$$F'(x) = \frac{-x\,F(x)}{b_0 + b_2 x^2} \qquad (61)$$

The function $F(x)$ satisfying the above equation will be the functional from which the idealised pressure model would be derived.

Integrating Eqn. (61), we obtain the general solution

$$F(x) = \frac{c}{(b_0 + b_2 x^2)^{1/2b_2}}$$

where $c$ is a constant and so

$$H(x) = \frac{c\Delta P}{(b_0 + b_2 x^2)^{1/2b_2}}$$

Thus $H(x)$ has three constants $b_0$, $b_2$ and $c$. The condition that $H(0) = \Delta P$ , eliminates one constant. Therefore without loss of generality, we can express $H(x)$ as

$$H(x) = \frac{\Delta P}{(1 + ax^2)^b} \qquad (62)$$

It is easily seen that the above function is exactly the same as the general model suggested by Bretschneider (1982) except that we have derived it through a logical concept. We now want to further generalise Eqn. (62) by substituting the exponent $x^n$ instead of $x^2$ where $n$ is an exponential and would be assigned a suitable value. Therefore we get

$$H(x) = \frac{\Delta P}{(1 + ax^n)^b} \qquad (63)$$

Thus Eqn. (63) is a specific case of Eqn. (62) and that the graphs of both have the same shape. The substitution of $x^n$ in lieu of $x^2$ has made the model more versatile and general. This returns us back to Sec. 4 and Eqn. (20).

It may be pointed out that there could be a few well defined pressure models to model MSLP in a TC field, which may not be derived from Pearson's equations. It also must be stated that we have not made use of the most general form of the Pearson's equation of Eqn. (60) but, have assumed a specific form of Eqn. (60) in Eqn. (61) to derive the pressure model. However, what is of importance is that the Pearson's equation, even in its restricted form, is capable of describing the pressure model faithfully and when the appropriate values of variables are assigned can reproduce most of the characteristics of derived parameters in the cyclone regime and so is adequate if not completely exhaustive.

## 11. Assumptions involved in the derivation of the pressure model and in the final estimation of *K*

We can now list forth the various assumptions that have been made in the estimation of the constant K, where $V_m$ = K$\sqrt{\Delta P}$ is the relation between maximum wind speed and pressure defect of a tropical cyclone.

(*i*) The forces acting on an air parcel in a TC regime are pressure gradient force, Coriolis force, centrifugal force, frictional force and the force due to movement of the cyclone.

(*ii*) The pressure gradient is zero at the centre and reaches maximum away from the centre of the TC.

(*iii*) In the neighbourhood of the TC centre, Coriolis force can be ignored and the wind can be derived based on cyclostrophic balance. In the outer region of the TC gradient wind balance is invoked.

(*iv*) For the curve / graph of pressure gradient, the tangent at the centre *viz*., $r = 0$ is the vertical axis and not the horizontal axis.

(*v*) The cumulative surface pressure drop from the centre of the TC to its outer extent is far greater than the value of the same measure in the outer region of the TC.

(*vi*) The quantum of depletion of mass in an annulus from the centre of the TC decreases beyond a certain radius as one moves away from the centre and approaches zero value as the radius becomes very large.

(*vii*) Positive relative vorticity should prevail atleast up to some distance from the centre of the TC. We have assumed that this distance should be atleast $3R$ where $R$ is the RMW.

(*viii*) The absolute vorticity should not become negative in the TC field.

(*ix*) A single mathematical expression for the surface pressure drop can describe the pressure profile in the entire TC field adequately. Further such an SPD profile which is maximum at the centre and decreases to near zero as one moves away from the centre to the periphery, could be modelled through the Pearson's differential equation.

All the above assumptions that we have made must be considered very reasonable and can be accepted. Some of the assumptions may have been based on observational data gathered over a long period of time from several cyclone basins, but, the derivation of K in this study has not been based on a specific set of observations and so can be considered theoretical.

## 12. A few remarks on the assumptions made

The derivation of the proportionality constant K has been carried out based on several concepts and assumptions as listed in Sec. 11. The idealised pressure model of a TC with the appropriate values of $n$ and $b$ should be capable of generating realistic profiles of pressure, pressure gradient, relative and absolute vorticity. Though not done in this study, it is possible to derive profiles of a few other measures such as absolute angular momentum, kinetic energy, advection of vorticity etc. Whether profiles of such measures based on the pressure model would turn out to be realistic as well and if so, whether a still better approximation of K could be obtained based on such profiles is an area that could be further explored. However, by and large, the measures already considered in the study appear to be adequate to achieve our objective.

Whether a single and same function can approximate the pressure profile of a TC in its entire field is another question that gets raised. Willoughby (1996) has made the observations that (*i*) Idealised pressure/wind profiles do not portray multiple wind maxima (which may be observed in actual specific cases) and (*ii*) in several cases of idealised pressure profiles, a single parameter determines the entire shape which is unrealistic.

In the pressure model assumed in the study, we have introduced two independent variables $n$ and $b$ thus providing more flexibility to the model. Such a model is expected to generate realistic profiles of the various parameters, better than a single parameter model. An idealised model may not generate each and every observed feature of the tropical cyclone. But as often done in meteorological research, such models are frequently assumed to begin with.

It must also be stated that aside from the derivation of profiles of measures such as pressure, pressure gradient, vorticity etc. and using certain properties of such profiles not much physics has gone into the present derivation. It is well known that distribution of temperature, moisture, convection etc. play important role in generating the internal structure of a TC both horizontal and vertical. The frictional forces within the boundary layer also play considerable role in generating cross isobaric flow without which convergence and vertical velocity would not get generated within the cyclone field. The deviation from gradient wind and the presence of super gradient wind speeds near the centre has been discussed in Anthes (1982). The formation and maintenance of eye at the centre is another important feature of an intense tropical cyclone. The idealised pressure model we have used in this study and the profiles of measures that we have derived alone do not obviously explain the physical processes behind the formation and presence of features such as eye, convective ring, secondary circulation, multiple wind maxima etc. in the TC regime.

However, to achieve our limited objective of deriving the proportionality constant from an idealised pressure profile, such detailed physical considerations perhaps appear superfluous. Basic measures such as pressure, wind, vorticity are manifestations of the various physical processes that take place within a cyclone field. It is reasonable to state that by invoking these and by using the various properties of such profiles into our derivations we have been able to bypass the numerous physical processes that are responsible for the formation, intensification, translation, dissipation of the TC and the development and maintenance of the various features associated with its three dimensional structure.

## 13. Concluding remarks and summary of results

The maximum wind in a tropical cyclone field is approximately linearly related to the square root of the pressure defect, the relation being $V_m = K\sqrt{\Delta P}$ . The value of K has been estimated in the literature from independent observations of $V_m$ and $\Delta P$. A large number of pressure models to model the surface pressure in a cyclone field are available in the literature, but it appears that constants of the models were assigned values such that the models generated acceptable and known values of K, which have already been empirically derived.

In this paper, we have handled the problem of determination of K from an entirely different angle. Several well known and reasonable assumptions have been made but at the same time the derivation did not utilise any specific set of observations of $V_m$ and $\Delta P$ and so was not empirical. It has been by and large based on theoretical considerations only. The theoretical derivation has been able to yield a K value of 11.0 which has been fairly close to the K values derived from empirical considerations based on observations taken from different basins such as north Indian Ocean, Pacific and Atlantic. To the best of knowledge of the author a similar type of attempt on derivation of K is not found in the literature. It can be argued that derivation based on actual values represent the reality which is what is needed. However, a theoretical derivation based on sound principles has its own merits and is also intrinsically attractive. That an empirical relation which has by and large stood the test of time could be derived through theoretical means testify to the robustness of the former and at the same time the sound assumptions based on which the latter has been derived.

The results of the study are summarised below:

(*i*) The determination of proportionality constant K between the pressure defect and maximum wind of a tropical cyclone, as carried out in various studies has been reviewed in detail.

(*ii*) Based on a generalised idealised surface pressure model derived from Pearson's distribution theory, the value of K has been theoretically estimated. Several well known properties of TC involving radius of maximum pressure gradient, validity of cyclostrophic assumption near the centre of the TC, existence of positive relative vorticity upto some distance from the centre and the concept of convergence of cumulative surface pressure anomaly in a TC regime have been invoked in the derivation.

(*iii*) Several cyclone measures such as radius of maximum pressure gradient, maximum wind, relative and absolute vorticities and related parameters were computed for different values of radius of maximum wind, pressure defect and latitude and their variation studied before the determination of K.

(*iv*) The final value of K has been estimated as 11.0 after incorporating effect of friction, environmental flow and direction of motion of the TC.

(*v*) The profiles of relative vorticity and convergence of cumulative pressure drop in respect of several other TC pressure models found in the literature have been derived and discussed.

(*vi*) Assumptions invoked in the derivation of the proportionality constant K have been listed and discussed in detail.

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