Parameterization of momentum and energy flux associated with mountain wave across the Assam - Burma hills

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सार – इस शोध–पत्र में आदर्शीकृत वायु धारा में असम बर्मा के पर्वतों (ए. बी. एच.) के पार पर्वतों से उत्पन्न होने वाले आंतरिक गुरूत्व तंरगों (आई.जी.डब्ल्यू.) से संबद्ध ऊर्जा फलक्स और संवेग फलक्स को प्राचलीकृत करने का प्रयास किया गया है। इसके लिए द्वि आयामी मेसोस्केल निदर्श का विकास किया गया है जिसका उपयोग आदर्शीकृत वायु धारा के लक्षणों के लिए ऊर्ध्वाधर रूप से संचरित पर्वत से उत्पन्न होने वाले आंतरिक गुरूत्व तंरगों से संबद्ध संवेग ⁄ ऊर्जा फलक्स का अध्ययन करने के लिए किया गया है। इस हालत में पवन और ऊर्जा की स्थिरता की स्थिति दोनों ऊँचाई के अनुरूप अपरिवर्ती रहते हैं। स्थिरता की इस स्थिति से क्रॉस–प्रिवेलिंग प्रवाह की दिशा लगभग अत्यधिक विस्तार में होती है।

इसके बाद उपर्युक्त निदर्श का अनुप्रयोग असम बर्मा के पर्वतों के पार फलक्सों का आकलन करने के लिए किया गया है। इस अध्ययन से दोनों फलक्सों की ऊँचाई के अलग–अलग होने ऊर्जा फलक्स के ऊर्ध्वमुखी होने और संवेग फलक्स के अधोमुखी होने का पता चलता है। इस अध्ययन से निम्नलिखित प्रमुख परिणाम पाए गए है:

असम बर्मा के पर्वतों के पार ऊर्ध्वाधर रूप से संचरित पर्वत के तरंग के लिए तरंग संवेग फलक्स अध्वार्धर रूप से अधोमुखी और तरंग ऊष्मा फलक्स ऊर्ध्वाधर रूप से ऊर्ध्वमुखी रहते हैं।

इसमें असम बर्मा के पर्वतों के दो रिजों के मध्य घाटी के पार ऊर्ध्वाधर फलक्सों की दिशायें दो रिज़ों के पार और पूरे असम बर्मा के पर्वत के पार की दिशाओं से प्रतिकूल पाई गई हैं।

इस अध्ययन से यह भी पता चला है कि लम्बी घाटी (घाटी से अलग किए हुए घाटी की लम्बाई प्रत्येक रिज की आधी चौड़ाई को मिलाकर उससे भी अधिक घाटी की लंबाई) वायुमंडलीय संवेग बजट में उद्गम के रूप में और वायुमंडलीय ऊर्जा बजट में धसाव के रूप में कार्य करती है।

मामलों का अध्ययन करने पर यह पता चला है कि पूरे असम बर्मा के पर्वतों के पार अधोमुखी संवेग फलक्स (प्रवाह के साथ 100 कि.मी. की लंबाई पर औसतन) में 4.5−10.0 एन.एम.$^{-2}$ के मध्य भिन्नता पाई गई है और दो रिजों के पार 2.0−7.0 एन.एम.$^{-2}$ तथा घाटी के पार अधोमुखी संवेग फलक्सों में 0.5−1.2 एन.एम.$^{-2}$ के मध्य भिन्नता पाई गई है। इसमें यह भी पाया गया है कि पूरे असम–बर्मा पर्वत के पार ऊर्ध्वमुखी संवेग फलक्स (प्रवाह के साथ 100 कि.मी. की लंबाई पर औसतन) में 45.0−180.0 डब्ल्यू.एम.$^{-2}$ के मध्य भिन्नता पाई गई है और दो रिज़ों के पार 20.0−125.0 डब्ल्यू.एम.$^{-2}$ के मध्य भिन्नता पाई गई है तथा घाटी के पार अधोमुखी ऊर्जा फलक्स में 5.0−21.0 डब्ल्यू.एम.$^{-2}$ के मध्य भिन्नता पाई गई है।

असम बर्मा पर्वतों के लिए दोनों फलक्सों के परिमाण में ऊँचाई के अनुरूप वृद्धि होती है किंतु इसकी मात्रा में कमी प्रत्येक रिज की कुल ऊँचाई की आधे के अनुरूप घटती है।

ABSTRACT. An attempt has been made to parameterize the energy flux and momentum flux associated with orographically excited internal gravity waves (IGW) across the Assam Burma Hills (ABH) in an idealized air stream. For this a two-dimensional meso-scale model has been developed and used to study the momentum/energy flux associated with vertically propagating orographically excited internal gravity waves for an idealized air stream characteristic, where both wind and stability of the prevailing flow remain invariant with height. Here the barrier is assumed to be extending semi infinitely in the cross-prevailing flow direction.

The above model is then applied to the ABH to compute the fluxes across it. The study shows that both the fluxes are independent of height, the energy flux is upward and momentum flux is downward. Following salient results are found from the study.

Wave momentum flux is vertically downward and wave energy flux is vertically upward for a vertically propagating mountain wave across the ABH.

It is found that the directions of vertical fluxes across the valley between the two ridges of the ABH are opposite to those across the two ridges and across the entire ABH.

This study also shows that a long valley (length of the valley exceeds the sum of the half widths of individual ridges, separated by the valley) acts as a source in the atmospheric momentum budget and as a sink in the atmospheric energy budget.

From the cases studied, it is found that the downward momentum flux (averaged over a length of 100 km along the flow) across the entire ABH varied between 4.5 – 10.0 Nm^{-2} and that across the two ridges varied between 2.0 - 7.0 Nm^{2} and upward momentum flux across the valley varied between 0.5 - 1.2 Nm^{-2}. It is also found that the upward energy flux (averaged over a length of 100 km along the flow) across the entire Assam- Burma hill varied between 45.0 - 180.0 Wm^{-2} and that across the two ridges varied between 20.0 - 125.0 Wm^{-2} and downward energy flux across the valley varied between 5.0 - 21.0 Wm^{-2}.

For the ABH, magnitude of both the fluxes increases with height but decreases with half with of the individual ridges.

Key words – Orographically excited internal gravity waves, ABH, Momentum flux, Energy flux.

1.   Introduction

We know that, when a stably stratified air-stream flows across an orographic barrier, gravity waves are excited which propagate upwards under certain conditions of thermal stability and airflow stratification. These orographic gravity waves can transport momentum from a stably stratified airstreams to the earth's surface in response to a net pressure drop between the windward and lee slopes of the barrier. It is also known that these gravity waves are also capable of transporting energy from surface to the mean flow at great height. It is also believed that the continuous extraction of momentum from the mean flow is one of the possible causes for breaking of gravity waves. But this transport of momentum or energy by orographic gravity waves is a sub grid scale phenomenon, yet its impact on large-scale motion is substantial. So the parameterization of these sub grid scale fluxes is important in the numerical weather prediction models. Now we consider the momentum and energy fluxes in association with orographic gravity waves.

Sawyar (1959) first pointed out the relative importance of this momentum loss of the mean flow due to continuous extraction of momentum from it by the orographic gravity waves. He examined the case of a two-dimensional (2-D) flow over a bell-shaped obstacle with half width (a) 2 km and height (b) 300 m and determined that the typical value of wave momentum flux is of the order 1-10 dynes/cm^{2}.

Eliassen and Palm (1961) showed that for 2-D linear gravity waves, the vertical flux of horizontal momentum, resulting from waves, is independent of height when the waves are steady and non-dissipative.

Bluemen (1965) noted that the magnitude of the wave drag is sensible to the vertical wavelength. He also showed that the maximum value of the drag is attained when the vertical wavelength is twice the maximum height of the mountain.

Bretherton (1969) reviewed the theories concerning the propagation of internal gravity waves (IGW) in a horizontally uniform shear flow. His computations showed that for a 19 m/s gradient wind over hilly terrain in north Welsh, the wave drag amounted to 4 dyne/cm^{2}, of which 3 dyne/cm^{2} probably acted on the atmosphere above 20 km.

A conclusive evidence of the importance of the wave-drag was obtained from the data collected over the Front Range of the Colorado Rockies by instrumented aircraft during field experiments, reported by Lilly (1972).

Smith (1978) determined the pressure drag on the Blue-ridge Mountain in the central Appalachians. During the first two weeks of January 1974 he observed several periods with significant wave drag with pressure differences typically of the order of 50 N/m^{2} across the ridge. The height of the section, studied by Smith (1978), of the above ridge is 300 m and length about 8km [Refer Figs. 1 (a&b) of Smith 1978]. Hence the above pressure differences of the order of 50 N/m^{2} across the ridge corresponds to a drag of 2N/m^{2}. Palmer et al., (1986) has pointed out the general westerly bias of the global general circulation models (GCM). They also pointed out that one way to reduce this general westerly bias is to incorporate the gravity wave drag parameterization scheme in the GCM. Gravity wave drag parameterization scheme proposed by Palmer et al., (1986), McFarlane (1987) reduced the westerly bias mainly in stratosphere.

Recently, lee waves have drawn the new attention from the viewpoint of the atmospheric momentum budged. Iwasaki et al., (1989) introduced a new type of gravity wave drag parameterization scheme to improve the

tropospheric westerly bias by including the effects of these tropospheric-trapped lee waves.

Duran (1992), using a simple 2-D model, showed the importance of non-hydrostatic trapped lee waves in the troposphere.

Satomura and Bougeault (1994) used a 2-D, non-hydrostatic, compressible model to simulate the airflow over the Pyrenees in connection with two lee wave events during PYREX experiment. In both cases, the simulated downward momentum fluxes agree well with the observed fluxes around 4km height. The simulated fluxes in their study were almost constant with height. The over estimation of simulated momentum flux in the upper half of the atmosphere was suggested to be due to the time evolution of the mean wind and the lateral momentum flux divergence found in the atmosphere.

Vosper and Mobbs (1998) showed that for steady waves, in the absence of dissipation, the vertical fluxes of both the components of horizontal momentum are constant with height.

Dutta (2001) studied momentum/energy flux associated with mountain wave across Mumbai-Pune section of the Western Ghats in an idealized air stream. He showed that both the fluxes were independent of height and the half width of the bell shaped part of the barrier.

De (1973) had shown that air stream characteristics across ABH during winter are favourable for the occurrence of orographic gravity wave across the ABH. Using satellite pictures, he also documented the observational evidence of orographic gravity wave across the ABH. But momentum/energy flux associated with orographic gravity wave across the ABH has not been studied so for. So in this paper, we aim at developing a modal to parameterize these fluxes across ABH.

2.   Data

For the present study, we have selected those dates on which mountain wave across the ABH was reported by De (1973). For those dates we have used the upper air soundings of Guwahati for 0000 UTC and 1200 UTC.

3.   Methodology

Let us consider a steady, adiabatic, hydrostatic laminar and Boussinesq flow across a two-dimensional north-south oriented ridge. Throughout the atmosphere, we take a constant value of the basic flow wind and stability, which are nothing but the vertically averaged of their actual value at each level. Under the above

assumptions the linearized governing equations may be written as :

$$U \frac{\partial u'}{\partial x} = -\frac{1}{\rho_0} \frac{\partial p}{\partial x} \tag{1}$$

$$U \frac{\partial w'}{\partial x} = -\frac{1}{\rho_0} \frac{\partial p'}{\partial x} + g \frac{\rho'}{\rho_0} \tag{2}$$

$$\frac{\partial u'}{\partial x} + \frac{\partial w'}{\partial z} = 0 \tag{3}$$

$$U \frac{\partial \theta'}{\partial x} + w' \frac{\partial \theta_0}{\partial z} = 0 \tag{4}$$

Where U is basic state wind speed. ρ_0 is basic state density, θ_0 is basic state potential temperature, both are function of z only.

u', w' are zonal and vertical components of perturbation wind.

p' and θ' are perturbation pressure and potential temperature respectively.

After performing Fourier transformation to the above equations and some algebraic simplification we obtain

$$\frac{\partial^2 \hat{w}}{\partial z^2} + \frac{1}{\rho_0} \frac{d\rho_0}{dz} \frac{\partial \hat{w}}{\partial z} + \left(l^2 - k^2\right)\hat{w} = 0 \tag{5}$$

Where, $\hat{w}(k, z)$ is the Fourier transform of $w'(x, z)$, $l = \frac{N}{U}$ is the Scorer's parameter and N is the Brunt-Vaisala frequency.

Now by the substitution,

$$\hat{w} = \sqrt{\frac{\rho_0(0)}{\rho_0(z)}} \hat{w}_1 \tag{6}$$

Equation (5) reduces to

$$\frac{\partial^2 \hat{w}_1}{\partial z^2} + \left\{ l^2 - k^2 + \frac{1}{4\rho_0^2} \left(\frac{d\rho}{dz}\right)^2 - \frac{1}{2\rho_0} \frac{d^2\rho_0}{dz^2} \right\} \hat{w}_1 = 0 \tag{7}$$

Now the last two terms in side bracket of equation (7) are very small compared to other terms, so they may be neglected. So equation (7) reduces to

$$\frac{\partial^2 \hat{w}_1}{\partial z^2} + \left\{ l^2 - k^2 \right\} \hat{w}_1 = 0 \tag{8}$$

Now for a vertically propagating hydrostatic wave, we have, k<< l. Hence the solution of equation (8) can be taken as

$$\hat{w}_1(k,z) = A\exp(ilz) + B\exp(-ilz) \tag{9}$$

Where A, B are constants, to be determined from the boundary conditions.

Lower boundary conditions :

At the ground the flow is assumed to follow the terrain, thus

$$w(x, z = 0) = U\frac{\partial \zeta}{\partial x} \tag{10}$$

Where, $\zeta(x)$ represents the terrain profile. The profile of ABH is a double ridge shaped, the analytical representation of which is given by

$$\zeta(x) = \frac{a^2 b_1}{a^2 + x^2} + \frac{a^2 b_2}{a^2 + (x - d)^2} \qquad \text{(De, 1973)} \tag{11}$$

Where a = 20.0 km, b_1 = 0.9 km, b_2 = 0.7 km, d = 55.0 km. Taking Fourier transform

$$\hat{\zeta}(k) = a\exp(-ak)\left[b_1 + b_2\exp(-ikd)\right]$$

Now

$$\hat{w}_1(k,0) = \hat{w}(k,0) = Uika\exp(-ak)\left[b_1 + b_2\exp(-ikd)\right] \tag{12}$$

From Eqns. (9) and (12), we get

$$A + B = Uika\exp(-ak)\left[b_1 + b_2\exp(-ikd)\right] \tag{13}$$

Upper boundary condition: We use radiation boundary condition. According to this, Orographic wave is assumed to propagate at a great distance in the vertical. For that, we must have B = 0.

Therefore,

$$A = Uika\exp(-ak)\left[b_1 + b_2\exp(-ikd)\right] \tag{14}$$

Thus the solution of equation (8) is given by

$$\hat{w}_1(k,z) = A\exp(ilz) = Uika\exp(-ak + ilz)$$
$$\left[b_1 + b_2\exp(-ikd)\right] \tag{15}$$

Now, we know that pressure drag per unit length along the ridge is given by

$$F = \int p'\mathrm{d}\varsigma = \int_{-\infty}^{\infty} p'\frac{\partial \varsigma}{\partial x}\mathrm{d}x = -\int_{-\infty}^{\infty} \rho_0 u'w'(z=0)\mathrm{d}x$$
$$\text{[Gill (1982)]} \tag{16}$$

Where $\int_{-\infty}^{\infty} \rho_0 u'w'\mathrm{d}x$ is the eddy momentum flux at any level associated with orographic gravity wave.

Thus from Eqn. (16) we see that pressure drag is equal and opposite to the wave momentum flux *i.e.*, the wave drag generated by mountain wave in the layer of the atmosphere bounded by the bottom and top of the mountain.

From the above Eqn. (16) it is seen that here horizontal momentum budget at any level contains only two terms: a momentum sink at the lower boundary and a momentum source at that level. The momentum source is a downward flux of momentum due to wave motion forced by the presence of the mountain in the stratified flow. This wave momentum flux is convergent in the layer below the level under consideration and the momentum is transferred to the surface *via* pressure force. The momentum had been extracted from the flow far aloft.

Now momentum flux

$$= \int_{-\infty}^{\infty} \rho_0 u'w'\mathrm{d}x = \rho_0 \int_{-\infty}^{\infty} u'w'\mathrm{d}x = 2\pi\rho_0 \int_{-\infty}^{\infty} \hat{u}\hat{w}^*\mathrm{d}k \tag{17}$$

Where $\hat{w}*$ is the complex conjugate $\hat{w}$. Since, here we consider only non-negative wave number, hence

$$\int_{-\infty}^{\infty} u'w' dx = 2\pi \int_0^{\infty} \hat{u}\hat{w}* dk = 2\pi \int_0^{\infty} \frac{-1}{ik} \hat{w}* \frac{\partial \hat{w}}{\partial z} dk$$

$$= 2\pi i \int_0^{\infty} \frac{1}{k} \hat{w}* \frac{\partial \hat{w}}{\partial z} dk \qquad (18)$$

Now,

$$\hat{w}_1(k,z) = Uika \exp(-ak + ilz)[b_1 + b_2 \exp(-ikd)]$$

$$\hat{w}(k,z) = \sqrt{\frac{\rho_0(0)}{\rho_0(z)}} Uika \exp(-ak + ilz)[b_1 + b_2 \exp(-ikd)]$$

$$\frac{\partial \hat{w}}{\partial z} = \sqrt{\frac{\rho_0(0)}{\rho_0(z)}} Uika \exp(-ak + ilz)[b_1 + b_2 \exp(-ikd)]$$

$$\left(il - \frac{\rho_0'(z)}{2\rho_0(z)}\right)$$

$$\hat{w}* = \sqrt{\frac{\rho_0(0)}{\rho_0(z)}} U(-i)ka \exp(-ak - ilz)[b_1 + b_2 \exp(ikd)]$$

Hence,

$$\hat{w}*\frac{\partial \hat{w}}{\partial z} = \frac{\rho_0(0)}{\rho_0(z)} U^2 k^2 a^2 \exp(-2ak)$$

$$\left(b_1^2 + b_2^2 + 2b_1 b_2 \cos dk\right)\left(il - \frac{\rho_0'(z)}{2\rho_0(z)}\right) \qquad (19)$$

Now using Eqn. (19) in Eqn. (18), we get

$$\int_{-\infty}^{\infty} u'w' dx = 2\pi \frac{\rho_0(0)}{\rho_0(z)} U^2 a^2 \left[-l - \frac{i\rho_0'(z)}{2\rho_0(z)}\right]$$

$$\int_0^{\infty} \left\{\left(b_1^2 + b_2^2\right)k + 2b_1 b_2 k \cos dk\right\} \exp(-2ak) dk \qquad (20)$$

Since the left hand side of the above equation is purely a real quantity, hence

$$\int_{-\infty}^{\infty} u'w' dx = -2\pi NU a^2 \frac{\rho_0(0)}{\rho_0(z)}$$

$$\int_0^{\infty} \left\{\left(b_1^2 + b_2^2\right)k + 2b_1 b_2 k \cos dk\right\} \exp(-2ak) dk$$

$$\int_{-\infty}^{\infty} u'w' dx = -2\pi NU a^2 \frac{\rho_0(0)}{\rho_0(z)}$$

$$\left[\left(b_1^2 + b_2^2\right)\frac{1}{4a^2} + 2b_1 b_2\left(4a^2 - d^2\right)\frac{1}{\left(4a^2 + d^2\right)^2}\right] \qquad (21)$$

So the momentum flux at any level in vertical is given by

$$\int_{-\infty}^{\infty} \rho_0(z) u'w' dx = -2\pi NU a^2 \rho_0(0)$$

$$\left[\left(b_1^2 + b_2^2\right)\frac{1}{4a^2} + 2b_1 b_2\left(4a^2 - d^2\right)\frac{1}{\left(4a^2 + d^2\right)^2}\right] \qquad (22)$$

Thus Eqn. (22) shows that for a vertically propagating (or decaying) mountain wave across Assam - Burma hills, the flux of wave momentum at any level is vertically downward (or upward). Eqn. (22) also shows that vertical wave momentum flux associated with mountain wave across Assam – Burma hills is independent of height, which is in conformity with earlier findings of Eliassen and Palm (1961), and Smith and Lin (1982).

From the profile of the first ridge of ABH is $\frac{a^2 b_1}{a^2 + x^2}$. Following Dutta (2001), expression for the wave momentum flux across the first ridge is given by

$$F_1 = \frac{-\pi NU \rho_0(0) b_1^2}{2} \qquad (23)$$

And the profile of the second ridge of ABH is $\frac{a^2 b_2}{a^2 + (x - d^2)}$. Similarly the expression of wave momentum flux across the second ridge is given by

$$F_2 = \frac{-\pi NU \rho_0(0) b_2^2}{2} \qquad (24)$$

From Eqns. (22), (23) and (24), it is clear that $F \neq F_1 + F_2$ and

$$F - (F_1 + F_2) = -4\pi NU \rho_0(0) a^2 b_1 b_2$$

$$(4a^2 - d^2)/(4a^2 + d^2)^2 = F_3 \qquad (25)$$

This may be attributed to the wave momentum flux across the valley between the two ridges of the ABH.

TABLE 1

Momentum flux and energy flux across the AB hill in different cases

S. No.	Date	U (ms^{-1})	N (s^{-1})	$F1$ (Nm^{-2})	$F2$ (Nm^{-2})	F (Nm^{-2})	$F3$ (Nm^{-2})	$E1$ (Wm^{-2})	$E2$ (Wm^{-2})	E (Wm^{-2})	$E3$ (Wm^{-2})
1	23 Nov 1966	11.37	0.03	-4.35	-2.63	-6.26	0.72	49.55	29.98	71.32	-8.215
2	1 Dec 1966	17.27	0.02	-6.87	-4.15	-9.87	1.14	122.6	74.2	176.45	-20.33
3	8 Jan 1967	13.99	0.03	-5.46	-3.3	-7.85	0.90	76.5	46.3	110.05	-12.68
4	9 Jan 1967	17.83	0.03	-6.92	-4.19	-9.97	1.15	124.19	75.13	178.73	-20.59
5	3 Feb 1967	9.60	0.025	-3.27	-1.98	-4.71	0.54	33.0	19.97	47.49	-5.47

Thus the momentum flux associated with orographic gravity wave across a valley between two ridges with height 'b_1' and 'b_2' half width 'a' and separated by a distance 'd', in a simplified flow is given by Eqn. (25). Vertical flux of wave energy is given by

$$E = \int_{-\infty}^{\infty} p'w' \mathrm{d}x \qquad \text{[Eliassen and Palm (1961)] (26)}$$

$$= 2\pi \int_{-\infty}^{\infty} \hat{p}\hat{w}^* \mathrm{d}k \qquad (27)$$

Again from Eqn. (7), we have $\hat{p} = -U\rho_0(z)\hat{u}$.

Hence vertical flux of wave energy

$$E = -U \int_{-\infty}^{\infty} \rho_0(z)u'w' \mathrm{d}x = -2\pi U\rho_0(z)\int_{0}^{\infty} \hat{u}\hat{w}^* \mathrm{d}k$$

$$= 2\pi N U^2 a^2 \rho_0(0)$$

$$\left[\left(b_1^2 + b_2^2\right)\frac{1}{4a^2} + 2b_1b_2\left(4a^2 - d^2\right)\frac{1}{\left(4a^2 + d^2\right)^2} \right]$$
(28)

Now the vertical flux of wave energy due to first ridge is given by

$$E_1 = \frac{1}{2}\pi N U^2 \rho_0(0)b_1^2 \qquad (29)$$

And that due to second ridge is given by

$$E_2 = \frac{1}{2}\pi N U^2 \rho_0(0)b_2^2 \qquad (30)$$

Equations (28), (29) and (30) show that $E \neq E_1 + E_2$

So,

$$E - (E_1 + E_2) = 4\pi N\rho_0(0)U^2 a^2 b_1 b_2 (4a^2 - d^2)/$$
$$(4a^2 + d^2)^2 = E \qquad (31)$$

Thus from Eqn. (28), it is clear that flux of wave energy is vertically upward (or downward) for a vertically propagating (or decaying) mountain wave. Thus the vertical fluxes of wave energy and wave momentum are oppositely directed. This is in conformity with the finding of Eliassen and Palm (1961) .The same equation also shows that wave energy flux is independent of height, which is in conformity with earlier findings of Eliassen and Palm (1961). From Eqns. (22) and (28), it appears that both the momentum flux and energy flux associated with orographic gravity wave across ABH are no longer independent of the half width as in Dutta (2001). This difference in result may be attributed to the presence of valley between the two ridges of the ABH.

4.   Results and discussion

(*i*)   Momentum flux and energy flux across the two separate ridges of the ABH, across the valley between them and across the entire ABH have been computed, using the formulae, discussed in the section 3, for different cases and they are shown in Table 1. From this table it is seen that wave momentum flux across the two ridges (F_1 and F_2) and across the entire ABH (F) are downward and that across the valley (F_3) is upward in all cases. For the cases studied, F_1, F_2 and F vary from –3.0 Nm^{-2} to –7. 0 Nm^{-2}, from –2.5 Nm^{-2} to –4.2 Nm^{-2} and from –4.5 Nm^{-2} to –10.0 Nm^{-2} respectively and F_3 varies from +0.5 Nm^{-2} to +1.2   Nm^{-2}. From the Table 1 we see that direction of computed vertical flux of momentum across

TABLE 2

Momentum flux and energy flux across the Western Ghats and across the AB hills for same air stream

S. No.	U (ms^{-1})	N (s^{-1})	F_{WG} (Nm^{-2})	F_{AB} (Nm^{-2})	E_{WG} (Wm^{-2})	E_{AB} (Wm^{-2})
1	11.37	0.03	-3.19	-6.26	36.27	71.32
2	17.27	0.02	-5.02	-9.87	89.75	176.45
3	13.99	0.03	-4.0	-7.85	55.97	110.05
4	17.83	0.03	-5.07	-9.97	90.91	178.73
5	9.60	0.025	-2.73	-4.71	26.86	47.49

the valley is opposite to that across the entire ABH and across the two ridges. This follows from the equations (22) to (25). These equations show that for d > 2 a, sign of F_3 is opposite to that of F_1, F_2 and F. In the present study a = 20 km and d = 55 km. From the table it is seen that wave energy flux across the two ridges (E_1 and E_2) and across the entire ABH (E) are upward and that across the valley (E_3) is downward in all cases. For the cases studied, E_1, E_2 and E vary from 33.0 Wm^{-2} to 125. 0 Wm^{-2}, from 19.0 Wm^{-2} to 76.0 Wm^{-2} and from 47.0 Wm^{-2} to 180.0 Wm^{-2} respectively and E_3 varies from –5.0 Wm^{-2} to –21.0 Wm^{-2}. Hence similar to computed momentum flux, direction of computed vertical flux of energy across the valley is opposite to that across the entire ABH and across the two ridges. This follows from the equations (28) to (31), which show that for d > 2 a, sign of E_3 is opposite to that of E_1, E_2 and E.

Hence, it follows that a long valley (length of the valley exceeds the sum of the half widths of individual ridges, separated by the valley) acts as a source in the atmospheric momentum budget and as a sink in the atmospheric energy budget.

However, the above results are from a computational study, which require observational verifications.

(*ii*)   Using the methodology, discussed in earlier section, we have computed the vertical flux of wave momentum and wave energy across ABH (F_{AB} and E_{AB}), for different values of 'a', 'b$_1$' and 'b$_2$' for a given air stream characteristics and are shown in Table 3. From this table it follows that both the fluxes increase with increase in 'b$_1$' and 'b$_2$' but with decrease in 'a'. But for the Mumbai-Pune section of the Western-Ghats (WG) both of them are independent of 'a' (Dutta 2001). This difference in the results may be attributed to the presence of valley in the ABH.   Maximum fluxes occur (F_{AB} = -1.98 Nm^{-2} and E_{AB} = 15.98 Wm^{-2}) for a = 2.0 km and b$_1$ = b$_2$ = 0.5 km.

(*iii*)   For a comparative study we have computed vertical flux of wave momentum and energy across the entire

TABLE 3

Momentum flux, energy flux across the AB hills for different values of a, b$_1$, b$_2$

S. No.	a (km)	b$_1$=b$_2$=b (km)	F_{AB} (Nm^{-2})	E_{AB} (Wm^{-2})
1	2	0.2	-0.37	3.61
2	5	0.2	-0.31	2.48
3	10	0.2	-0.29	2.33
4	2	0.3	-0.71	5.73
5	5	0.3	-0.69	5.58
6	10	0.3	-0.65	5.24
7	2	0.5	-1.98	15.91
8	5	0.5	-1.93	15.51
9	10	0.5	-1.81	14.56

ABH and across the Mumbai-Pune section of the WG, for the same air stream characteristics and are shown in Table 2. In all the cases it is seen that direction of these fluxes are same for both the barriers, but magnitudes of the fluxes across the ABH are nearly double the magnitudes of the fluxes across the WG. This may be attributed to the fact that in the profile of ABH there are two ridges separated by the valley, whereas in the profile of the Mumbai-Pune section of the WG there is only one ridge and a plateau. The plateau part does not contribute to the fluxes of energy or momentum (Dutta 2001). The vertical flux of energy/momentum across the ABH is the combination of those across the two ridges with a slight (less than even 10%) lick of them across the valley, where as the vertical flux of energy/momentum across the WG comes from only one ridge.

5.   Conclusions

(*i*)   Wave momentum flux is vertically downward and wave energy flux is vertically upward for a vertically propagating mountain wave across the ABH.

(*ii*)   This study also shows that a long valley (length of the valley exceeds the sum of the half widths of individual ridges, separated by the valley) acts as a source in the atmospheric momentum budget and as a sink in the atmospheric energy budget. From the cases studied, it is found that the fluxes across the valley between the two ridges of the ABH are opposite to those across the two ridges and across the entire ABH.

(*iii*)   From the cases studied, it is found that the downward momentum flux (averaged over a length of 100 km along the flow) across the entire ABH varied between 4.5 - 10.0

Nm^{-2} and that across the two ridges varied between 2.0 - 7.0 Nm^2 and upward momentum flux across the valley varied between 0.5 - 1.2 Nm^{-2}.

(*iv*) From the cases studied, it is found that the upward energy flux (averaged over a length of 100 km along the flow) across the entire Assam- Burma hill varied between 45.0 - 180.0 Wm^{-2} and that across the two ridges varied between 20.0 - 125.0 Wm^{-2} and downward energy flux across the valley varied between 5.0 - 21.0 Wm^{-2}.

(*v*) For the ABH, magnitude of both the fluxes increases with height but decreases with half width of the individual ridges.

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