

Derivation of the schemes of lateral and vertical dispersion parameters: Application in Gaussian plume model

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(Received 7 September 2013)

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सार – इस शोध पत्र का मुख्य उद्देश्य पवन गति का पावर लॉ और चक्रवात विसरणशीलता के विन्यास का उपयोग करते हुए अस्थिर दशा में पिच्छक विसर्जन प्राचलों का पार्श्विक दिशा (σ_y) तथा उर्ध्वधर दिशा (σ_z) में आकलन करना है। इसमें हमारे मॉडल और अल्जेब्रीक (लीडियन बुलिगन आदी, 2008) एवं इंटीग्रल (पासक्वील और स्मीथ 1983) सूत्रों की तुलना की गई है। हमें पता चला है कि हमारे मॉडल और अन्य दो मॉडल, प्रेक्षित किए गए डेटा के अनुरूप हैं।

ABSTRACT. The main objective of this paper is to estimate the plume dispersion parameters in lateral (σ_y) and vertical (σ_z) direction by using power law of wind speed and the scheme of eddy diffusivity in unstable condition. Comparison among our model and algebraic (Lidiane Buligon *et al.*, 2008) and integral (Pasquill and Smith, 1983) formulations were made. We find that besides our model two other models are in agreement with observed data.

Key words – Dispersion parameters, Gaussian plume model, Eddy diffusivity.

1. Introduction

The study and employment of operational short-range atmospheric dispersion models for environmental impact assessment have demonstrated to be of wide use in the evaluation of ecosystems perturbation in many distinct scales, (Meyer and Diniz, 2007).

In operational applications, the classical Gaussian diffusion models are largely employed in assessing the impacts of existing and proposed sources of air contaminants on local and urban air quality (Arya, 1999). Simplicity, associated with the Gaussian analytical model, makes this approach particularly suitable for regulatory usage in mathematical modeling of the air pollution, such models are quite useful in short term forecasting. The lateral and vertical dispersion parameters, respectively σ_y and σ_z represent the key turbulent parameterization in this approach. They contain the physical parameters that describe the dispersion process and, consequently, express the spatial extent of the contaminant plume under the effect of the turbulent motion in the planetary boundary layer (PBL) (Abdul-Wahab, 2006).

In this work, we estimated the schemes of dispersion parameters in the lateral direction (σ_y) and the

vertical direction (σ_z) in unstable conditions by using wind speed in power law and comparing between our work and (algebraic and integral formulations) with observed data of sulfur hexafluoride (SF_6) taken from Copenhagen in Denmark.

2. Model formulation

The concentration associated from point source of strength Q , is expressed as (Akula Venkatram 2004):

$$\frac{C(x, y, z)}{Q} = \frac{A}{\sqrt{2\pi} \sigma_y \bar{U} \bar{Z}} \exp\left[-\left(\frac{Bz}{\bar{Z}}\right)^s\right] \exp\left(-\frac{y^2}{2\sigma_y^2}\right) \quad (1)$$

where,

C is the average concentration of diffusing point (x, y, z) (kg/m^3),

U is mean wind velocity along the x -axis (m/s),

x is along-winds coordinate measured in wind direction from the source (m),

y is cross-wind coordinate direction (m),

z is vertical coordinate measured from the ground (m) and

σ_y is the plume dispersion parameter in the lateral directions.

where, the value of the parameter, s, depends on the stability ($s = 0.75$ and $A = 1.42$ in unstable case, (Sven – Erik *et al.*, 1983).

The mean plume height, \bar{Z} , is defined by

$$\bar{Z}(x) = \frac{\int_0^\infty z C(x, y, z) dz}{\int_0^\infty C(x, y, z) dz} \tag{2}$$

and the mean plume velocity, \bar{U} , is defined by

$$\bar{U} = \frac{\int_0^\infty U(z) C(x, y, z) dz}{\int_0^\infty C(x, y, z) dz} \tag{3}$$

We assume that the mean wind speed, $U(z)$, can be described by a power law so that:

$$U(z) = U_r \left(\frac{z}{Z_r} \right)^p \tag{4}$$

U_r is a reference velocity at height Z_r , the value of the power, p, lies between 0.15 and 0.20 in unstable case (Irwin, 1979).

Let, B, be a non-zero constant, then taking:

$$\left(\frac{Bz}{\bar{Z}} \right)^s = X \Rightarrow X^{\frac{1}{s}} = \frac{Bz}{\bar{Z}} \tag{5}$$

$$\therefore z = \frac{\bar{Z}}{B} X^{\frac{1}{s}} \Rightarrow dz = \frac{\bar{Z}}{Bs} X^{\frac{1}{s}-1} dX$$

Substitution from equations (1) and (5) in equation (2) one gets:

$$B = \frac{\Gamma(2/s)}{\Gamma(1/s)} \tag{6}$$

$$\Gamma(p) = \int_0^\infty x^{p-1} \exp(-x) dx$$

where, $\Gamma(p)$ is the gamma function.

Substituting from equations (4), (1) and (5) in equation (3), we obtain the mean plume velocity:

$$\bar{U}(z) = U_r \left(\frac{\bar{Z}}{Z_r B} \right)^p \left[\frac{\Gamma\left(\frac{p+1}{s}\right)}{\Gamma\left(\frac{1}{s}\right)} \right] \tag{7}$$

Van Ulden (1978) shows that the mean plume height, \bar{Z} , can be calculated from:

$$\frac{d\bar{Z}}{dx} = \frac{K(q\bar{Z})}{U(q\bar{Z})q\bar{Z}} \tag{8}$$

and

$$q = \left\{ s \left[\frac{\Gamma\left(\frac{2}{s}\right)}{\Gamma\left(\frac{1}{s}\right)} \right] \right\}^{\frac{1}{(1-s)}} \tag{9}$$

where $K(z)$ is the eddy diffusivity parameterization that is led to the K- theory assumption.

According to Pleim and Chang (1992), the form of $K(z)$ in an unstable case is:

$$K(z) = k w_* z \left(1 - \frac{z}{h} \right) \tag{10}$$

where, k is the von Karman constant which is set to 0.4, w_* is the convection scaling parameter and h is the effective height of release above the ground estimated from (Briggs, 1969):

$$h = h_s + \Delta h$$

where h_s is physical stack height (115 m).

$$\Delta h = 3 (W/U_{115}) D$$

where, W is the exit velocity (4 m/s)

D is the internal stack diameter (1 m) and

$$U_{115} = U_{10} (h_s / 10)^p$$

U_{10} is the wind speed at 10 m height.

Substituting from equations (4), (10) in equation (8) and integrating equation (8), we obtain the mean plume height \bar{Z} :

$$\bar{Z} = \left[\frac{h^2 k w_* Z_r^p x}{q^2 \left(1 - \frac{p}{p+1}\right) U_r q^p} \right]^{\frac{1}{p+1}} \quad (11)$$

Substituting from equation (11), in equation (7), we obtain the mean plume velocity \bar{U} :

$$\bar{U}(z) = \left[\frac{h^2 k w_* Z_r^p x}{q^2 \left(1 - \frac{p}{p+1}\right) U_r q^p} \right]^{\frac{p}{p+1}} \frac{U_r}{(Z_r B)^p} \left[\frac{\Gamma\left(\frac{p+1}{s}\right)}{\Gamma\left(\frac{1}{s}\right)} \right] \quad (12)$$

We estimate the horizontal spread σ_y using Eckman's (1994) hypothesis that

$$\frac{d\sigma_y}{dx} = \frac{\sigma_v}{\left[\frac{h^2 k w_* Z_r^p x}{q^2 \left(1 - \frac{p}{p+1}\right) U_r q^p} \right]^{\frac{p}{p+1}} \frac{U_r}{(Z_r B)^p} \left[\frac{\Gamma\left(\frac{p+1}{s}\right)}{\Gamma\left(\frac{1}{s}\right)} \right]} \quad (13)$$

where,

$$\bar{U}(z) = \left[\frac{h^2 k w_* Z_r^p x}{q^2 \left(1 - \frac{p}{p+1}\right) U_r q^p} \right]^{\frac{p}{p+1}} \frac{U_r}{(Z_r B)^p} \left[\frac{\Gamma\left(\frac{p+1}{s}\right)}{\Gamma\left(\frac{1}{s}\right)} \right]$$

where, σ_v is the standard deviation of the wind speed in the lateral direction.

By Integrating the equation (13) with respect to x , we obtain the plume dispersion parameter in the lateral direction (σ_y) as follows:

$$\sigma_y(x) = \frac{\sigma_v x (Z_r B)^p}{\left[\frac{h^2 k w_* Z_r^p x}{q^2 \left(1 - \frac{p}{p+1}\right) U_r q^p} \right]^{\frac{p}{p+1}} U_r \left[\frac{\Gamma\left(\frac{p+1}{s}\right)}{\Gamma\left(\frac{1}{s}\right)} \right]} \quad (14)$$

Also estimating the vertical spread σ_z using Eckman's (1994) hypothesis that :

$$\frac{d\sigma_z}{dx} = \frac{\sigma_w}{\left[\frac{h^2 k w_* Z_r^p x}{q^2 \left(1 - \frac{p}{p+1}\right) U_r q^p} \right]^{\frac{p}{p+1}} \frac{U_r}{(Z_r B)^p} \left[\frac{\Gamma\left(\frac{p+1}{s}\right)}{\Gamma\left(\frac{1}{s}\right)} \right]} \quad (15)$$

where, σ_w is the standard deviation of the wind speed in the vertical direction. Integrating equation (15) with respect to x , we obtain the plume dispersion parameter in the vertical direction (σ_z) as follows:

$$\sigma_z(x) = \frac{\sigma_w x (Z_r B)^p}{\left[\frac{h^2 k w_* Z_r^p x}{q^2 \left(1 - \frac{p}{p+1}\right) U_r q^p} \right]^{\frac{p}{p+1}} U_r \left[\frac{\Gamma\left(\frac{p+1}{s}\right)}{\Gamma\left(\frac{1}{s}\right)} \right]} \quad (16)$$

Then Gaussian expressions for the ground crosswind-integrated concentration and the normalized ground-level concentration along the plume centerline respectively are given by (Arya, 1999) on the forms:

$$\frac{C_y(x,0)}{Q} = \sqrt{\frac{2}{\pi}} \frac{1}{U \sigma_z} \exp\left(\frac{-h^2}{2\sigma_z^2}\right) \quad (17)$$

$$\frac{C_y(x,0,0)}{Q} = \frac{1}{\pi U \sigma_z \sigma_y} \exp\left(\frac{-h^2}{2\sigma_z^2}\right) \quad (18)$$

From the previous works, the plume dispersion parameters in the vertical and lateral directions (σ_z and σ_y) respectively are given by Lidiane *et al.* (2008) in the form:

$$\frac{\sigma_z^2}{z_i^2} = \frac{0.42\psi^{2/3}x^2}{1 + (2.9\psi^{1/3}x)} \quad (19)$$

and

$$\frac{\sigma_y^2}{z_i^2} = \frac{0.55\psi^{2/3}x^2}{1 + (2.24\psi^{1/3}x)} \quad (20)$$

TABLE 1

Observed and model ground-level centerline concentration $C(x, 0, 0)/Q$ at different distances, wind speed and effective height from the source

Run no.	h(m)	U_{115}	distance (x) (m)	W_*	$C(x,0,0)/Q$ (s/m ²) Observed	$C(x,0,0)/Q$ (s/m ²)		
						Our model Equations (14), (16), (18)	Previous work equations (19), (20), (18)	Previous work equations (21), (22), (18)
1	119	3	1900	1.8	10.5	8.61	5.34	6.37
1	119	3	3700	1.8	2.14	7.42	2.17	2.55
2	117	8	2100	1.8	9.85	5.55	7.67	8.71
2	117	8	4200	1.8	2.83	1.80	2.93	3.48
3	118	4	1900	1.3	16.33	14.56	13.74	15.87
3	118	4	3700	1.3	7.95	5.77	5.95	6.98
3	118	4	5400	1.3	3.76	1.09	3.72	4.32
5	117	5	2100	0.7	15.71	15.49	17.51	19.36
5	117	5	4200	0.7	12.11	10.49	20.94	21.73
5	117	5	6100	0.7	7.24	4.94	11.49	13.14
6	116	11	2000	2	4.75	5.43	7.52	8.69
6	116	11	4200	2	7.44	2.94	8.02	8.91
6	116	11	5900	2	3.37	8.29	3.24	3.8
7	117	7	2000	2.2	1.74	2.74	2.07	2.44
7	117	7	4100	2.2	9.48	9.78	5.55	6.54
7	117	7	5300	2.2	2.62	4.12	2.03	2.41
8	117	7	1900	2.2	1.15	1.74	1.44	1.7
8	117	7	3600	2.2	9.76	3.22	8.43	9.62
8	117	7	5300	2.2	2.64	1.96	4.06	4.69
9	116	8	2100	1.9	0.98	3.34	2.59	2.96
9	116	8	4200	1.9	8.52	1.52	6.86	7.85
9	116	8	6000	1.9	2.66	6.75	2.55	3.04

Also, the plume dispersion parameters in the vertical and lateral directions (σ_z and σ_y) respectively are given by Pasquill and Smith (1983) as follows:

$$\frac{\sigma_z^2}{z_i^2} = \frac{0.29}{\pi^2} \int_0^\infty \frac{\sin^2(0.98 \pi \Psi^{1/3} x n')}{n'^2 (1+n')^{5/3}} dn' \quad (21)$$

$$\frac{\sigma_y^2}{z_i^2} = \frac{0.66}{\pi^2} \int_0^\infty \frac{\sin^2(0.75 \pi \Psi^{1/3} x n')}{n'^2 (1+n')^{5/3}} dn' \quad (22)$$

3. Results and discussion

The data set used was observed from the atmospheric diffusion experiments conducted at the northern part of Copenhagen, Denmark, under unstable conditions

(Gryning and Lyck, 1984; Gryning *et al.*, 1987). The tracer sulfur hexafluoride (SF₆) was released from a tower at a height of 115 m without buoyancy. There are two Gaussian models, The First is measured at ground surface and the other at the plume centerline. In this work, there are three predicated normalized concentrations (our model and two previous models) as shown in Tables (1&2).

Fig. (1&2) show that the observed and predicated scatter diagram of crosswind integrated concentrations of centerline and ground level respectively using Gaussian model with vertical and lateral dispersion parameters given by [Equations (14) and (16), our model] and [Equations (19), (20), algebraic formulation], [Equations (21), (22), integral formulation] respectively. From the two figures one finds that there are some predicated data which are in agreement with observed data (one to one) and others lie inside the factor of two.

TABLE 2

Observed and model ground-level concentration $C_y(x, 0)/Q$ at different distances, wind speed and effective height from the source

Run no.	h (m)	U_{115}	distance (x) (m)	W_*	$C(x, 0)/Q$ (s/m ²) Observed	$C(x, 0)/Q$ (s/m ²)		
						Our model Equations (16), (17)	Previous work Equations (19), (17)	Previous work Equations (21), (17)
1	119	3	1900	1.8	6.48	3.72	6.06	6.58
1	119	3	3700	1.8	2.31	4.61	3.96	4.28
2	117	8	2100	1.8	5.38	4.66	3.64	3.79
2	117	8	4200	1.8	2.95	2.46	2.48	2.68
3	118	4	1900	1.3	8.2	7.92	7.35	7.72
3	118	4	3700	1.3	6.22	1.23	5.22	5.6
3	118	4	5400	1.3	4.3	2.21	4.22	4.52
5	117	5	2100	0.7	6.72	8.24	8.54	8.77
5	117	5	4200	0.7	5.84	2.63	6.04	5.71
5	117	5	6100	0.7	4.97	4.65	5.73	5.96
6	116	11	2000	2	3.96	3.30	4.9	5.19
6	116	11	4200	2	2.22	4.41	3.14	3.18
6	116	11	5900	2	1.83	1.06	2.31	2.47
7	117	7	2000	2.2	6.7	4.63	1.9	2.04
7	117	7	4100	2.2	3.25	1.09	3.69	4.25
7	117	7	5300	2.2	2.23	1.48	2.14	2.73
8	117	7	1900	2.2	4.16	5.16	4.12	2.31
8	117	7	3600	2.2	2.02	2.91	3.12	4.28
8	117	7	5300	2.2	1.52	2.50	2.56	3.31
9	116	8	2100	1.9	4.58	3.04	3.53	2.71
9	116	8	4200	1.9	3.11	2.19	2.34	3.7
9	116	8	6000	1.9	2.59	4.04	1.85	2.54

4. Statistical method

Now, the statistical method is presented and comparison among analytical, statistically and observed results will be presented (Hanna, 1989). The following standard statistical performance measures that characterize the agreement between prediction ($C_P = C_{pred}/Q$) and observations ($C_o = C_{obs}/Q$):

$$\text{Normalized mean square error (NMSE)} = \frac{\overline{(C_P - C_o)^2}}{\overline{(C_P C_o)}}$$

$$\text{Fractional Bias (FB)} = \frac{(\overline{C_o} - \overline{C_P})}{0.5(\overline{C_o} + \overline{C_P})}$$

$$\text{Correlation coefficient (COR)} = \frac{1}{N_m} \sum_{i=1}^{N_m} (C_{Pi} - \overline{C_P}) \times \frac{(\overline{C_{oi}} - \overline{C_o})}{(\sigma_P \sigma_o)}$$

$$\text{Factor of two (FAC2)} = 0.5 \leq \frac{C_P}{C_o} \leq 2.0$$

where, σ_P and σ_o are the standard deviations of C_P and C_o respectively. Here the over bars indicate the average over all measurements (N_m). A perfect model would have the following idealized performance:

$$\text{NMSE} = \text{FB} = 0 \text{ and } \text{COR} = \text{FAC2} = 1.0$$

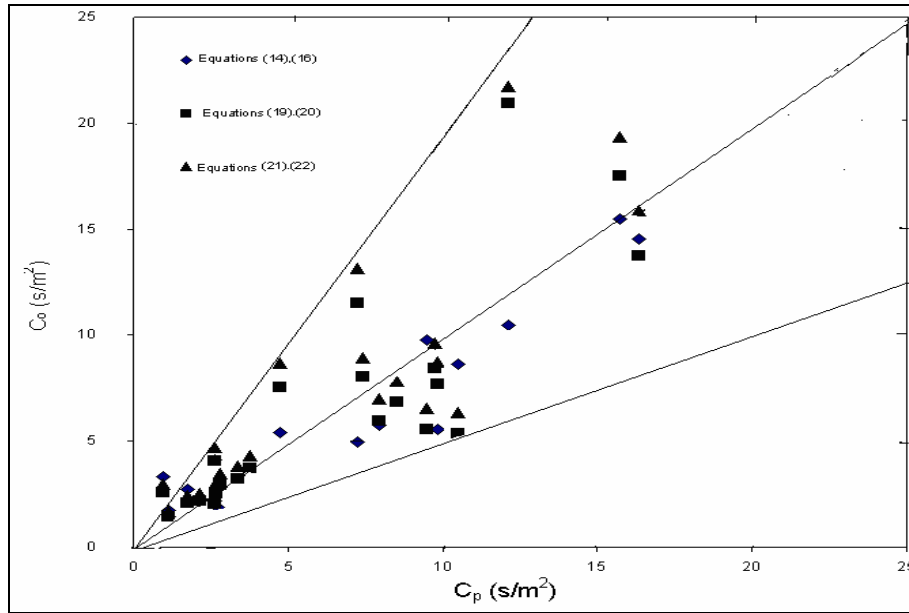


Fig. 1. Observed and predicted ground crosswind integrated centerline concentration, normalized with emission $C_y(x, 0, 0)/Q$: scatter diagram for the solution of equation (18) using equations (14), (16), (19), (20), (21) and (22)

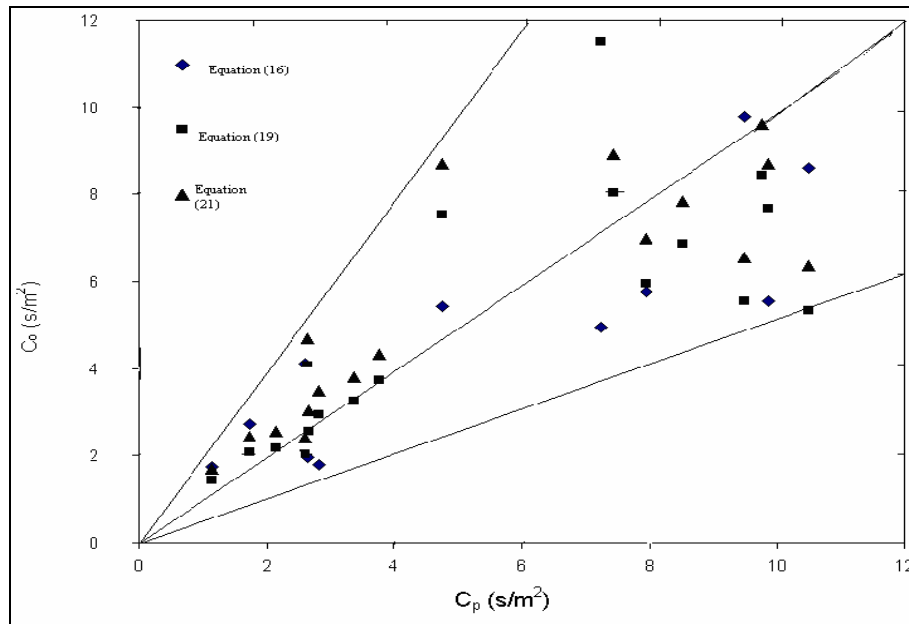


Fig. 2. Observed and predicted ground crosswind integrated concentration, normalized with emission $C_y(x, 0)/Q$: scatter diagram for the solution of equation (17) using equations (16), (19) and (21)

From the statistical method of Table (3), we find that the predicted concentrations for all models lie inside factor of 2 with observed data. Regarding NMSE, we find

that two previous works are better than our model. Regarding FB and correlation coefficient all models are in agreement with observed data.

TABLE 3

Comparison between different models ground – level centreline concentration $C(x, 0, 0)/Q$ and observed concentrations

Predicted models $C(x, 0, 0)/Q$	NMSE	FB	COR	FAC2
Our model Equations (14), (16), (18)	0.28	0.12	0.72	1.22
Previous work Equations (19), (20), (18)	0.18	-0.02	0.83	1.12
Previous work Equations (21), (22), (18)	0.18	-0.14	0.86	1.28

TABLE 4

Comparison between different models ground – level concentration $C(x, 0)/Q$ and observed concentrations

Predicted models $C_y(x, 0)/Q$	NMSE	FB	COR	FAC2
Our model Equations (16), (17)	0.13	0.07	0.90	1.09
Previous work Equations (19), (17)	0.11	0.03	0.72	1.05
Previous work Equations (21), (17)	0.14	-0.03	0.64	1.15

In Table 4, we find that the predicted concentrations by all models lie within a factor of 2 with observed data. Regarding NMSE, all the predicted concentrations are better than the observed data. Regarding FB and correlation coefficient all methods are in agreement with observed data.

5. Conclusion

We estimated new schemes of dispersion parameters in the lateral direction (σ_y) and the vertical direction (σ_z) in unstable condition by using power law wind speed and calculating Gaussian plume model at ground and at plume centerline.

We used observed data of the tracer sulfur hexafluoride (SF_6) which was released from a tower at a height of 115m without buoyancy at the northern part of Copenhagen, Denmark, under unstable conditions (Gryning and Lyck, 1984; Gryning *et al.*, 1987). There are two Gaussian models, The First is measured at ground surface and the other at the plume centerline. In this work, there are three predicted normalized concentrations (our model and two previous models).

From the two Fig. (1&2) one finds that there are some predicted data which are in agreement with observed data (one to one) and others lie within a factor of two.

From the statistical method, we find that the predicted concentrations for all models lie within a factor of 2 with observed data. Regarding NMSE, all the predicted concentrations are in agreement with the

observed data. Regarding FB and correlation coefficient of all methods agree with observed data.

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