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Derivation of the Gaussian plume model in three dimensions

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सार – गाउसीय प्लम मॉडल एक सामान्य मॉडल है जिसके द्वारा अभिवहन विसरण समीकरण का अध्ययन किया गया है। सतत भंवर विसरणशीलता और पवन गति घात नियम का विवेचन करते हुए लाप्लास रूपांतरण के उपयोग द्वारा तीन आयामों में इसे हल किया गया है। क्रॉस पवन समाकलित सांद्रता को प्राप्त करने के लिए इरविन, पावर लॉ, ब्रिगस और मानक पद्धतियों जैसी विभिन्न योजनाओं का उपयोग किया गया है। इस शोध में यह पता करने के लिए सांख्यिकीय उपायों का उपयोग किया गया है कि कोपनहेगन, डेनमार्क से प्राप्त किए गए प्रेक्षित सांद्रता के साथ बेहत्तर ढंग से मेल खाने वाली कौन सी सर्वोत्तम योजना है। इस मॉडल से प्राप्त किए गए परिणामों की तुलना प्रेक्षित किए गए आँकड़ों के साथ की गई है।

ABSTRACT. Gaussian plume model is a common model to study advection diffusion equation which is solved in three dimensions by using Laplace transformation considering constant eddy diffusivity and wind speed power law. Different schemes such as Irwin, Power Law, Briggs and Standard methods are used to obtain crosswind integrated concentration. Statistical measures are used in this paper to know which is the best scheme which agrees with the observed concentration data obtained from Copenhagen, Denmark. The results of model are compared with observed data.

Key words – Gaussian-plume model, Dispersion parameter scheme, Downwind distances, Copenhagen.

1. Introduction

Our preoccupation about air pollution is a consequence of the explicit evidence that air contaminants adversely affect the health and the welfare of human beings. Air contaminants concentration affects the health of humans and animals; damage vegetation and materials; reduces visibility and solar radiation; and affects weather and climate (Arya, 1999). The study and the employment of operational short-range atmospheric dispersion models for environmental impact assessment have demonstrated to be of large use in the evaluation of ecosystems perturbation in many distinct scales (Meyer, *et al*., 2007). Therefore, short-range atmospheric dispersion models, including the physical description of the Planetary Boundary Layer (PBL), are fundamental tools to evaluate the noxious effect of air pollutants on human health and on urban and agricultural environments (Gokhale, *et al*., 2004). Generally, such air quality short-range models can be useful in predicting contaminants concentration magnitudes in atmospheric boundary layer generated by different forcing mechanisms and consequently distinct degrees of complexity.

An atmospheric dispersion model predicts downwind airborne concentrations of contaminants from a known release (the release location, altitude, amount of agent released, and meteorological data). Gaussian plume model is an example (Hanna, *et al*., 1982).

Gaussian-plume models are widely used, well understood, easy to apply, and until more recently have received international approval. Even today, from a regulatory point of view ease of application and consistency between applications is important. Also, the assumptions, errors and uncertainties of these models are generally well understood, although they still suffer from misuse.

Gaussian-plume models play a major role in the regulatory arena. However, they may not always be the best models to use and it was noted at the 15th International Clean Air Conference 2000 – Modeling Workshop that particular models are not always chosen on an objective scientific basis (Ross, 2001). Gaussian-plume models are generally applicable when:

(*i*) The pollutants are chemically inert, a simple first-order mechanism is appropriate, or the chemistry may be carried out as a post-processing step.

(*ii*) The terrain is not steep or complex.

(*iii*) The meteorology may be considered uniform spatially.

(*iv*) There are few periods of calm or light winds.

A careful choice of Gaussian-plume model is needed if the effects of deposition, chemistry or fumigation need to be simulated (Ministry for the Environment, 2004).

Laplace transformation technique has been used to get desired solutions. In addition to this method, Hankel transform method, Airs moment method, perturbation approach, method using Green's function, superposition method have also been used to get the analytical solutions of the advection–diffusion equations in one, two and three dimensions. But Laplace transformation technique has been commonly used because of being simpler than other methods and the analytical solutions using this method being more reliable in verifying the numerical solutions in terms of the accuracy and the stability (Kumar *et al.*, 2009).

2. Mathematical model

Under steady state dispersion from a line source neglecting diffusion along *x*-axis as compared to advection and the wind speed *u*, and considering wind speed and eddy diffusivity are constants then the equation of diffusion take the form:

$$u\frac{\partial C}{\partial x} = K\frac{\partial^2 C}{\partial y^2} + K\frac{\partial^2 C}{\partial z^2} \tag{1}$$

where u is the wind speed; (m/s)

C is the concentration; (g/m^3) or (Bq/m^3)

K is the constant eddy diffusivity and x, y, z are the Cartesian coordinates.

Equation (1) is solved under the boundary conditions as follows:

(a) Mass continuity exists

$$C\left(0,y,z\right) = Q\delta\left(y\right)\ \delta\left(z-H\right)$$

where, Q is the emission rate and δ is Dirac delta function.

(b) $C\left(x,y,z\right) = 0$ at $x, y, z \to \infty$

(c) Flux at ground equals zero $K\frac{\partial C}{\partial z}\left(x,y,0\right) = 0$.

Equation (1) can be written in the form:

(*i*) $$u\frac{\partial C(x,y)}{\partial x} = K\frac{\partial^2 C(x,y)}{\partial y^2} \qquad 0 \le x < \infty, -\infty < y < \infty$$

where, (*i*a) $c(0, y) = \delta(y)$

(*i*b) $c(x, y) = 0$ at $x \to \infty$, $y \to \pm\infty$

(*ii*) $$u\frac{\partial C(x,z)}{\partial x} = K\frac{\partial^2 C(x,z)}{\partial z^2}$$

where, (*ii*a) $c(0, z) = \delta(z - H)$

(*ii*b) $\frac{\partial C}{\partial z}(x,z) = 0$ at $z = 0, H$

H is the mixing height.

Then the general solution is in the form:

$$C(x,y,z) = \frac{Q}{u}C(x,y)C(x,z) \tag{2}$$

First taking Laplace Transform for partial differential equation (*i*) with respect to *x*, we get

$$um\hat{c} - uc(0,y) = K\frac{\partial^2 \hat{C}}{\partial y^2}$$

Since $\hat{c}(m,y) = L[c(x,y)] = \int_0^\infty e^{-mx}c(x,y)\mathrm{d}x$

Where "m" is the transform variable. Applying the condition (*i*a) c(0, *y*) = δ (*y*), we obtain:

$$\frac{\partial^2 \hat{C}}{\partial y^2} - \frac{u}{K} m\hat{C} = -\frac{u}{K}\delta(y) \quad (3)$$

Taking Laplace transform in *y* we get the formula:

$$l^2\hat{\hat{c}}(m,l) - l\hat{C}(m,0) - \frac{\partial \hat{C}(s,0)}{\partial y} - \frac{u}{k} s\hat{\hat{C}}(s,l) = -\frac{u}{k}\int_0^\infty e^{-ly}\delta(y)dy = -\frac{u}{k}$$

where,

$\hat{\hat{C}}(s,l) = \int_0^\infty e^{-ly}\hat{C}(s,y)dy; 1$ is the transform variable. We restrict ourselves to value 0 ≤ y ≤ ∞ for the symmetry over the entire range -∞ < y < ∞ equation (3) becomes:

$$\left(l^2 - \frac{u}{K}m\right)\hat{\hat{c}}(m,l) = l\hat{c}(m,0) + \frac{\partial \hat{c}(m,0)}{\partial y} - \frac{u}{k}$$

$$\therefore \hat{\hat{c}}(m,l) = \frac{l\hat{c}(m,0) + \dfrac{\partial \hat{c}(m,0)}{\partial y} - \dfrac{u}{k}}{l^2 - \dfrac{u}{K}m} = \frac{lp_1 - p_2}{l^2 - \dfrac{u}{K}m}$$

where $p_1 = \hat{c}(m,0)$, $p_2 = \dfrac{\partial \hat{C}(m,0)}{\partial y} - \dfrac{u}{K}$, let um/k=s.

Taking the inverse in *y*, we obtain:

$$\hat{c}(s,y) = p_1 \cosh\sqrt{s}y - \frac{p_2}{\sqrt{s}}\sinh\sqrt{s}y$$

$$\hat{C}(s,y) = \frac{p_1}{2}\left(e^{\sqrt{s}y} + e^{-\sqrt{s}y}\right) - \frac{p_2}{2\sqrt{s}}\left(e^{\sqrt{s}y} - e^{-\sqrt{s}y}\right)$$

Using the condition (ib) we get:

$$0 = \frac{p_1}{2}e^{\sqrt{s}y} - \frac{p_2}{2\sqrt{s}}e^{\sqrt{s}y} = \left(\frac{p_1}{2} - \frac{p_2}{2\sqrt{s}}\right)e^{\sqrt{s}y} \Rightarrow p_1 = \frac{p_2}{\sqrt{s}}$$

Then,

$$\hat{C}(s,y) = \frac{p_2}{2\sqrt{s}}\left(e^{\sqrt{s}y} + e^{-\sqrt{s}y}\right) - \frac{p_2}{2\sqrt{s}}\left(e^{\sqrt{s}y} - e^{-\sqrt{s}y}\right)$$

$$\therefore \hat{c}(s,y) = \frac{p_2}{\sqrt{s}}e^{-\sqrt{s}y}$$

Also taking the inverse in s (assuming that p_2 is independent of *s*), we get:

$$c(x,y) = \frac{p_2}{\sqrt{\pi x}}e^{-\frac{y^2}{4x}}$$

Using the condition $(0,y) = \delta(y)$

$$\therefore c(0,y) = \delta(y) = \frac{p_2}{\sigma_y\sqrt{\pi}}\lim_{x\to\infty} e^{\frac{-y^2}{2\sigma_y^2}}\sigma_y\sqrt{4\pi}$$

$$\Rightarrow 2p_2 = 1 \Rightarrow p_2 = \frac{1}{2}$$

$$c(x,y) = \frac{e^{\frac{-y^2}{2\sigma_y^2}}}{\sigma_y\sqrt{2\pi}} \quad (4)$$

Taking Laplace transformation (ii) with respect to *x* we get:

$$\frac{\partial^2 \hat{c}}{\partial z^2} - \frac{u}{K}f\hat{c} + \frac{u}{K}\hat{c}(0,z) = 0$$

Using the condition (iia), we obtain as follows:

$$\frac{\partial^2 \hat{c}}{\partial z^2} - \frac{u}{K}f\hat{c} = -\frac{u}{K}\delta(z-H)$$

Taking Laplace transform in *z* we get:

$$m^2\hat{\hat{c}}(s,m) - m\hat{c}(f,0) - \frac{\partial \hat{c}(f,0)}{\partial z} - uf\hat{\hat{c}}(f,m) = -\frac{u}{K}e^{-mH}$$

Let uf/K=s, Appling condition (iib) where $\dfrac{\partial \hat{C}(s,0)}{\partial z} = 0$,

we obtain that,

$$\left(m^2 - us\right)\hat{\hat{c}}(s,m) = -m\hat{C}(s,0) - e^{-mH}$$

TABLE 1

Values of wind speed at 10 m and 115 m and downwind distance through unstable and neutral stabilities in northern part of Copenhagen

Run no.	Stability	*U* 10 (m.sec^{-1})	*U* 115(m.sec^{-1})	Downwind distance (m)
1	Very unstable (A)	2.1	3.029172	1900
1	Very unstable (A)	2.1	3.029172	3700
2	Slightly unstable (C)	4.9	7.986117	2100
2	Slightly unstable (C)	4.9	7.986117	4200
3	Moderately unstable (B)	2.4	3.461911	1900
3	Moderately unstable (B)	2.4	3.461911	3700
3	Moderately unstable (B)	2.4	3.461911	5400
4	Slightly unstable (C)	2.5	4.074549	4000
5	Slightly unstable (C)	3.1	5.052441	2100
5	Slightly unstable (C)	3.1	5.052441	4200
5	Slightly unstable (C)	3.1	5.052441	6100
6	Slightly unstable (C)	7.2	11.7347	2000
6	Slightly unstable (C)	7.2	11.7347	4200
6	Slightly unstable (C)	7.2	11.7347	5900
7	Moderately unstable (B)	4.1	5.914098	2000
7	Moderately unstable (B)	4.1	5.914098	4100
7	Moderately unstable (B)	4.1	5.914098	5300
8	Neutral (D)	4.2	7.734349	1900
8	Neutral (D)	4.2	7.734349	3600
8	Neutral (D)	4.2	7.734349	5300
9	Slightly unstable (C)	5.1	8.312081	2100
9	Slightly unstable (C)	5.1	8.312081	4200
9	Slightly unstable (C)	5.1	8.312081	6000

$$\therefore \hat{c}(s,m) = \frac{m\hat{c}(s,0) - e^{-4mH}}{(m^2 - us)}$$

Applying the inverse in *m*, we get:

$$\hat{c}(s,z) = \hat{c}(s,0)\cos h\sqrt{s}z - \frac{1}{\sqrt{s}}\sinh\sqrt{s}(z-H)$$

$$\hat{c}(s,z) = \frac{u\hat{c}(s,0)}{2}\left(e^{\sqrt{s}z} + e^{-\sqrt{s}z}\right) - \frac{1}{2\sqrt{s}}\left(e^{\sqrt{s}(z-H)} - e^{-\sqrt{s}(z+H)}\right)$$

Since $\hat{c}(s,z) \to 0 \,\text{as}\, z \to \infty$ we get:

$$\left[\frac{\hat{c}(s,0)}{e^{-\sqrt{s}H}} - \frac{1}{\sqrt{s}}\right]\frac{e^{\sqrt{s}(z-H)}}{2} = 0 \Rightarrow \hat{c}(s,0) = \frac{e^{-\sqrt{s}H}}{\sqrt{s}} \text{ then:}$$

$$\hat{c}(s,z) = \frac{1}{2\sqrt{s}}\left[e^{\sqrt{s}(z-H)} + e^{-\sqrt{s}(z-H)} - e^{\sqrt{s}(z-H)} + e^{-\sqrt{s}(z+H)}\right]$$

$$\hat{c}(s,z) = \frac{1}{2\sqrt{s}}\left(e^{-\sqrt{s}(z-H)} + e^{-\sqrt{s}(z+H)}\right) \qquad (5)$$

TABLE 2

The values of the standard deviation of the wind direction in lateral and vertical directions for different stability classes

Stability Classes	A	B	C	D	E	F
σ_θ (deg)	25	20	15	10	5	2.5
σ_ϕ (deg)	10	8	6.5	5.5	2.5	1

Finally applying the inverse in *s* yield

$$C(x,z)=\frac{1}{\sigma_z\sqrt{2\pi}}\left[e^{\frac{-(z-H)^2}{2\sigma_z^2}}+e^{\frac{-(z+H)^2}{2\sigma_z^2}}\right]$$

Substituting from (4) and (5) in (2) we obtain the solution in the form:

$$C(x,y,z)=\frac{Q}{2\pi\sigma_y\sigma_z}e^{\frac{-y^2}{2\sigma_z^2}}\left[e^{\frac{-(z-H)^2}{2\sigma_z^2}}+e^{\frac{-(z+H)^2}{2\sigma_z^2}}\right] \quad (6)$$

3. Case study

The data set used was observed from the atmospheric diffusion experiments conducted at the northern part of Copenhagen, Denmark, under unstable conditions (Gryning and Lyck, 1984; Gryning *et al*., 1987). The tracer, sulfur hexafluoride (SF_6) was released from a tower at a height of 115 m without buoyancy. The values of different parameters such as stability, wind speed at 10 m. (*U*10), wind speed at 115m (*U*115), and downwind distances are different at the same stability classes because of the measuring concentrations of SF_6 at different downwind distances through the same stability classes during the experiment (Table 1).

4. Dispersion parameters schemes

Since the Gaussian plume model is expressed in terms of the dispersion parameters σ_y and σ_z, appropriate selection of lateral and vertical dispersion parameters is much targeted. We select four different methods namely, Irwin, Power-Law, Brigg's and Standard method, for calculating σ_y and σ_z to select the most accurate one.

4.1. *Irwin method*

Irwin (Irwin, 1983) proposed the standard deviations of the lateral and vertical crosswind concentration distribution of pollutant σ_y and σ_z respectively, as follows:

$$\sigma_y(x)=\sigma_v t f_y \quad \text{and} \quad (7)$$

$$\sigma_z(x)=\sigma_w t f_z \quad (8)$$

where, *t* is the travel time of the pollutant (sec) and equals to $t = x/\text{U115}$

f_y and f_z are non-dimensional function of travel time and are given by Irwin (Irwin, 1983) as,

$$f_y=\frac{1}{1+0.9\sqrt{\frac{t}{1000}}}, \quad f_z = 1 \text{ for unstable condition} \quad (9)$$

$$\text{and } f_z=\frac{1}{1+0.9\sqrt{\frac{t}{50}}} \text{ for stable condition.} \quad (10)$$

σ_v and σ_w are the standard deviation of the wind speed in the lateral and vertical directions respectively. For small angles they can give as,

$$\sigma_v = \sigma_\theta . U_{115} \quad (11)$$

and

$$\sigma_w = \sigma_\phi U_{115} \quad (12)$$

where σ_θ and σ_ϕ are the standard deviations of the wind direction in lateral and vertical, respectively. Specifications of σ_θ and σ_ϕ can be found in Gifford (1976) and Hanna (1982). Based on the Pasquill stability classes from A to F, they are given in Table 2.

So the values σ_y and σ_z are obtained by the following equations,

$$\sigma_y(x)=\frac{\sigma_\theta x}{1+0.9\sqrt{\frac{x}{1000U_{115}}}} \text{ For stable and unstable conditions,} \quad (13)$$

TABLE 3

Brookhaven National Laboratory Parameters

Stability Classes	Moderately unstable (B1)	Slightly unstable (B2)	Neutral (D)	Moderately stable (F)
a	0.36	0.40	0.32	0.31
b	0.86	0.91	0.78	0.71
c	0.33	0.41	0.22	0.06
d	0.86	0.91	0.78	0.71

TABLE 4

Formulas produced by Briggs (1973) for σ*y* (*x*) and σ*z* (*x*) (102 < x < 104 m)

Stability Classes	Very unstable (A)	Moderately unstable (B)	Slightly unstable (C)	Neutral (D)	Slightly stable (E)	Moderately stable (F)
σ_y (x)	$0.32x(1+0.0004x)^{-1/2}$	$0.32x(1+0.0004x)^{-1/2}$	$0.22x(1+0.0004x)^{-1/2}$	$0.16x(1+0.0004x)^{-1/2}$	$0.11x(1+0.0004x)^{-1/2}$	$0.11x(1+0.0004x)^{-1/2}$
σ_z (x)	$0.24x(1+0.001x)^{1/2}$	$0.24x(1+0.001x)^{1/2}$	0.20x	$0.14x(1+0.0003x)^{-1/2}$	$0.08x(1+0.00015x)^{-1/2}$	$0.08x(1+0.00015x)^{-1/2}$

TABLE 5

Values of the dispersion parameters for the Pasqual stability classes

Stability Classes	Very unstable (A)	Moderately unstable (B)	Slightly unstable (C)	Neutral (D)	Slightly stable (E)	Moderately stable (F)
r (m/km)	250	202	134	78.7	56.6	37
s (m/km)	102	96.2	72.2	47.5	33.5	22
a (km)	0.927	0.370	0.283	0.707	1.07	1.17
p	0.189	0.162	0.134	0.135	0.137	0.134
q	-1.918	-0.101	0.102	0.465	0.624	0.70

$$\sigma_z(x) = \sigma_\phi x \quad \text{For unstable condition,} \qquad (14)$$

and

$$\sigma_z(x) = \frac{\sigma_\phi x}{1 + 0.9\sqrt{\frac{x}{50U_{115}}}} \quad \text{For stable condition} \qquad (15)$$

The final results of normalized crosswind-integrated concentration, Cy/Q (10^{-4} sm^{-2}), after calculating σ_y and σ_z by using Irwin method, are presented in Table 6.

4.2. *Power-Law method*

Smith (1968) worked out analytical Power-Law formulae for σ_y and σ_z to be used easily than using a graph or a table. He used the Brookhaven National Laboratory (BNL) formulas, which are defined by him using wind direction θ recorded over one hour as follows:

A : fluctuations of θ exceed 90°. (Very Unstable conditions)

B_1 : fluctuations of θ from 40 to 90°. (Moderately Unstable)

TABLE 6

The comparison between observed, at ground in diffusion experiment in northern part of Copenhagen, and different calculated crosswind-integrated concentrations Cy/Q (10^{-4} m^{-2}) obtained from the used dispersion schemes in the case of Gaussian

Distance (*x*) (m)	Stability	Observed (C/Q)	Calculated Irwin (C/Q)	Calculated Power-Law (C/Q)	Calculated Briggs (C/Q)	Calculated Standard (C/Q)
1900	A	6.48	2.54	4.67	6.70	1.1E-07
3700	A	2.31	1.31	1.68	2.73	1.6E-08
2100	C	5.38	9.33	6.03	4.57	2.086
4200	C	2.95	4.67	2.84	2.36	1.119
1900	B	8.2	3.63	4.09	5.86	0.127
3700	B	6.22	1.87	1.47	2.39	0.061
5400	B	4.3	1.28	0.82	1.40	0.040
4000	C	11.7	2.50	5.87	1.82	0.597
2100	C	6.72	5.91	9.51	7.22	1.320
4200	C	5.84	2.95	4.48	3.72	0.708
6100	C	4.97	2.03	2.92	2.58	0.506
2000	C	3.96	14.40	4.31	3.26	3.202
4200	C	2.22	6.86	1.93	1.60	1.645
5900	C	1.33	4.88	1.31	1.15	1.212
2000	B	6.7	5.90	2.21	1.40	0.206
4100	B	3.25	2.88	0.74	3.01	0.093
5300	B	2.23	2.23	0.50	1.17	0.070
1900	D	4.16	11.81	10.62	9.93	53.703
3600	D	2.02	6.23	8.25	7.64	38.173
5300	D	1.52	4.23	6.18	6.39	31.044
2100	C	4.58	9.72	5.79	4.40	2.171
4200	C	3.11	4.86	2.73	2.26	1.165
6000	C	2.59	3.40	1.81	1.59	0.846

TABLE 7

Comparison among different models according to standard statistical performance measure

Case	Models	NMSE	FB	COR	FAC2
Gaussian model	Irwin method	0.74	-0.02	-0.10	1.39
	Power-Law method	0.54	0.21	0.32	1.02
	Briggs method	0.68	0.19	0.09	1.06
	Standard method	6.80	0.13	-0.35	2.01

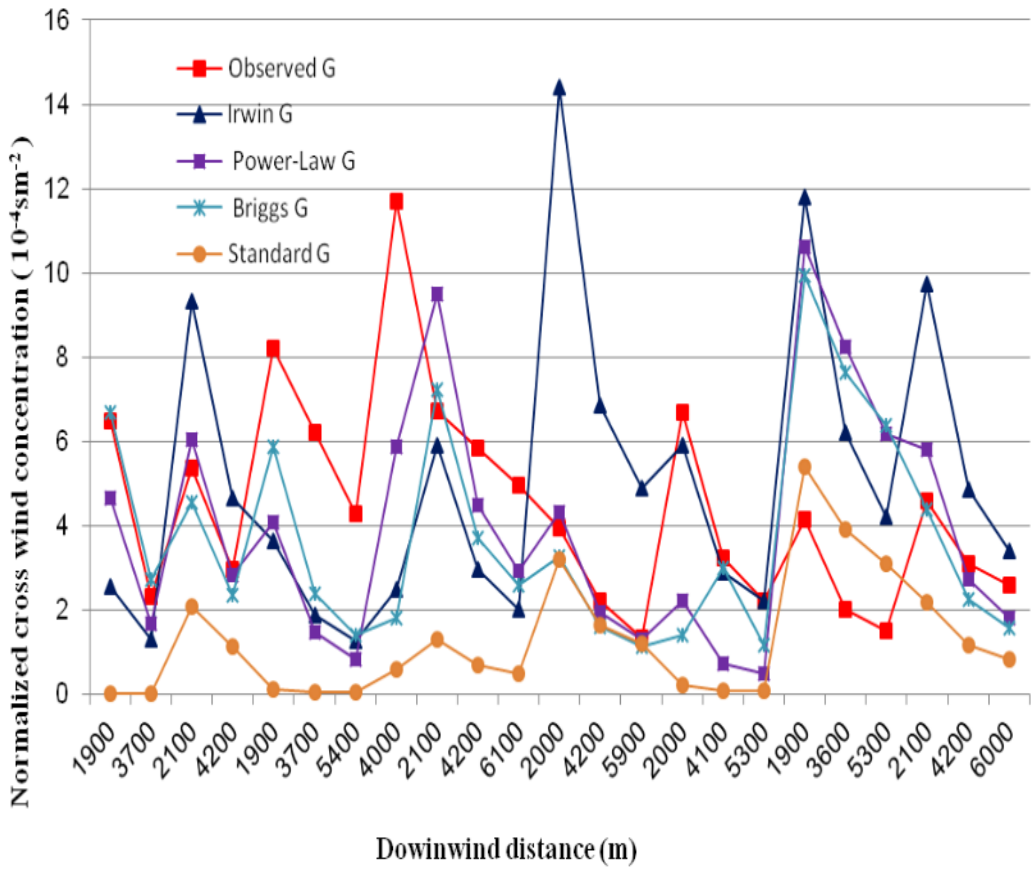

Fig. 1. The relation between the downwind distances and the observed and predicted concentrations of SF_6 in Gaussian

B_2 : fluctuations of θ from 15 to 40°. (Slightly Unstable)

C : fluctuations of θ greater than 15° with strip chart showing an unbroken solid core in the trace. (Neutral)

D : Trace in a line, short-term fluctuations in θ less than 15° (Moderately stable).

He summarized the BNL formulas which are based on hourly average measurements of diffusion to about 10 km of a no buoyant plume released from a height of 108 m:

$$\sigma_y = ax^b \quad (16)$$

$$\sigma_z = cx^d \quad (17)$$

Values of the parameters a, b, c, and d are given in Table 3.

Because of the absence of the Very Unstable condition in the solution of Smith (1968), here we use values of the Moderately Unstable condition parameters to calculate cases of the Very Unstable condition. The final results of crosswind-integrated concentration Cy/Q (10^{-4} sm^{-2}), after calculating σ_y and σ_z by using Power-Law method, are presented in Table 6.

4.3. *Briggs method*

Briggs (1973) used theoretical concepts of the related formulas to get set of formulas that can be used in common practices. According to these formulas σ_y and σ_z is proportional to *x* at all stability conditions. Also σ_y and σ_z are independent of release height and roughness in these formulas. The values of σ_y and σ_z in urban conditions are given in Table 4.

The final results of crosswind-integrated concentration Cy/Q (10^{-4} sm^{-2}) after calculating σ_y and σ_z by using Briggs method are presented in Table 6.

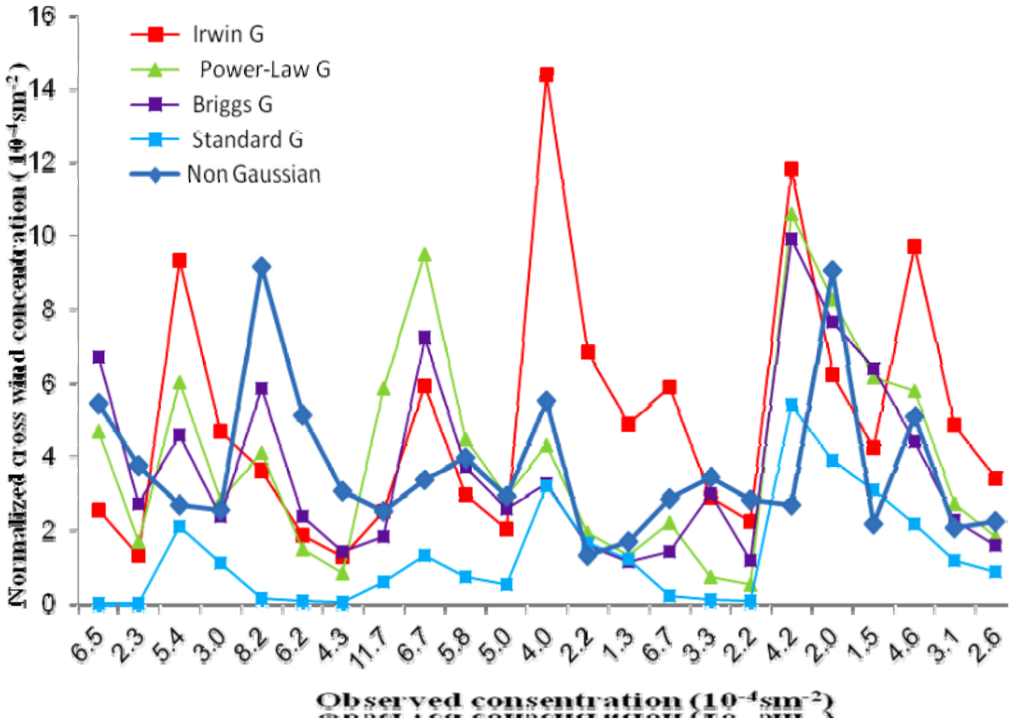

Fig. 2. Relation between observed and calculated concentrations for different Gaussian models

4.4. *Standard method*

In this method, σ_y and σ_z can be analytically expressed, based on (P-G) curves, using the following forms:

$$\sigma_y = \frac{r\,x}{\left(1+\frac{x}{a}\right)^p} \quad \text{and} \tag{18}$$

$$\sigma_z = \frac{s\,x}{\left(1+\frac{x}{a}\right)^q} \tag{19}$$

where r, s, a, p, and q are constants depending on the atmospheric stability. Their values are given in Table 5.

The final results of crosswind-integrated concentration Cy/Q (10^{-4} sm^{-2}) after calculating σ_y and σ_z as estimated using Standard method are given in Table 6.

5. Comparison between the used methods

In this section, we compare between the final results obtained using the five different schemes. We look for which is the most optimum method to be used. Fig. 1 shows the relation between the observed and calculated crosswind concentrations of the tracer sulfur hexafluoride (SF_6) with downwind distances from continuous source.

In Fig. 1, we can notice that the observed concentrations line is not nearby any one identified line. So each model has some points near the observed results while the others are not. In the Fig. 2, we plot the normalized crosswind concentrations calculated using different Gaussian models versus the observed concentrations.

In Fig. 2, we can observe that the Standard method graph-line is almost near zero line. But it is still difficult to know which is the most accurate among Gaussian, Power, Briggs, Irwin methods.

5.1. *Statistical method*

Here we try to know which method's results are the nearest to the observed concentrations. So to solve this problem, we have used the following standard statistical performance measures that characterize the agreement between model prediction (C_p = Cpred/Q) and observations (C_o=Cobs/Q) by Willmott (1981).

Normalized Mean Square Error (NMSE)

$$= \frac{\overline{\left(C_p - C_0\right)^2}}{\left(\overline{C_p}\,\overline{C_0}\right)} \qquad (20)$$

$$\text{Fractional Bias (FB)} = \frac{\left(\overline{C_0} - \overline{C_p}\right)}{\left[0.5\left(\overline{C_0} + \overline{C_p}\right)\right]} \qquad (21)$$

Correlation Coefficient (COR)

$$= \frac{1}{N_m} \sum_{i=1}^{N_m} \left(C_{pi} - \overline{C_p}\right) \times \frac{\left(C_{oi} - \overline{C_0}\right)}{\left(\sigma_p \sigma_o\right)} \qquad (22)$$

$$\text{Factor of two (FAC2)} = 0.5 \leq \frac{C_p}{C_o} \leq 2.0 \qquad (23)$$

Where σ_p and σ_o are the standard deviations of Cp and Co respectively. Here the over bars indicate the average over all measurements (Nm). A perfect model would have the following idealized performance:

NMSE = FB = 0 and COR = FAC2 = 1.0

From the statistical method, in the Gaussian model, we find that the Irwin, Power Law and Briggs are factors of 2 with observed data. Regarding NMSE, the mentioned methods can be considered as good models except for standard method which is relatively far away. Irwin method is the best relating to FB, while the Power Law and Standard method have the best correlation with observed data (Table 7).

6. Conclusion

Gaussian plume model of advection diffusion equation is solved in three dimensions by using Laplace transformation considering wind speed and eddy diffusivity are constants. Different schemes such as Irwin, power law, Briggs, and Standard method are used to obtain crosswind integrated concentration. Also wind speed in Power Law, plume rise are used in this work. We used observed Normalized concentration data for sulfur hexafluoride (SF_6) from the atmospheric diffusion experiments conducted at the northern part of Copenhagen, Denmark, under unstable conditions to compare with predicted concentration data using different schemes of dispersion parameters.

Each model has some points near the observed results while the others are not. It is difficult to know which is the most accurate among Gaussian, Power, Briggs, Irwin methods.

In the Gaussian model, we find that the Irwin, Power Law and Briggs are factors of two with observed data. Regarding NMSE, the mentioned methods can be considered as good models except for Standard method which is relatively far. Irwin method is the best relating to FB, while the Power Law and Standard methods have the best correlation with observed data.

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