

Representative scales of LASPEX wind data

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सार – इस शोध पत्र में उन केन्द्रों के पवन आँकड़े तैयार करने के लिए समुचित पैमाने की पहचान करने का प्रयास किया गया है जिनका परीक्षण लैसपैक्स द्वारा किया गया है। ऐसे पैमाने को जिस क्षेत्र का परीक्षण किया जाता है उस समूचे क्षेत्र के पवन पैटर्न को प्रस्तुत करने के लिए प्रतिनिधि पैमाना माना जा सकता है। इसके लिए विभिन्नताओं वाले क्षेत्रों को चुना गया है। चुने गए सभी पॉइंटों के अक्षांशों और देशांतरों के आँकड़ों के औसत को परिकल्पित किया गया है। जिससे केन्द्र बिन्दु के क्रमशः अक्षांश और देशांतर का पता चलता है। केन्द्र बिन्दु से तरंग के दैर्घ्य और इसकी दूरी के त्रिकोणमितीय फलन द्वारा सभी पॉइंटों केन्द्रों की पवनों के घटकों को स्वैच्छिक आधार पर प्रस्तुत किया गया है जिसे मूल तथ्य माना गया है। प्रत्येक केन्द्र के विभिन्नताओं वाले मानों को आकल्पित किया गया है जिससे सभी केन्द्रों के औसत मान का पता चला है। यनाई इत्यादि (1973) द्वारा तैयार की गई पद्धति के प्रयोग से उस क्षेत्र में बृहत मान वाले विभिन्नताओं वाले क्षेत्र का पता लगाने के लिए पवनों के समरूपी मानों का उपयोग किया गया है। इसमें यह भी पाया गया है कि 2830 कि.मी. से अधिक के तरंग के दैर्घ्य की दो विभिन्नताओं का अनुपात 0.9 से अधिक था, जिससे तरंग के दैर्घ्य के सीमित मान का पता चला है और जिसको समूचे क्षेत्र के द्योतक के रूप में मापने पर विचार किया गया है।

ABSTRACT. In this study, an attempt is made to identify the appropriate scales for the wind data over the stations covered in the LASPEX experiment, which can be considered to represent the large-scale wind pattern, representative of the whole area, covered by the field experiment. The divergence field was chosen for this purpose. The mean of the latitudes and longitudes of all the five stations was calculated and this represented the latitude and longitude respectively, of the central point. The wind components at each of the five stations were arbitrarily represented by a trigonometric function of wavelength and distance from the central point, which was considered as the origin. The divergence values at each station were computed and the mean value over all the stations was found. The same values of winds were used to compute the large scale divergence field over the area, by applying the method devised by Yanai *et al.* (1973). It was seen that for wavelengths greater than 2830 km, the ratio of the two divergences was greater than 0.9, suggesting that this was the limiting value of the wavelength which can be considered as representative of the entire area.

Key words – Representative scales, Limiting wavelength, LASPEX

1. Introduction

A major field experiment (LASPEX) was conducted in the semi-arid Sabarmati river basin in Gujarat, during the period January 1996 to March 1999. The details of this experiment are given in Verneker *et al.* (2003). Its main objective was to collect a complete surface and sub-surface atmospheric hydrological data base, against which parameterized models for land-surface process *i.e.*, energy exchange, radiative, sensible and latent heat fluxes can be tested for improvement and further development. However, the general circulation and climate models can respond only to larger scales of motion due to their coarse resolutions. Since wind data is involved in the computation of most of the land-surface parameters, it is important to compute the wind data of the appropriate scale, from the raw data, so that it matches the model resolution.

The latitudes and longitudes of the stations covered by LASPEX, are given in Fig. 1. We define the wind components of the wind in a trigonometric form, as functions of wavelength and distance from the central

point. The coordinates of each station is now redefined in terms of its X and Y distances from the central point, which is chosen as the origin. The X and Y ordinates are represented by the east and north directions respectively from the origin. The next step is the computation of the divergences analytically at each station. The mean value of the divergence, considering all the stations is computed for different sets of wavelengths. We now use the method of Yanai *et al.* (1973) to compute the large scale divergences over the area covered by LASPEX, using the same values of the wind and wavelengths. The aim is to identify the wavelengths for which the ratio of the two divergences are greater than 0.9. These wavelengths can be taken to be representative of the large scale wind field over the area.

2. Computation of divergences

The latitudinal and longitudinal wind fields are expressed as :

$$U_i = A \sin(2\pi/L_x X_i) \quad (1)$$

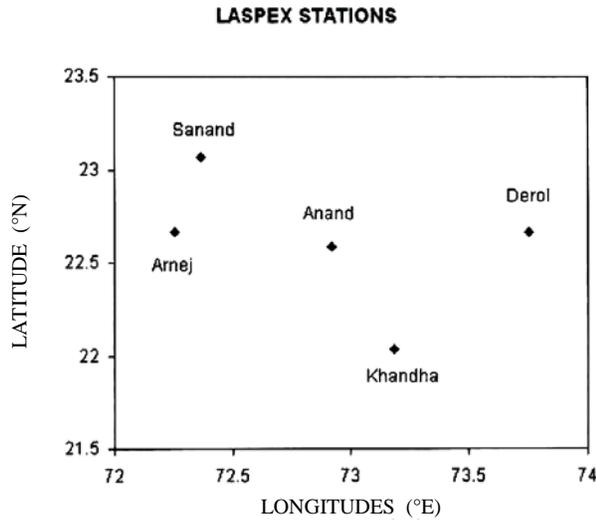


Fig. 1. Latitudes and longitudes of the stations which were identified for LASPEX

$$V_i = A \sin(2\pi/L_y Y_i) \tag{2}$$

'A' is the arbitrary amplitude of the wave, L_x , L_y are the wavelengths in the X and Y direction, respectively, and X_i , Y_i are the coordinates of the particular station. The divergences DA_i , are given by :

$$DA_i = \partial U_i / \partial X_j + \partial V_i / \partial Y_j \tag{3}$$

Thus, the divergences at each of the five points can be expressed as :

$$DA_i = 2\pi A \{ \cos(2\pi/L_x X_i) / L_x + \cos(2\pi/L_y Y_i) / L_y \} \tag{4}$$

By the method of Yanai *et al.* (1973), the wind components are expressed as :

$$U_i = A_1 X_i^2 + B_1 X_i Y_i + C_1 Y_i^2 + D_1 X_i + E_1 Y_i + F_1 \tag{5}$$

$$V_i = A_2 X_i^2 + B_2 X_i Y_i + C_2 Y_i^2 + D_2 X_i + E_2 Y_i + F_2 \tag{6}$$

$A_1, A_2, B_1, B_2, \dots$, are the six coefficients of the regression equations which are to be determined from the five values of U_i, V_i . Since there are only five stations, where values of U_i, V_i are defined, one constant has to be eliminated. A constraint is imposed that the curvature of

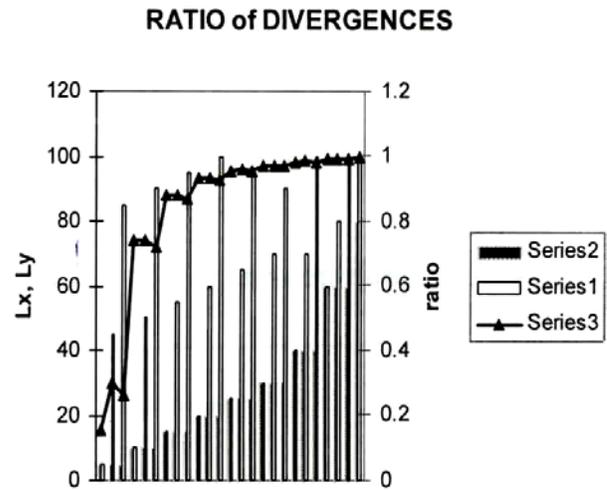


Fig. 2. Wave components L_x (Series 1) and L_y (Series 2) (scaled by the factor 10^5 meters) and the corresponding ratios (Series 3) of the divergences

the quadratic surface should be minimized, as in the case of skilled hand analysis. Thus, we minimize the quantity :

$$\left[2 \frac{\partial^2 U}{\partial X \partial Y} \right]^2 + \left[\frac{\partial^2 U}{\partial X^2} - \frac{\partial^2 U}{\partial Y^2} \right]^2 = 4[B^2 + (A - C)^2] \tag{7}$$

where the subscripts for the constants and variables, respectively, have been removed for convenience. Equation (5) can be rewritten as :

$$A_1 X_i^2 + B_1 X_i Y_i + C_1 Y_i^2 + D_1 X_i + E_1 Y_i = U_i - F_1 \tag{8}$$

Differentiating the above equation *w.r.t.* the coefficient F_1 , we get :

$$\left(\frac{\partial A}{\partial F} \right) X^2 + \left(\frac{\partial B}{\partial F} \right) XY + \left(\frac{\partial C}{\partial F} \right) Y^2 + \left(\frac{\partial D}{\partial F} \right) X + \left(\frac{\partial E}{\partial F} \right) Y = -1 \tag{9}$$

where, as usual, the subscripts have been removed for convenience. We now minimize the R.H.S. of Equation (6), *w.r.t.* F, to obtain :

$$B \frac{\partial B}{\partial F} + (A - C) \left(\frac{\partial A}{\partial F} - \frac{\partial C}{\partial F} \right) = 0 \tag{10}$$

From the above equation we get an expression for the coefficient 'B' in terms of other coefficients and their derivatives, *w.r.t.* F. We now have five unknown coefficients which can be calculated from the five known

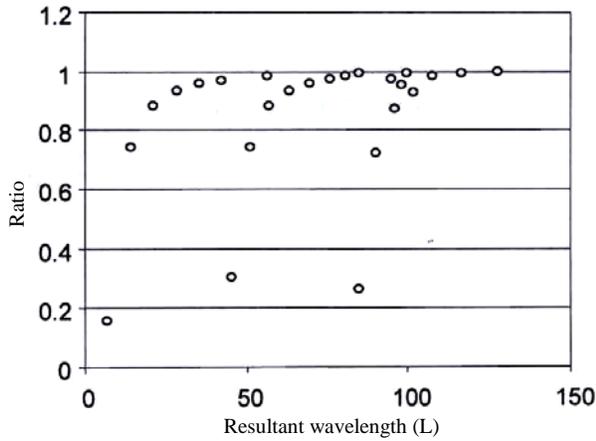


Fig. 3. Total wavelength L and corresponding ratios of divergences

values of ‘ U ’, at the five comers of the pentagon. Thus, the equation for the U component is as follows :

$$U_i = A(X_i^2 + b X_i Y_i) + C(Y_i^2 - b X_i Y_i) + D X_i + E Y_i + F_i \quad (11)$$

where $b = B/(A-C)$. The value of ‘ b ’ is known from Eqn. 10. Thus, there are five unknown coefficients which are to be evaluated from five station data. A similar expression can be obtained for the component V_i . The values of the coefficients, $A_1, A_2, b, C_1, C_2, D_1, D_2, \dots$, can be computed by solving the matrix, as shown in Appendix 1. The large scale divergence was given by Eqn. 3, computed at the origin. Thus,

$$\partial U_i / \partial X_i + \partial V_i / \partial Y_i = D_1 + E_2 \quad (12)$$

3. Results

Fig. 2 shows a plot between various combinations of the wavelengths L_x and L_y (meters/ 10^5) and the divergence ratio. In the legends, L_x is shown as series 1, L_y is shown as series 2 and the ratio of the divergences for a particular combination of wavelengths, as series 3. The wavelengths are to be multiplied by a factor 10^5 to give the actual wavelengths in meters. Adjacent columns touching each other represent the components of the wavelengths and the point where it cuts the ratio curve (after being extended if necessary) the value of the corresponding ratio is obtained. It may be seen that even though the L_y component of the wavelength is large in some cases, the corresponding ratio is less than 0.9, suggesting that it is not representative of the large scale field. It is therefore, the combination of the two components of the wavelength That decides large-scale representativeness of the wave. This point is made clearer in Fig. 3, which plots the total

wavelength L and the corresponding divergence ratio. It seen that the first value of L for which the ratio is above 0.9, is $L = 2830$ km. There are several values of L which are greater than this value, but the corresponding ratios are below 0.9.

4. Conclusion

This study indicates that the effective scales of the wind, used in derived parameters, such as divergence, depend upon both the components of the scales. The effective scale is representative of the large-scale field if the component scales combine in the proper proportion to generate the large-scale field. This aspect should be considered while examining field data for verification purposes by large-scale modelers.

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References

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Appendix 1

The derivatives of the coefficients *w.r.t.* F, can be expressed as a product of an inverse matrix with (-1), as shown :

$$\begin{bmatrix} X_1^2 & X_1 Y_1 & Y_1^2 & X_1 & Y_1 \\ X_2^2 & X_2 Y_2 & Y_2^2 & X_2 & Y_2 \\ X_3^2 & X_3 Y_3 & Y_3^2 & X_3 & Y_3 \\ X_4^2 & X_4 Y_4 & Y_4^2 & X_4 & Y_4 \\ X_5^2 & X_5 Y_5 & Y_5^2 & X_5 & Y_5 \end{bmatrix} \begin{bmatrix} \partial A / \partial F \\ \partial B / \partial F \\ \partial C / \partial F \\ \partial D / \partial F \\ \partial E / \partial F \end{bmatrix} = -1 \quad (A1)$$

The first matrix represents the X, Y, distances of the respective stations from the origin. The second matrix represents the unknown derivative of the coefficients in

Eqns. 5 & 6, *w.r.t.* 'F'. Thus, it can be evaluated as the product of inverse of matrix 1 with (-1), after which the value of 'b' in Eqn. (11), can be evaluated.

Now the coefficients in Eqn. 8 can be evaluated as follows :

$$[A] = [X]^{-1} [U] \quad (A2)$$

where [A] is the matrix of coefficients as in Eqn. 11, [X] represents the elements of distances expressed by the right hand side of Eqn. 11 and [U] represents the *u* - components of the velocities at each station. In a similar way the coefficients for the *v* - components of the velocity field can be evaluated and the large scale divergence field is computed from Eqn. 3.
