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Simulation of internal waves using GM model and comparison against measurements during ARMEX-I

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सार – गारेट–मंक (इसके बाद से जी. एम.) मॉडल स्पेकट्रम के आधार पर रैखिक आंतरिक तरंग मॉडल तैयार किया गया है। यह मॉडल महासागर में आंतरिक तरंगों के कारण उत्पन्न होने वाली ध्वनि वेग संरचना को अनुकरित करता है। आरमेक्स–I प्रयोग के दौरान संग्रहीत किए गए सी. टी.. डी. योयो (निरंतर ऊपर और नीचे जाना) आँकड़ों का उपयोग मॉडल की प्रमाणिकता जाँचने के लिए किया गया है। प्रयोग के स्थान में आंतरिक तरगों का विस्तार एक मी. की अपेक्षा >1 मी. पाया गया है। ई. ओ. एफ. और मॉडल वियोजन विश्लेषण से पता चलता है कि मॉडल बैंड विस्तार प्राचल $j_* = 3$ और मॉडल ऊर्जा अवमंदन प्राचल $P = 1$ या 2 है और यह जी. एम. द्वारा प्राप्त मानों के अनुरूप रहे है। ध्वनि वेग के मापे गए और अनुकरित प्रोफाइलों की तुलना से यह पता चला है कि जी. एम. मॉडल गहरे समुद्रों में आई. डब्ल्यू. के कारण अधिकांश परिवर्तनशीलता उत्पन्न करने में सक्षम है। प्रेक्षित और अनुकरित क्षोभों के मध्य अच्छा सामंजस्य रहा है।

ABSTRACT. A linear internal wave model based on the Garrett-Munk (hereafter GM) modal spectrum has been developed. The model simulates sound speed structure due to internal waves in the ocean. CTD yoyo (go up and down continuously) data collected during the ARMEX-I experiment has been utilized to validate the model. Internal waves amplitudes at the experimental site are found to be >1m. EOF and modal decomposition analyses reveal that the modal bandwidth parameter $j_* = 3$ and modal energy damping parameter $P = 1$ or 2 and follows the GM suggested values. Comparison of the measured sound speed profiles against the simulated reveals that the GM model is capable of reproducing most of the variability due to IW in the deep-waters. The agreement between the observed and simulated perturbations is quite good.

Key words – ARMEX, Internal wave, GM model, EOF, Simulation, Comparison.

1. Introduction

In a stratified medium, when a fluid parcel is displaced from its equilibrium level, it will experience a restoring force due to reduced gravity and the disturbance propagates in the form of internal waves (IWs). IWs induce vertical motions in the water column and change the temperature and salinity at a fixed depth. Extend of these variations is proportional to the energy of the IWs as well as the density gradient. IWs are one of the most energetic components in the ocean current spectrum. Since the presence of IWs changes the temperature and salinity profile at a given position and time, the sound speed structure also changes accordingly, which in turn effects the acoustic transmission. Despite numerous applications of the IWs, they are still the least studied feature in the seas around India. In order to understand the impact of these waves on sound propagation, a simulation algorithm based on GM model has been developed. In series of papers in the 1970s Garrett and Munk formulated and refined an empirical model of the internal wave modal spectrum (Garrett and Munk 1972, 1975; Munk, 1981). During the ARMEX-I experiment CTD yo-yo data was collected in deep waters, which helped us in validating the model. In this paper, we have carried out simulation of IWs and compared the results with measurement.

2. Observations

During the ARMEX-I experiment apart from all the regular oceanographic measurements, CTD yo-yo data was collected (15° 23.11′ N, 72° 16.18′ E) in the deep-waters. Before collecting the CTD yo-yo data, a vertical CTD profile up to a depth of 1345m has been collected for initializing the model. CTD yo-yo was carried out for 2hr

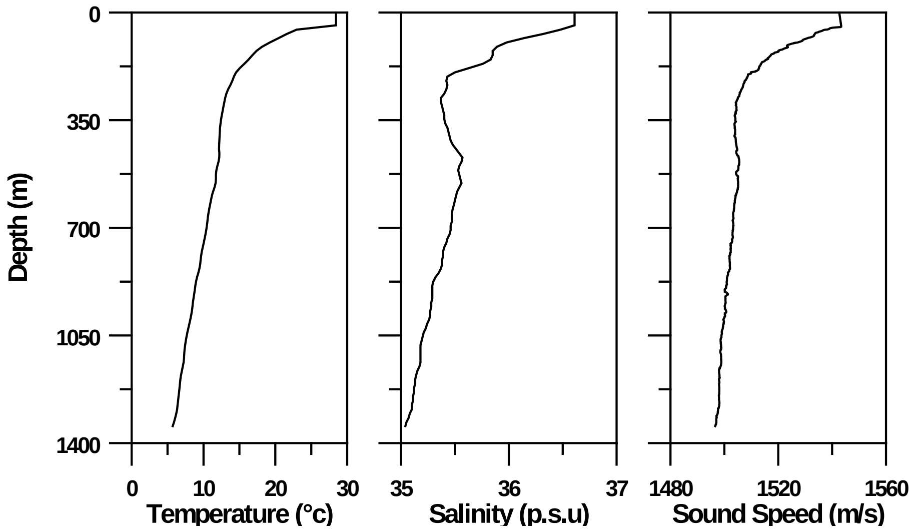

Fig. 1. Vertical profiles of temperature, salinity and sound speed used for the simulations

and 10 min. from 0735 to 0945 at 7 min. interval approximately on 10th August 2002. A total of 18 profiles were collected between 50 and 200 m depth of water column. Sound speed profiles were estimated from the measured CTD profiles. Sound speed profiles were chosen to present, since they account for variations in the temperature as well as salinity profiles. The vertical structure at the observational site is illustrated in Fig. 1. Temperature decreased linearly from a surface maximum of 28.5° C to a value around 5.7° C at 1345 m depth. As far as salinity is concerned it has a maximum value of 36.5 psu at the surface and 35.0 psu at 1345 m depth. Though salinity has well marked variations in its vertical structure, sound speed structure mostly followed the temperature pattern only. Mixed-layer is seen up to 45 m in both temperature as well as salinity followed by a thermocline up to 300 m.

Depth-time section of the sound speed profiles collected during the 2.1 hr CTD yo-yo experiment is shown in Fig. 2. It is apparent from the depth-time section that internal waves of low frequency components are present at the experimental site. In general, iso-sound speed line displacements were oscillatory about their mean vertical position. Also there is an evidence of an upward displacement of the deeper water between 150 m and 200 m.

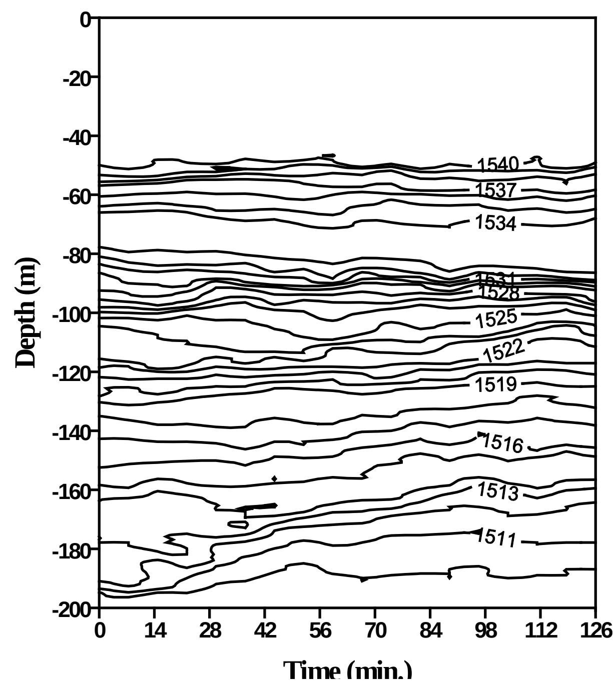

Fig. 2. Sound speed variations as a function of depth and time derived form the 126 min. CTD yo-yo experiment on 10 August 2002

3. Internal wave model

A simple and efficient simulation algorithm for simulating the internal wave field was developed. In this simulation algorithm, the GM spectral model was chosen to estimate the modal amplitude spectrum. The model, which has since been compared to measured data sets, has been shown to be remarkably universal in the deep waters. The following assumptions were made either explicitly or

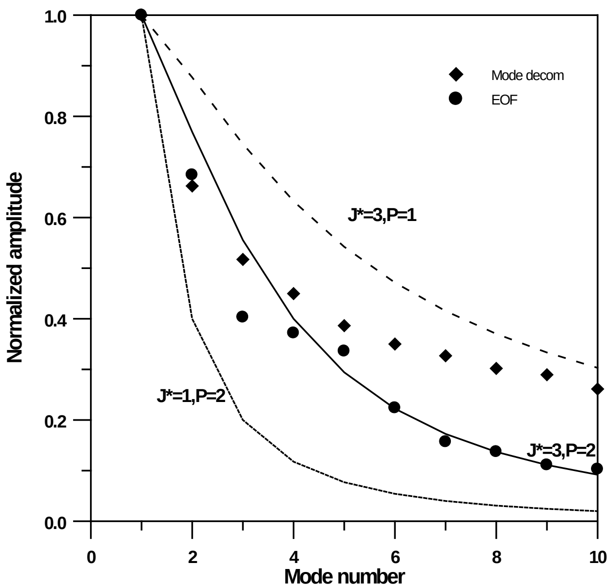

Fig. 3. Mode amplitudes normalized by the first mode amplitude. Results of EOF analysis for the entire data set are indicated by filled circles and results using mode decomposition are indicated by filled diamonds

implicitly in the derivation of the model, linear (wave filed is assumed to be a linear combination of plane waves), no shear (shear is ignored because more complex (Tylor-Goldstien) equation is to be solved for the vertical modal structure), flat (or slowly varying) bottom, density is a function of depth only and a random internal wave field. The vertical displacement of the internal wave field is expressed as a weighted double sum over mode number j and frequency ω of the form

$$\zeta(r,z,t)=\sum_{\omega=f}^{N_{\max}}\sum_{j=1}^{j_{\max}}A(\omega,j)W(\omega,j,z)e^{i[\,k(\omega,j)\,r-\omega t+\psi\,]} \tag{1}$$

where $W(\omega,j,z)$ is the vertical mode structure as a function of frequency, mode number and depth, $A(\omega,j)$ is the zero mean Gaussian random variable with an associated power spectrum defined by GM model, z is depth, r is range, t is time and ψ is phase.

The depth dependent vertical modal function $W(\omega,j,z)$ and the dispersion relation $k(\omega,j)$ satisfy the eigenvalues equation (Gill, 1983).

$$\frac{\mathrm{d}^2}{\mathrm{d}z^2}W(\omega,j,z)+k^2\left[\frac{N^2(z)-\omega^2}{\omega^2-f^2}\right]W(\omega,j,z)=0 \tag{2}$$

subject to the rigid boundary conditions $W(\omega,j,0)=W(\omega,j,H)=0$, where H is the water depth, f is inertial frequency and is given by $f=2\Omega\sin(\text{Latitude})$, Ω represents the earth's angular velocity and $N(z)$ is buoyancy frequency. The buoyancy frequency is given by $N^2(z)=-\frac{g}{\bar{\rho}}\frac{d\rho}{dz}$ where g is acceleration due to gravity, ρ is density and $\bar{\rho}$ is average density.

The method of Saunders and King (1991) was used to specify the amplitudes $A(\omega,j)$, where an expression for the modal amplitude is obtained by equating the horizontal kinetic energy derived from both the internal wave equations and GM equations. This allows the amplitude to be written as

$$A(\omega,j)=\left[2R\int_0^H N(z)\mathrm{d}z.\,B(\omega).\,H(j)\right]^{1/2} \tag{3}$$

where

$$B(\omega)=\frac{2f}{\pi\,\omega\sqrt{\omega^2-f^2}}\qquad\int_f^N B(\omega)\mathrm{d}\omega=1 \tag{4}$$

$$H(j)=\frac{\left(j^2+j_*^2\right)^{-\frac{P}{2}}}{\sum_{j=1}^{\infty}\left(j^2+j_*^2\right)^{-\frac{P}{2}}}\qquad\sum_{j=1}^{\infty}H(j)=1 \tag{5}$$

R, j_* and P are the energy, characteristic mode number and spectral slope parameters. These parameters are to be determined from the data. Usually for deep waters the values suggested by GM are R = 320 m^2.cph (an equivalent of 0.4 J/cm^2), j_* = 3 and P =2. A set of frequencies between the inertial frequency f and the average buoyancy frequency N was selected to represent the continuous frequency spectrum of GM formulation. These frequencies were placed equally on a logarithmic scale between f and N.

When the vertical displacement structure due to internal wave modes has been calculated from an observed buoyancy profile it can be used in a vertical advection model (horizontal advection ignored) to simulate the time dependent temperature and salinity at fixed grid points in the vertical using a numerical scheme

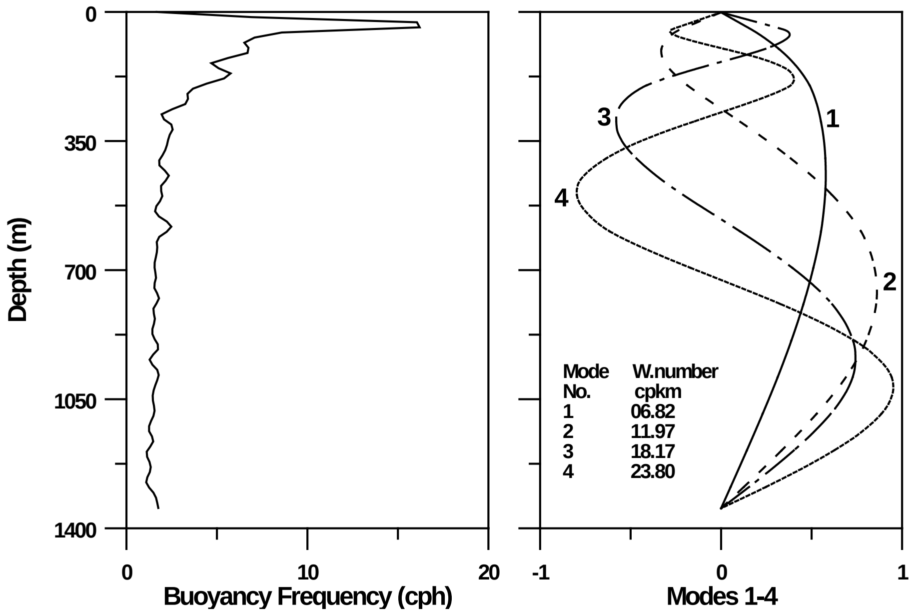

Fig. 4. Profile of the buoyancy frequency (N) and first four internal wave modes [vertical velocity arbitrary scale (m/s)] for the corresponding profile at internal tidal period

based on a particle tracking method suggested by Jackson and Elliott (2002) and Elliott and Jackson (1998). In this method initial temperature and salinity profiles were represented by a set of equally spaced water particles, which were allowed to move in response to the vertical velocity field associated with the internal wave modes. Each particle retains its initial temperature and salinity and the vertical structure of the water column at a future time was found by interpolating particle parameters back on to the initial grid.

3.1. *EOF and modal decomposition analyses*

We have carried out EOF (Empirical Orthogonal Functions) and modal decomposition analyses to estimate the values of j_* and P of GM model (Eqn. 5). The parameters j_* and P determines the energy distribution among different modes. Though GM suggested global values for these parameters they remain to be estimated from the data for the experimental area (since they are space and time specific). Detailed discussions about afore said techniques can be found in Yang and Kwang (1999). In the EOF analysis, first we computed the sound speed covariance matrix for the measured sound speed profiles and then estimated the eigenvalues and eigenvectors of the covariance matrix. Modal decomposition analysis is done by solving the dynamical equations (Eqn. 2) along with the boundary conditions. The ratio of the eigenvalues (mode amplitudes) is a measure of the energy distribution among different modes. We used ratio of mode amplitudes (instead of amplitudes itself) for the comparison, as the ratio is independent of the modal energy. The mode amplitudes were normalized with the first mode amplitude since it has the maximum energy. The measured amplitudes are fitted with the GM model in Fig. 3 with different values of j_* and P. It is apparent from EOF's of the measured data that the GM suggested values for j_* and P (j_* = 3 and P = 2) hold good at the experimental site. However, in the mode decomposition method first five modes follow GM suggested values and later five modes (higher order) favor j_* = 3 and P = 1. We conclude from these analyses that j_* = 3 and P = 1 or 2 provide the best fit for the data set. Energy parameter R could not be estimated since the information on vertical displacements is not available. However, we have used the GM suggested global value for the simulations.

4. Simulations and comparisons

Based on the expressions and procedures given in the previous section, an algorithm was developed (Krishna Kumar and Murthy 2002; Krishna Kumar and Balasubramanian 2003) which can be used both in deep as well as in shallow waters. A total of 16 frequencies (including the internal tide) were placed between the average buoyancy and inertial frequency. The number of modes chosen is 10. In the absence of directional

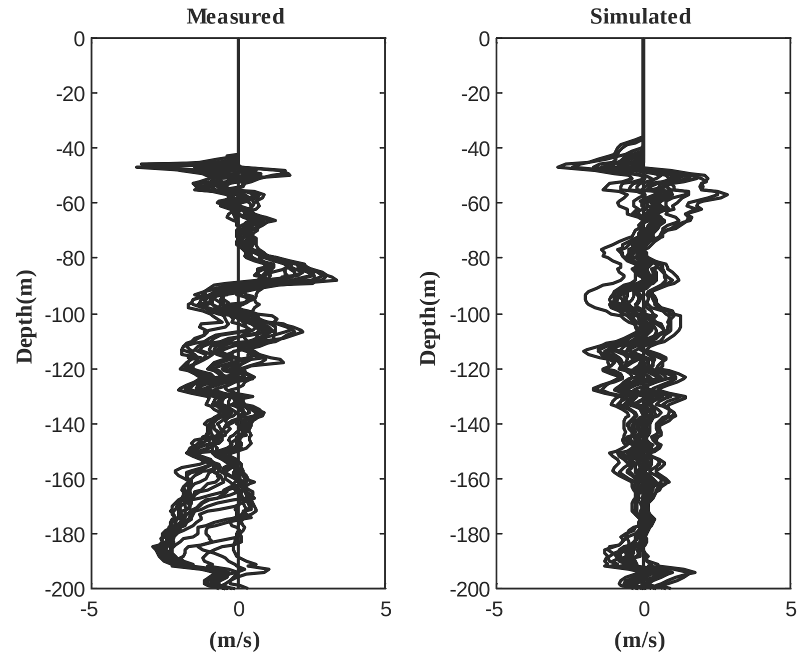

Fig. 5. Comparison between measured and simulated sound speed perturbations for the CTD yo-yo experiment

spectrum of IW, a random phase was assigned to each frequency and mode between 0 and π. The model has been initialized with the first CTD profile. Equation (2) has been solved for the observed buoyancy profile using the central finite difference technique, and QR method for the eigenvalues and eigenvectors. Fig. 4 shows an observed buoyancy profile and the corresponding first four internal wave modes computed for an internal tide period (the buoyancy frequency profile shown in Fig. 4 corresponds to the temperature and salinity profiles presented in Fig. 1). Though mode shapes are function of frequency they did not change much with frequency.

The model does not represent solitons and other oceanographic features like fronts, eddies and thermohaline fluctuations of the water column. The model gives an idea of typical variability of sound speed as a function of range, depth and time due to internal waves. Comparisons were done only in the time domain since we do not have measurements in range. The CTD yo-yo data set collected during the experiment allows a comparison to be made between the measured and simulated sound speed perturbations due to internal waves. The particle tracking vertical advection model was used to compute the sound speed profiles. A total of 18 profiles are presented in Fig. 5. In each case, the initial sound speed profile has been subtracted from each subsequent profile.

It can be seen from Fig. 5 that all variability in the observed profiles from top to bottom could not be reproduced by the model; however general level of sound speed variability could reasonably be well modeled. The deviations could be attributed to the frequency scale chosen energies supplied to the model (estimate of energy levels are not available) and also the distribution energy between the modes (it could be seen in the previous section that the values of j_* and P suggest two different values for the experimental site). Energy distributions among the frequencies and modes could be different than the GM (approximate) model.

The initial measured profile (to initialize the model) itself introduces additional uncertainty in the predictions, since it does not contain any information as to the current state of internal wave field (direction). This has been circumvented by taking ensemble average of the realizations. However a long period data at high resolution would be required to calibrate the model in deep waters.

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